Question 1.

a). $x = 2\sin(t)$ $\quad y = 5\sin(t)\cos(t)$

$y = \frac{5}{2} x\cos(t)$

$y^2 = \frac{25}{4}x^2\cos^2(t)$

$= \frac{25}{4}x^2(1-\sin^2(t))$

$= \frac{25x^2}{4}(1-\frac{x^2}{4})$

$= \frac{25x^2}{4} - \frac{25x^4}{16}.$

$y^2 - \frac{25x^2}{4} + \frac{25x^4}{16} = 0.$

b). tangent:

$x'(t) = 2\cos(t)$

$y'(t) = 5\sin(t)(-\sin(t)) + 5\cos(t)\cos(t)$

$= -5\sin^2(t) + 5\cos^2(t)$

$[x'(t), y'(t)] = [2\cos(t), 5\cos^2(t) - 5\sin^2(t)]$

normal:

~~$[x'(t),$~~ $[5\cos^2(t) - 5\sin^2(t), -2\cos(t)].$

c). Since the formula is $y^2 - \frac{25x^2}{4} + \frac{25x^4}{16}$, with all exponents being even, replacing x with $-x$ and y with $-y$ gives the same result in all cases, as there is no translation. The curve is symmetric around the x and y axis.

d). $y = (\frac{25}{4}x^2 - \frac{25}{16}x^4)^{1/2}$

$= 5x(\frac{1}{4} - \frac{1}{16}x^2)^{1/2}$ $\qquad 10/3 \times 4 = 40/3$

$\int_0^2 (\frac{25}{4}x^2 - \frac{25}{16}x^4)^{1/2}$

$= \left. \frac{5(-x^2(x^2-4))^{3/2}}{12x^3} \right|_0^2$

$= 16/3$

e). $\int (x'^2 + y'^2)^{1/2} dt = \int_0^{2\pi} (4\cos^2(t) + 25\cos^4(t) - 50\cos^2(t)\sin^2(t) + 25\sin^4(t))^{1/2}$

Question 2.

a). Commute

$$\begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx_2 + tx_1 \\ 0 & 1 & ty_2 + ty_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx_2 + tx_1 \\ 0 & 1 & ty_2 + ty_1 \\ 0 & 0 & 1 \end{bmatrix}$$

b). Don't Commute

$$\begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos t & -\sin t & tx \\ \sin t & \cos t & ty \\ 0 & 0 & 1 \end{bmatrix}$$

c). Don't Commute

$$\begin{bmatrix} Sx & 0 & 1-Sx \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx\cos t & -Sx\sin t & 1-Sx \\ Sy\sin t & Sy\cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Sx & 0 & 1-Sx \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx\cos t & -Sy\sin t & \cos t - Sx\cos t \\ Sx\sin t & Sy\cos t & \sin t - Sx\sin t \\ 0 & 0 & 1 \end{bmatrix}$$

d). Commute

$$\begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx_1Sx_2 & 0 & 0 \\ 0 & Sy_1Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx_2Sx_1 & 0 & 0 \\ 0 & Sy_2Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Question 3.

a). $\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Solving for T gives us:

$$T = \begin{bmatrix} -1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

b). (5, 7).

Question 4.

a) Take the cross product of $[v_1 - V_0]$ and $[q - V_0]$ (edge1)
Repeat with $[V_2 - V_0]$, $[q - v_1]$ and $[V_0 - V_2]$, $[q - V_2]$
Take the dot product of every combination of edges, if all dot products ≥ 0, point is inside triangle.

b) If one of the dot products is $= 0$ for an edge, point is on edge.

c). The centroid is the average of all coordinates of vertices.
The area can be found by finding the lengths of each side $(\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2})$, then dividing the sum in half.
area = $\sqrt{\text{halfsum}(\text{halfsum} - \text{side1})(\text{halfsum} - \text{side2})(\text{halfsum} - \text{side3})}$