1).

a. $\begin{bmatrix} A_{11} & A_{12} & t_1 \\ A_{21} & A_{22} & t_2 \\ 0 & 0 & 1 \end{bmatrix}$ If the points are parallel at infinity, the transformation is too.

b. 3, 2.

c. Affine transformations will preserve midpoints for lines, and since the centroid is defined by midpoints to vertices for the 3 sides, the centroid is affine invariant. The circumcenter relies on distance from vertex to the center, and so a scaling transformation may change it. The circumcenter isn't affine invariant

2).

a. Light can only travel through the center of projection, and so any point that isn't at the same level will travel at an angle and end up on the opposite side.

b. $$\begin{bmatrix} u_x & u_y & u_z & 0 \\ (C_x-P_x)/\|C_x-P_x\| \times u_x & (C_y-P_y)/\|C_y-P_y\| \times u_y & (C_z-P_z)/\|C_z-P_z\| \times u_z & 0 \\ (C_x-P_x)/\|C_x-P_x\| & (C_y-P_y)/\|C_y-P_y\| & (C_z-P_z)/\|C_z-P_z\| & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

c. As long as they are parallel to the image plane.

d. They will converge at the vanishing point.

3).

a. $f'(x,y,z) = \left(\frac{-2xR}{\sqrt{x^2+y^2}} + 2x, \frac{-2yR}{\sqrt{x^2+y^2}} + 2y, 2z\right)$.

b. $f'(x,y,z) \cdot (x-x_0, y-y_0, z-z_0)$

$= \left(2x - \frac{2xR}{\sqrt{x^2+y^2}}, 2y - \frac{2yR}{\sqrt{x^2+y^2}}, 2z\right) \cdot (x-x_0, y-y_0, z-z_0)$.

c. $(R - \sqrt{R\cos\lambda^2 + R\sin\lambda^2})^2 + r^2 - r^2$

$= (R - \sqrt{R^2\cos^2\lambda + R^2\sin^2\lambda})^2 = (R - \sqrt{R^2(\cos^2\lambda + \sin^2\lambda)})^2$

$= (R-R)^2 = 0$.

d. $(R\cos\lambda', R\sin\lambda', r') = (-R\sin\lambda, R\cos\lambda, 0)$.

4).

a. $B_1' = 3(P_4 - P_3)$

$B_2' = 3(P_5 - P_4)$.

b. $B_1'' = 3(1-1) = 0$

$B_2'' = 3(1-1) = 0$.

c. Any arbitrary value will do, since $B_1'' = B_2'' = 0$

d. Can express continuous paths

Can express any shape

Easy to compute

Relaetively simple for higher dimensions.