Forecasting the monthly supply of new houses in the U.S.

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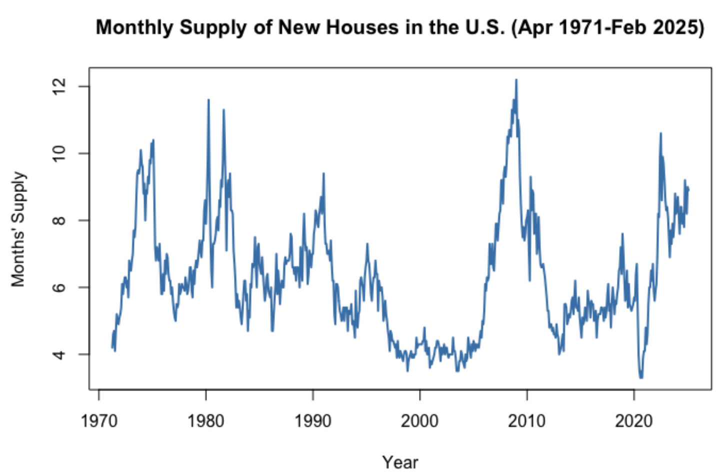
1 Introduction

The goal of this project is to forecast the monthly supply of new houses in the U.S. for the next 12 periods. We will explore AR, ARMA, ADL, and GARCH models, and use the best-performing model to generate the 12-month forecast.

The months' supply is the ratio of new houses for sale to new houses sold. This statistic provides an indication of the size of the new for-sale inventory in relation to the number of new houses currently being sold. The months' supply indicates how long the current new for-sale inventory would last given the current sales rate if no additional new houses were built.

Supply of New Houses in the U.S.

We downloaded the series from FRED, which originally covers data from January 1963 to February 2025. However, we had to drop the first 99 observations, since one of the X variables we are using (30-year mortgage rates) starts from April 1971. We are using the seasonally adjusted version of the series.



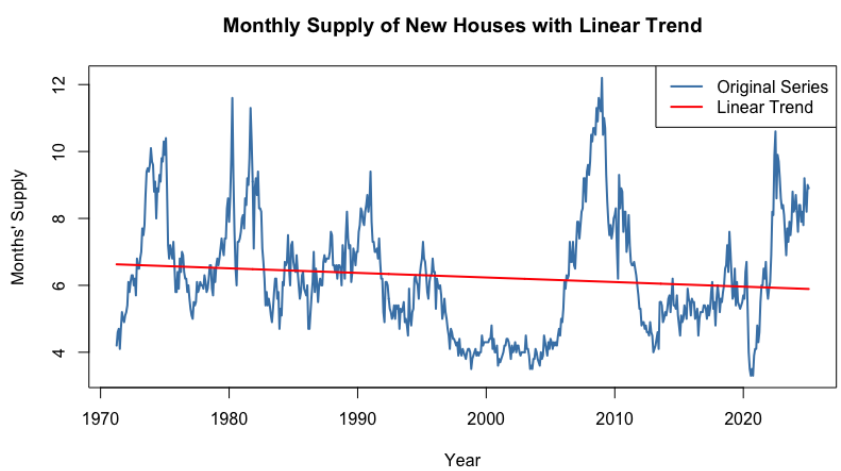
2 Modeling

Now, let us move to the modeling stage of the project.

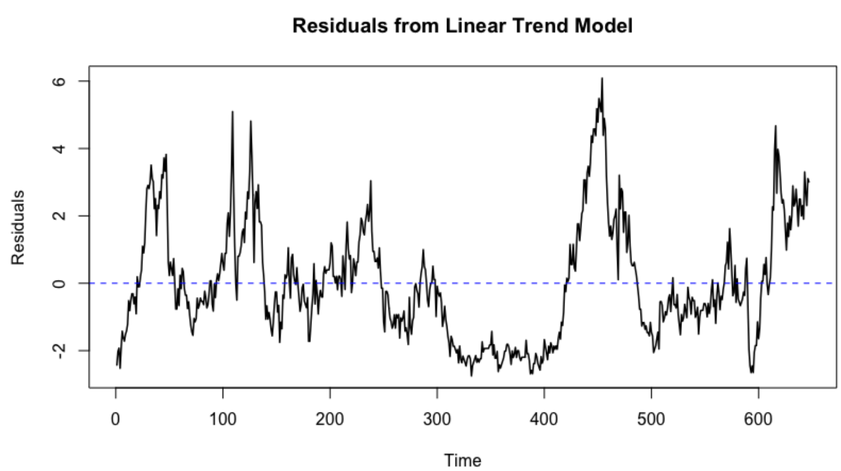
2.1 Trend

We begin by assessing whether the data exhibits a deterministic trend. Although the series does not really appear to be trending visually, we confirm this by fitting a linear model of the form y on constant and t.

The regression results indicate that the intercept and the time variable t are statistically significant (Fig. 3 in the Appendix). Although, in practical terms, the coefficient on t is quite small (-0.0011). A plot of the fitted linear trend shows that it is a slightly downward-sloping line.



To further investigate, we examine the residuals from the fitted trend model. These residuals resemble the original data, indicating that the trend component did not really capture much of the pattern in our series.

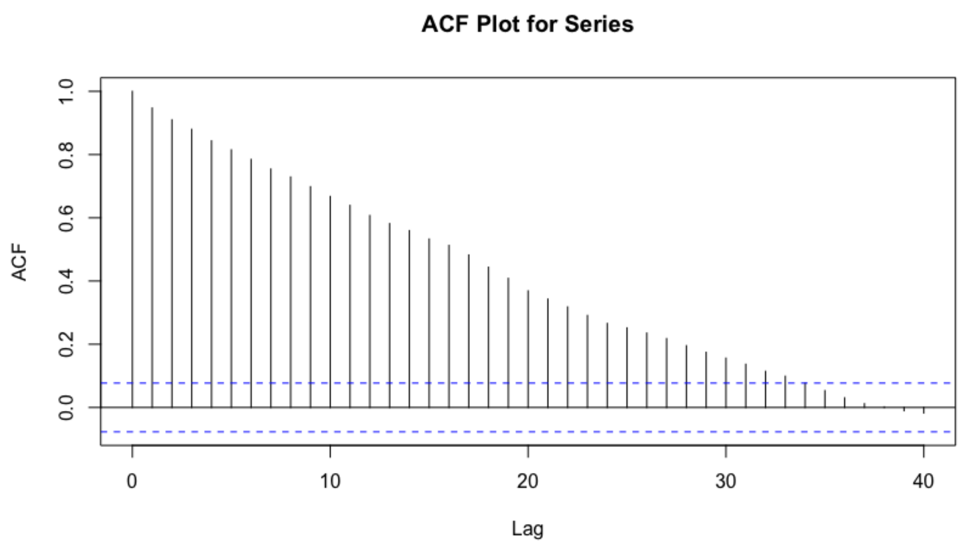


Taking everything into account, we decided not to rule out the possibility of a deterministic trend. Omitting a trend that truly exists could lead to model misspecification.

Additionally, we have not yet checked for a unit root in our series, which we will test for using the Augmented Dickey-Fuller (ADF) test later.

2.2 Stationarity

Next, we assess autocorrelation in the series using an ACF plot. The ACF plot reveals significant and persistent autocorrelation up to approximately the 33rd lag. This confirms that the series is not stationary, which is important to consider in subsequent hypothesis testing. Moreover, the strong autocorrelation suggests that we may need to fit an AR or ARMA model to appropriately capture the temporal dynamics in the data.



To formally test for stationarity, we conduct an ADF test (Fig. 4 in the Appendix). The results illustrate that the time series is non-stationary. Specifically, the p-values for the test with type = ”no drift, no trend” are large across all lag lengths, meaning we cannot reject the null hypothesis of non-stationarity. Interestingly, for types ”with drift, no trend” and ”with drift and trend”, the p-values become statistically significant, implying that the series becomes stationary once a drift and/or a trend term are included.

Based on the results above, we decided to difference our data to make it stationary. As we will demonstrate below, the independent variables we plan to include in our model are also non-stationary. To avoid the risk of spurious regression, we chose to transform all variables to be stationary before modeling. This is also supported by the fact that when we were experimenting with different models, the one estimated on our data in levels resulted in a very high R2 (around 92%) - even after including a trend variable and lags of Y - which we assume might have been due to a spurious regression. Hence, to be on the safe side, we decided to continue working with stationary data only.

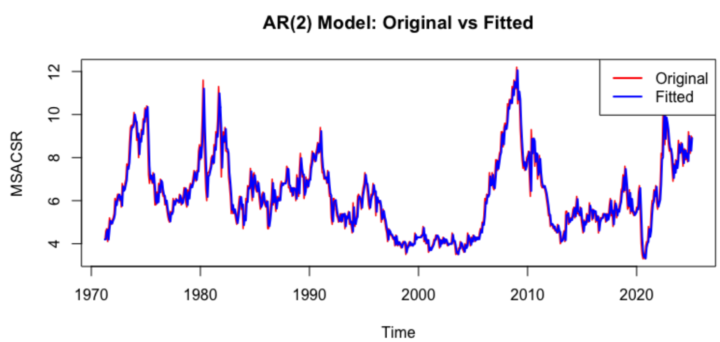
2.3 Seasonality

The second component we need to consider is seasonality. However, we do not actually model it in this project, as the data has already been seasonally adjusted.

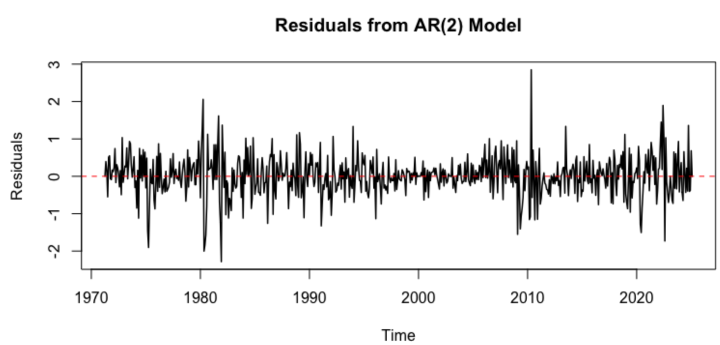
2.4 Cyclicality

The third component is cyclicality. Visually, it appears that the data may exhibit cyclical behavior. To explore this, we begin by fitting a basic AR model as our baseline. We determine the optimal lag length, p, by calculating AIC values for AR models with up to 12 lags of the differenced dependent variable. From the Fix. 5 in the Appendix, we observe that the lowest AIC corresponds to an AR(2) model, with a value of 1053.

A plot of the fitted values shows that this simple AR(2) model already does pretty well with capturing the patterns in our data.

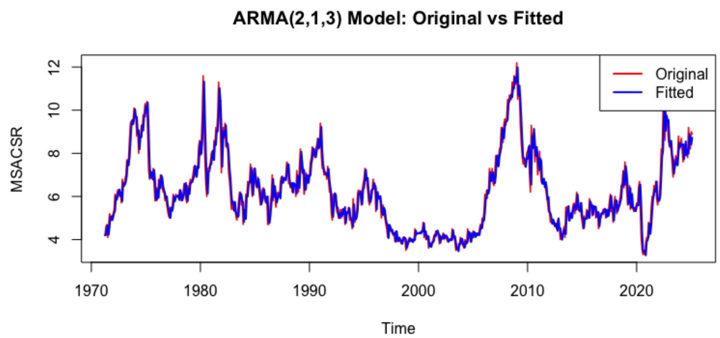


The residuals plot for AR(2) looks a bit more like white noise, compared to the residuals after fitting a linear trend.

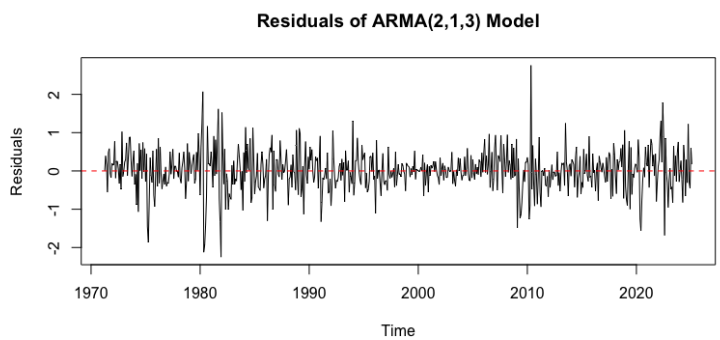


Next, we fit an ARMA model to potentially capture the cyclical component more precisely. Using the auto.arima() function in R with d = 1, we allow the function to difference the data to ensure stationarity. The model selection is based on AIC, and auto.arima() identifies an ARIMA(2,1,3) specification as the best fit (Fig. 6 in the Appendix). This model gives as an AIC of 1048.28, which is a pretty small improvement from AR(2).

The fitted values from the ARMA(2,1,3) model show that it captures the data patterns as well as AR(2), but not significantly better.



The residuals plot indicates that the ARMA(2,1,3) model extracts a substantial portion of the signal from the series, as the residuals resemble white noise. However, we still observe notable spikes at specific time points (like during Covid-19) as well as some volatility clustering.



To further improve our model, we now consider incorporating independent variables that have a sound economic rationale for influencing our target variable. While ARIMA demonstrated reasonable performance, we now explore ADL models to assess whether they can offer improved performance.

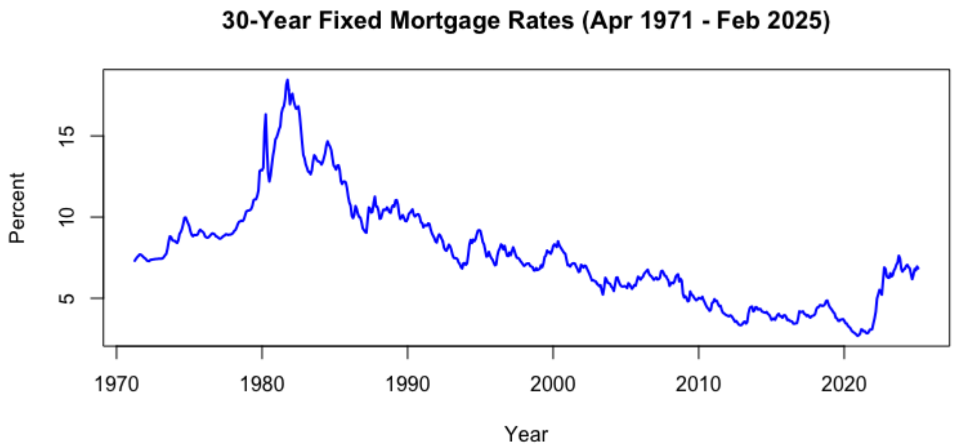
2.5 Independent Variables

Below is a list of potential predictors, each with a bit more detail on how they might drive the months' supply and what the data looks like at a glance.

1. **Mortgage Rate**

   Borrowing costs directly affect buyer affordability. When rates rise, monthly payments get higher, which can push some buyers out of the market, which usually means more new homes sit unsold, so the months' supply goes up. But when rates drop, more people can afford to buy, and homes get snapped up faster.

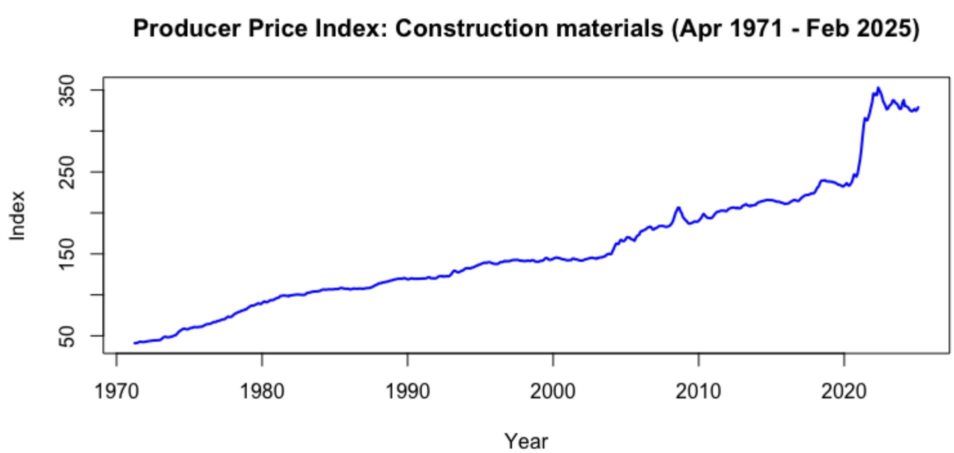
Visually, the mortgage rate series shows a clear downward trend since the 1980s. There's no obvious seasonal pattern, but it does seem like the data has some cyclicality to it.



2. **Producer Price Index for Construction**

   Construction input costs influence builders' willingness to start or finish projects. When material prices jump, margins compress, so builders may pause projects, meaning fewer new homes get added to the supply.

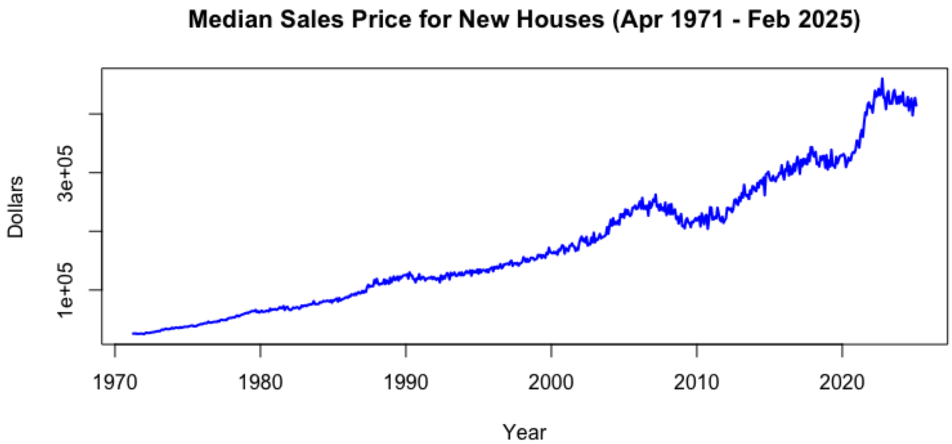
   The PPI series clearly trends upward. There's no regular seasonal pattern or cyclical behaviour.



3. **Median Sales Price of New Homes**

   Sales price reflects market valuation and affordability. If prices shoot up too fast, some buyers get priced out, sales slow down, and more homes stay on the market. But if prices grow steadily or stay stable, it helps keep demand strong.

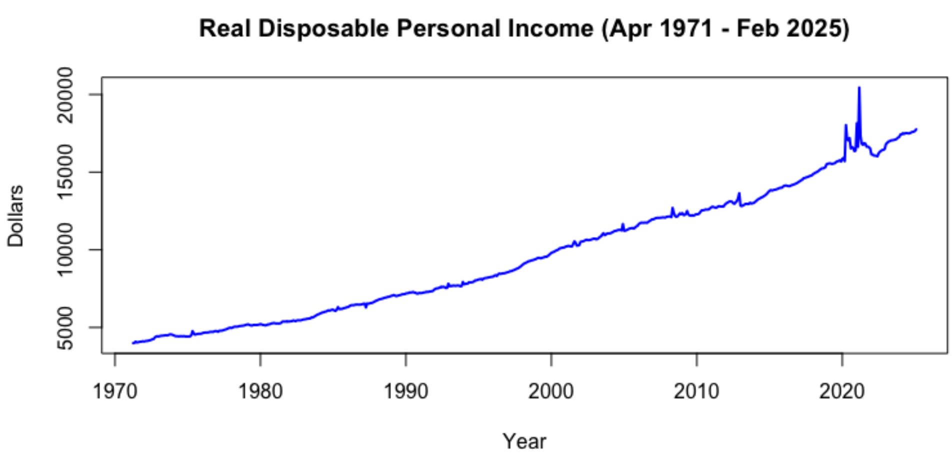
The median price trend is strongly upward over 30 years, with occasional cooling in downturns. Visually, it looks like there might be some seasonality as well as cyclicality to this data.



4. **Real Disposable Personal Income**

   Household income sets buying power. When real disposable income rises, more households can afford new homes, helping absorb supply; stagnant or falling income can leave inventory unsold.

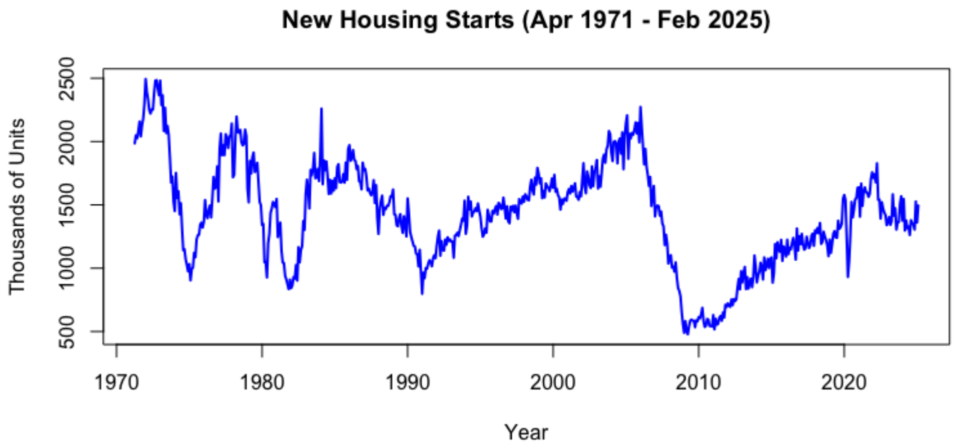
   The income series shows a long-term upward trend, and no clear signs of seasonality or cyclicality.



5. **Housing units started**

   Housing starts represent the beginning of construction on new residential buildings and serve as a leading indicator of future housing supply. An increase in housing starts typically signals that more new homes will enter the market in the coming months, which can raise the monthly supply of new houses.

This time series shows clear signs of cyclicality and there's an indication of some seasonality as well. However, it is difficult to say anything about trend - visually, it looks pretty flat.



Now, we proceed to selecting variables that Granger cause the monthly supply of new houses in the U.S.

2.6 Granger causality

Since our variables are non-stationary, we begin by differencing them to achieve stationarity and conduct Wald tests for Granger causality (Fig. 7-9 in the Appendix). The results indicate that for mortgage rates, PPI, and real disposable personal income, the p-values are tiny, hence we reject the null hypothesis, suggesting that these variables Granger-cause the monthly supply of new houses in the U.S. Therefore, we proceed with these three predictors in modeling our ADL.

2.7 Cointegration

Since we decided to work with differenced data, we do not test for cointegration.

2.8 Lag selection for ADL

Next, we determine the appropriate number of lags for Y and each X variable in the ADL model. To do this, we employ a grid search approach that selects the model with the lowest AIC among all combinations of lags (up to a maximum of 12). The optimal model, with an AIC of 970.79, includes:

1. 4 lags of monthly housing supply (Y)

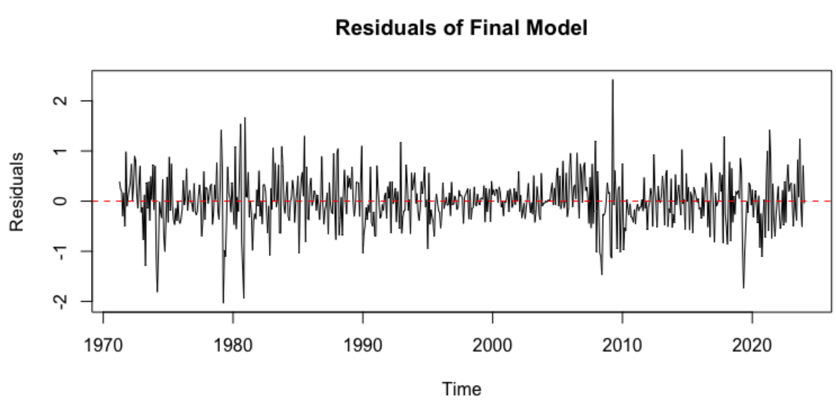
2. 12 lags of mortgage rates

3. 8 lags of PPI

4. 0 lag of real disposable income - meaning, the model does not benefit from this variable at all, so we drop it.

The regression output shows that each of our predictor variables has at least one statistically significant lag (Fig. 10 in the Appendix). Our target variable has statistically significant lags at periods 1, 2, and 4; mortgage rates show significance at lags 1, 3, and 12; and PPI exhibits significance at lags 5 and 7. Given this, we chose to keep all lags in the model, including those that are not individually significant, in order to preserve the full dynamic structure. Since we are working with differenced series, interpreting individual coefficients becomes less straightforward. Nevertheless, it is reassuring to see a reasonably adequate R2 - while the model explains only about 16% of the variation in the differenced target variable, it provides a more reliable specification. Most importantly, it helps us avoid the risk of spurious regression, which we suspect may have occurred when experimenting with an ADL model using the data in levels (since it gave us an R2 of around 92%).

The residuals plot of the final model reveals periods of both higher and lower variance, suggesting potential volatility clustering. To account for this, we think it is be a good idea to model the residuals using GARCH, but first let's forecast our target variable using our baseline ADL model.

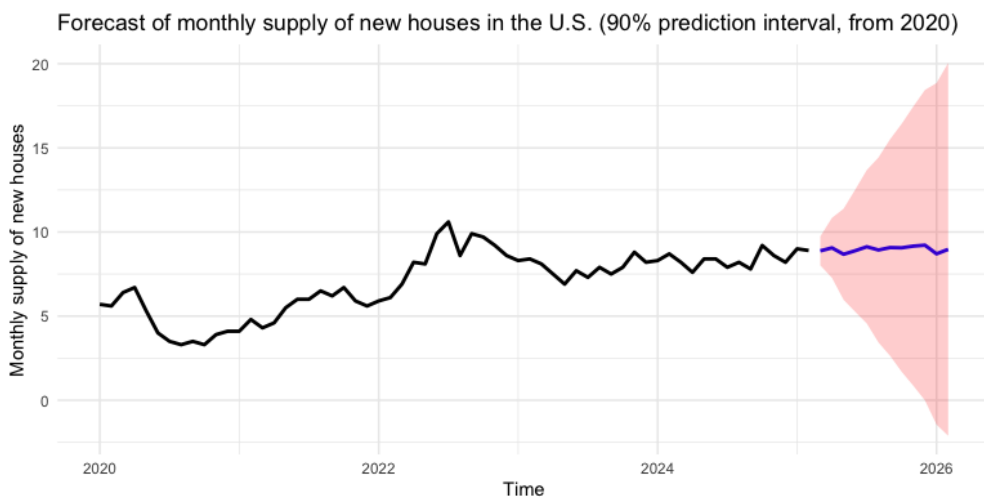


2.9 Forecast using ADL

For the forecasting stage, we estimate our final model and generate forecasts for the next 12 periods (h = 12) using the direct forecasting approach. The results are presented below:

	horizon	level_forecast	level_lower_90	level_upper_90
1	1	8.877073	8.01545120	9.738695
2	2	9.057548	7.27671196	10.838384
3	3	8.675631	5.97224506	11.379016
4	4	8.886191	5.25896569	12.513416
5	5	9.122211	4.56228128	13.682141
6	6	8.930458	3.43678518	14.424131
7	7	9.074354	2.64304764	15.505660
8	8	9.059484	1.69010796	16.428860
9	9	9.161023	0.86524507	17.456801
10	10	9.215764	-0.00710543	18.438633
11	11	8.701109	-1.44968952	18.851908
12	12	8.967079	-2.11004841	20.044206

Figure 1: Forecasted values in levels (ADL model)

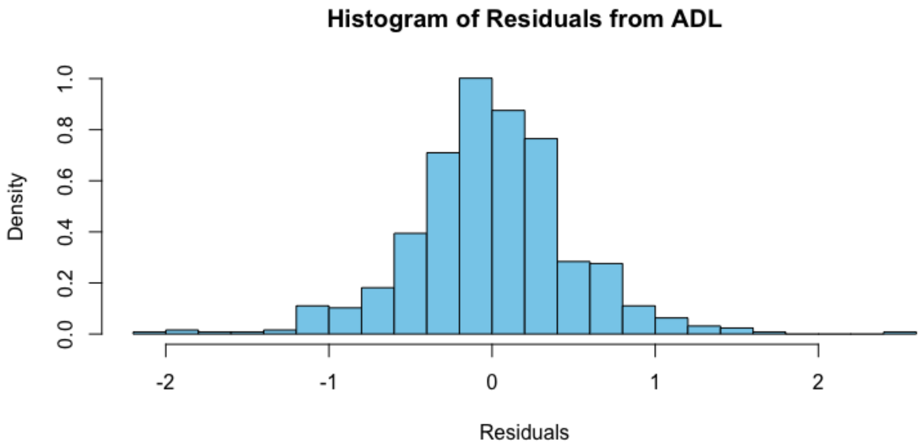


The main issue with these results is that, to revert forecasts from differences back to levels, we must cumulatively sum the predicted changes across periods. This process compounds uncertainty over time, as each forecast error adds to the previous one. As a result, the prediction intervals in levels become increasingly wide and less reliable the further out we forecast.

To address this issue and to capture the presence of volatility clustering in our data, we estimate a GARCH model using the residuals from our ADL model.

2.10 GARCH

First, we examine the distribution of residuals from the ADL model. The histogram reveals that the residuals exhibit heavier tails than those of a normal distribution. To formally test for normality, we apply the Shapiro-Wilk test, which yields a very small p-value. We reject the null hypothesis of normality, confirming that the residuals are not normally distributed. This finding is important, as it justifies our choice of the Generalized Error Distribution (GED) when specifying the GARCH(1,1) model.

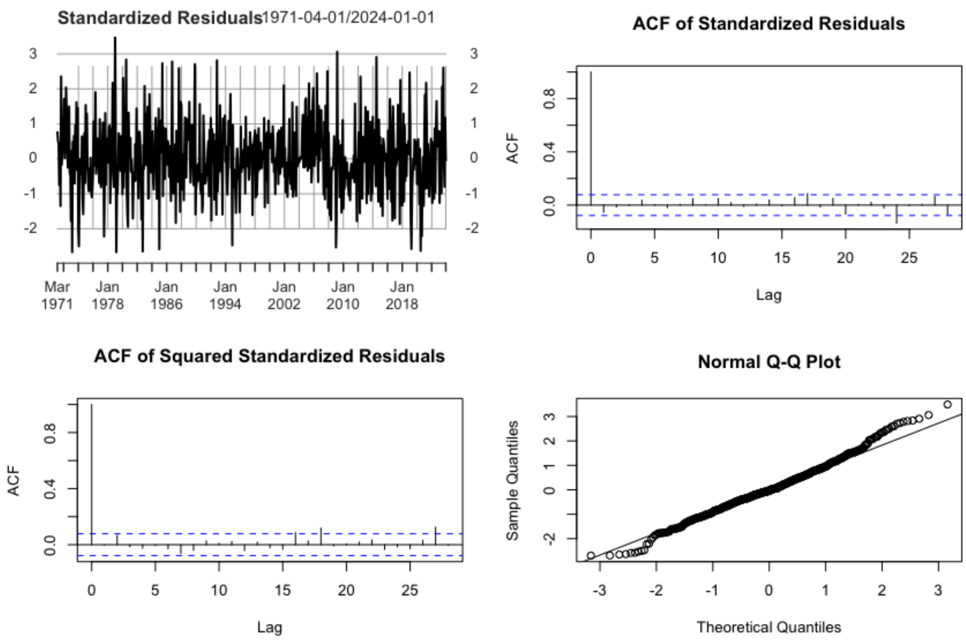


The results of our GARCH(1,1) model are shown in Fig. 11 in the Appendix. We observe strong volatility persistence, as indicated by the high sum of alpha1 and beta1, which approaches 1. The Ljung-Box tests on both residuals and squared residuals suggest no remaining autocorrelation, and the ARCH LM tests confirm that the model has effectively captured conditional heteroskedasticity.

2.11 Forecast using ADL+GARCH(1,1)

We model the mean behavior of the series using ADL and conditional variance using GARCH(1,1). Essentially, the GARCH component helps us capture time-varying volatility and generate more reliable and robust prediction intervals. We use weighted least squares (WLS) to re-estimate our original ADL coefficients, where weights are derived from the inverse of the GARCH-estimated conditional variances.

Below are the standardized residuals from the GARCH(1,1) model. Looking at these results, we can conclude that our residuals now resemble white noise even more closely, with much more reduced volatility clustering. Moreover, the absence of significant autocorrelation in both the residuals and squared residuals indicates that the GARCH model has effectively captured the conditional heteroskedasticity in the data.

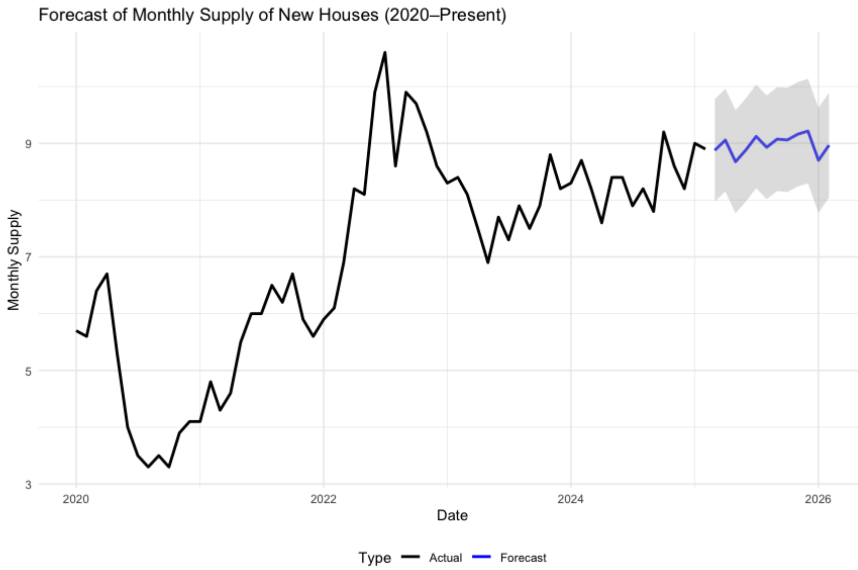


Our WLS regression results (Fig. 12 in the Appendix) reinforce the conclusions drawn from the original ADL model - particularly, that each variable included in the model has at least one statistically significant lag. This new ADL+GARCH model gives us an AIC of 850.16, which is a pretty decent improvement from the baseline ADL (AIC=970.09).

Just as with ADL, we forecast the differenced target variable and then revert it back to levels. Below are our point and interval forecasts for the next 12 periods. These results appear much more reliable compared to the baseline ADL, since prediction intervals are noticeably narrower. This improvement stems from the GARCH model's ability to explicitly model time-varying volatility, allowing us to adjust uncertainty estimates dynamically over time.

	Horizon	Forecast	Lower90	Upper90
1	1	8.877073	7.976418	9.777728
2	2	9.057548	8.154177	9.960919
3	3	8.675631	7.769709	9.581552
4	4	8.886191	7.977874	9.794508
5	5	9.122211	8.211644	10.032779
6	6	8.930458	8.017776	9.843140
7	7	9.074354	8.159684	9.989023
8	8	9.059484	8.142947	9.976021
9	9	9.161023	8.242731	10.079315
10	10	9.215764	8.295822	10.135706
11	11	8.701109	7.779615	9.622603
12	12	8.967079	8.044126	9.890031

Figure 2: Forecasted values in levels (ADL+GARCH)



2.12 Out-of-sample forecast accuracy evaluation

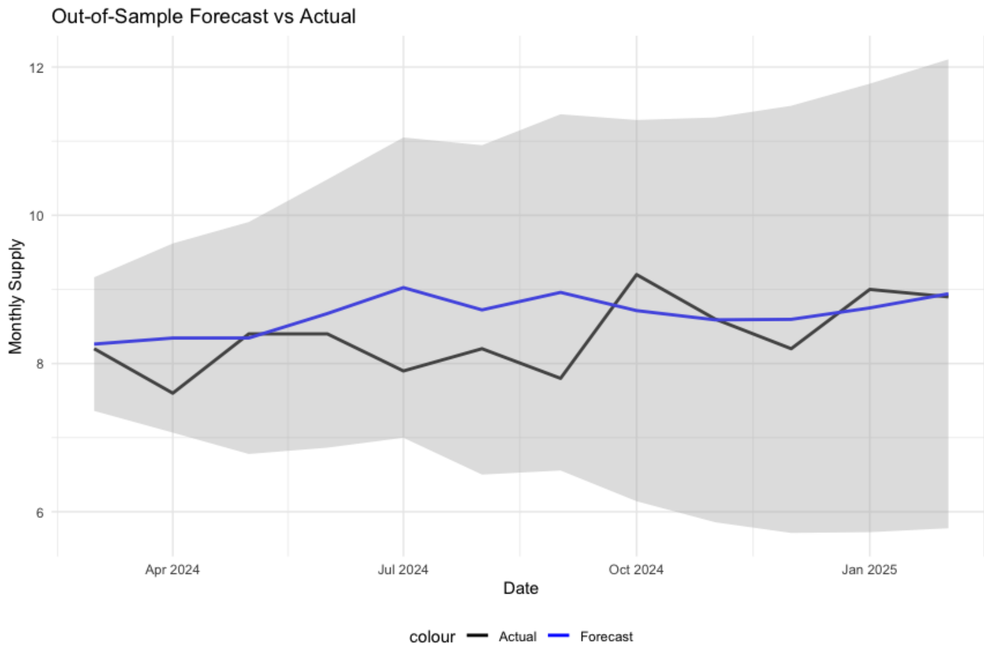
One additional step we perform is evaluate the out-of-sample performance of our final model. We split the dataset into training and test sets, holding out the last 12 months as the test period. Using only the training data, we estimate the model and generate 12-step-ahead forecasts, which we then compare against the actual values in the test set.

The table below summarizes our forecast performance. On average, our forecast errors are about 0.58 units off from the actual values. The average absolute forecast error is within half a unit off the true values.

RMSE	**MAE**	**MAPE**
0.576	0.427	5.29%

Table 1: Forecast Accuracy Metrics

Below is the plot of our OOS forecasts and actual values. The forecast line stays close to the actual values throughout the 12-month horizon. Although our forecasted values do not perfectly match the actual observations, they consistently fall within the prediction intervals, indicating reasonably decent performance. One thing to note is that the forecasted values appear smoother than the actual data, so the model slightly underreacts to short-term fluctuations.



3 Conclusion

We initially started out with a very simple autoregressive model, progressed to ARMA, followed by an ADL model, and finally integrated a GARCH component into the ADL. At each step, we saw performance improvements, starting from an AIC of 1053 in the AR(2) model and reaching 850.16 in the final ADL+GARCH specification. The out-of-sample forecast accuracy of our final model also looks quite promising, so we're excited to see how well it predicts the next period's value.

4 Appendix

```
Call:
lm(formula = series ~ t)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.7509 -1.2595 -0.3150  0.9449  6.0882 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  6.629237   0.136872  48.434  < 2e-16 ***
t           -0.001140   0.000366  -3.114  0.00193 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.739 on 645 degrees of freedom
Multiple R-squared:  0.01481,	Adjusted R-squared:  0.01328 
F-statistic: 9.697 on 1 and 645 DF,  p-value: 0.001927
```

Figure 3: Regression results of fitting a linear trend do the data

```
Type 1: no drift no trend
     lag    ADF p.value
[1,]   0 -0.744   0.413
[2,]   1 -0.552   0.481
[3,]   2 -0.454   0.513
[4,]   3 -0.465   0.510
[5,]   4 -0.392   0.531
[6,]   5 -0.448   0.515
[7,]   6 -0.434   0.519
```

```
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -4.01  0.0100
[2,]   1 -3.42  0.0110
[3,]   2 -3.14  0.0245
[4,]   3 -3.31  0.0161
[5,]   4 -3.04  0.0334
[6,]   5 -3.10  0.0282
[7,]   6 -3.10  0.0284
```

```
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -4.03  0.0100
[2,]   1 -3.43  0.0491
[3,]   2 -3.15  0.0973
[4,]   3 -3.33  0.0650
[5,]   4 -3.05  0.1335
[6,]   5 -3.10  0.1147
[7,]   6 -3.10  0.1142
```

Figure 4: ADF test results

```
    p   AIC
<int> <dbl>
    1 1056.
    2 1053.
    3 1055.
    4 1053.
    5 1055.
    6 1057.
    7 1058.
    8 1059.
    9 1061.
   10 1063.
   11 1064.
   12 1064.
```

```
Series: series 
ARIMA(2,1,0) 

Coefficients:
          ar1      ar2
      -0.1645  -0.0857
s.e.   0.0392   0.0392

sigma^2 = 0.2972:  log likelihood = -523.7
AIC=1053.4   AICc=1053.43   BIC=1066.81

Training set error measures:
                      ME      RMSE      MAE        MPE    MAPE      MASE
Training set 0.009002766 0.5438607 0.397621 -0.2523654 6.25281 0.3490315
                    ACF1
Training set 0.002226009
```

Figure 5: AIC table for AR(p) and AR(2) results

```
ARIMA(2,1,3)

Coefficients:
          ar1      ar2     ma1     ma2      ma3
      -1.3294  -0.9594  1.1748  0.7372  -0.1276
s.e.   0.1167   0.0703  0.1227  0.0644   0.0843

sigma^2 = 0.2933:  log likelihood = -518.14
AIC=1048.28   AICc=1048.41   BIC=1075.11

Training set error measures:
                      ME      RMSE       MAE        MPE     MAPE      MASE
Training set 0.008684372 0.5390902 0.3950627 -0.2373173 6.227233 0.3467858
                    ACF1
Training set 0.005899017
```

Figure 6: ARMA(2,1,3) results

```
$hous_st_diff
Linear hypothesis test

Hypothesis:
x_lag1 = 0
x_lag2 = 0
x_lag3 = 0
x_lag4 = 0
x_lag5 = 0
x_lag6 = 0
x_lag7 = 0
x_lag8 = 0
x_lag9 = 0
x_lag10 = 0
x_lag11 = 0
x_lag12 = 0

Model 1: restricted model
Model 2: y ~ y_lag1 + y_lag2 + y_lag3 + y_lag4 + y_lag5 + y_lag6 + y_lag7 +
    y_lag8 + y_lag9 + y_lag10 + y_lag11 + y_lag12 + x_lag1 +
    x_lag2 + x_lag3 + x_lag4 + x_lag5 + x_lag6 + x_lag7 + x_lag8 +
    x_lag9 + x_lag10 + x_lag11 + x_lag12

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F Pr(>F)
1    621
2    609 12 1.5522 0.1014
```

```
$mortgage_rate_diff
Linear hypothesis test

Hypothesis:
x_lag1 = 0
x_lag2 = 0
x_lag3 = 0
x_lag4 = 0
x_lag5 = 0
x_lag6 = 0
x_lag7 = 0
x_lag8 = 0
x_lag9 = 0
x_lag10 = 0
x_lag11 = 0
x_lag12 = 0

Model 1: restricted model
Model 2: y ~ y_lag1 + y_lag2 + y_lag3 + y_lag4 + y_lag5 + y_lag6 + y_lag7 +
    y_lag8 + y_lag9 + y_lag10 + y_lag11 + y_lag12 + x_lag1 +
    x_lag2 + x_lag3 + x_lag4 + x_lag5 + x_lag6 + x_lag7 + x_lag8 +
    x_lag9 + x_lag10 + x_lag11 + x_lag12

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F    Pr(>F)
1    621
2    609 12 5.7067 2.277e-09 ***
```

Figure 7: Wald test for Granger causality for housing starts and mortgage rate

```
$ppi_diff
Linear hypothesis test

Hypothesis:
x_lag1 = 0
x_lag2 = 0
x_lag3 = 0
x_lag4 = 0
x_lag5 = 0
x_lag6 = 0
x_lag7 = 0
x_lag8 = 0
x_lag9 = 0
x_lag10 = 0
x_lag11 = 0
x_lag12 = 0

Model 1: restricted model
Model 2: y ~ y_lag1 + y_lag2 + y_lag3 + y_lag4 + y_lag5 + y_lag6 + y_lag7 +
    y_lag8 + y_lag9 + y_lag10 + y_lag11 + y_lag12 + x_lag1 +
    x_lag2 + x_lag3 + x_lag4 + x_lag5 + x_lag6 + x_lag7 + x_lag8 +
    x_lag9 + x_lag10 + x_lag11 + x_lag12

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F    Pr(>F)
1    621
2    609 12 3.1396 0.0002359 ***
```

```
$hous_pr_diff
Linear hypothesis test

Hypothesis:
x_lag1 = 0
x_lag2 = 0
x_lag3 = 0
x_lag4 = 0
x_lag5 = 0
x_lag6 = 0
x_lag7 = 0
x_lag8 = 0
x_lag9 = 0
x_lag10 = 0
x_lag11 = 0
x_lag12 = 0

Model 1: restricted model
Model 2: y ~ y_lag1 + y_lag2 + y_lag3 + y_lag4 + y_lag5 + y_lag6 + y_lag7 +
    y_lag8 + y_lag9 + y_lag10 + y_lag11 + y_lag12 + x_lag1 +
    x_lag2 + x_lag3 + x_lag4 + x_lag5 + x_lag6 + x_lag7 + x_lag8 +
    x_lag9 + x_lag10 + x_lag11 + x_lag12

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F Pr(>F)
1    621
2    609 12 1.2366 0.2536
```

Figure 8: Wald test for Granger causality for PPI and housing prices

```
$disp_inc_diff
Linear hypothesis test

Hypothesis:
x_lag1 = 0
x_lag2 = 0
x_lag3 = 0
x_lag4 = 0
x_lag5 = 0
x_lag6 = 0
x_lag7 = 0
x_lag8 = 0
x_lag9 = 0
x_lag10 = 0
x_lag11 = 0
x_lag12 = 0

Model 1: restricted model
Model 2: y ~ y_lag1 + y_lag2 + y_lag3 + y_lag4 + y_lag5 + y_lag6 + y_lag7 +
    y_lag8 + y_lag9 + y_lag10 + y_lag11 + y_lag12 + x_lag1 +
    x_lag2 + x_lag3 + x_lag4 + x_lag5 + x_lag6 + x_lag7 + x_lag8 +
    x_lag9 + x_lag10 + x_lag11 + x_lag12

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F   Pr(>F)
1    621
2    609 12 2.3369 0.006242 **
```

Figure 9: Wald test for Granger causality for real disposable personal income

```
Residuals:
     Min       1Q   Median       3Q      Max
-2.03576 -0.27575 -0.01412  0.27726  2.42354

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)             -0.026389   0.021953  -1.202 0.229806
y_lag1                  -0.290496   0.041901  -6.933 1.05e-11 ***
y_lag2                  -0.154412   0.043522  -3.548 0.000418 ***
y_lag3                  -0.025226   0.043626  -0.578 0.563320
y_lag4                  -0.089314   0.042043  -2.124 0.034043 *
mortgage_rates_diff_lag1   0.664425   0.089265   7.443 3.36e-13 ***
mortgage_rates_diff_lag2  -0.146365   0.102370  -1.430 0.153297
mortgage_rates_diff_lag3   0.310570   0.105794   2.936 0.003455 **
mortgage_rates_diff_lag4  -0.175426   0.106985  -1.640 0.101579
mortgage_rates_diff_lag5   0.191594   0.103135   1.858 0.063694 .
mortgage_rates_diff_lag6  -0.059178   0.103588  -0.571 0.568019
mortgage_rates_diff_lag7  -0.061882   0.103486  -0.598 0.550080
mortgage_rates_diff_lag8   0.041060   0.103103   0.398 0.690592
mortgage_rates_diff_lag9  -0.025383   0.102460  -0.248 0.804421
mortgage_rates_diff_lag10 -0.134894   0.101976  -1.323 0.186397
mortgage_rates_diff_lag11  0.177919   0.098007   1.815 0.069961 .
mortgage_rates_diff_lag12 -0.262004   0.085694  -3.057 0.002330 **
ppi_diff_lag1            0.021136   0.014722   1.436 0.151603
ppi_diff_lag2            0.003927   0.016848   0.233 0.815783
ppi_diff_lag3           -0.008727   0.017031  -0.512 0.608525
ppi_diff_lag4            0.025670   0.017077   1.503 0.133297
ppi_diff_lag5            0.037051   0.017138   2.162 0.031012 *
ppi_diff_lag6            0.018866   0.017099   1.103 0.270315
ppi_diff_lag7           -0.043038   0.017089  -2.518 0.012043 *
ppi_diff_lag8            0.022103   0.015025   1.471 0.141793
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5096 on 609 degrees of freedom
Multiple R-squared:  0.191,     Adjusted R-squared:  0.1591
F-statistic: 5.991 on 24 and 609 DF,  p-value: < 2.2e-16
```

Figure 10: Regression coefficients for the optimal model

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
omega   0.019254    0.012873   1.4957 0.134735
alpha1  0.239454    0.072512   3.3023 0.000959
beta1   0.702367    0.103408   6.7922 0.000000
shape   1.513145    0.126950  11.9192 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
omega   0.019254    0.022307  0.86314 0.388061
alpha1  0.239454    0.090964  2.63240 0.008479
beta1   0.702367    0.166381  4.22144 0.000024
shape   1.513145    0.110545 13.68804 0.000000

LogLikelihood : -403.5363
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.789  0.1811
Lag[2*(p+q)+(p+q)-1][2]     1.852  0.2885
Lag[4*(p+q)+(p+q)-1][5]     2.221  0.5672
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.001374  0.9704
Lag[2*(p+q)+(p+q)-1][5]  2.442878  0.5181
Lag[4*(p+q)+(p+q)-1][9]  4.126441  0.5676
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1254 0.500 2.000  0.7233
ARCH Lag[5]    0.4338 1.440 1.667  0.9030
ARCH Lag[7]    1.7729 2.315 1.543  0.7653
```

Figure 11: GARCH(1,1) on ADL residuals

```
Weighted Residuals:
     Min       1Q   Median       3Q      Max
-2.84741 -0.61711 -0.07483  0.57286  3.05653

Coefficients:
                             Estimate Std. Error t value Pr(>|t|)
X(Intercept)                 0.010772   0.018605   0.579  0.56283
Xy_lag1                     -0.348730   0.048551  -7.183 2.00e-12 ***
Xy_lag2                     -0.161982   0.049693  -3.260  0.00118 **
Xy_lag3                     -0.018287   0.048399  -0.378  0.70568
Xy_lag4                     -0.064480   0.044879  -1.437  0.15130
Xmortgage_rates_diff_lag1    0.536842   0.088115   6.092 1.97e-09 ***
Xmortgage_rates_diff_lag2   -0.008992   0.100350  -0.090  0.92863
Xmortgage_rates_diff_lag3    0.311630   0.102169   3.050  0.00239 **
Xmortgage_rates_diff_lag4    0.014773   0.102403   0.144  0.88534
Xmortgage_rates_diff_lag5    0.037721   0.099820   0.378  0.70564
Xmortgage_rates_diff_lag6   -0.038390   0.098049  -0.392  0.69553
Xmortgage_rates_diff_lag7   -0.013958   0.096660  -0.144  0.88523
Xmortgage_rates_diff_lag8    0.041873   0.095612   0.438  0.66158
Xmortgage_rates_diff_lag9   -0.007456   0.094897  -0.079  0.93740
Xmortgage_rates_diff_lag10 -0.138053   0.092999  -1.484  0.13820
Xmortgage_rates_diff_lag11  0.161243   0.087253   1.848  0.06509 .
Xmortgage_rates_diff_lag12 -0.169254   0.078605  -2.153  0.03169 *
Xppi_diff_lag1               0.018435   0.013541   1.361  0.17389
Xppi_diff_lag2               0.003095   0.016127   0.192  0.84789
Xppi_diff_lag3              -0.011297   0.016727  -0.675  0.49970
Xppi_diff_lag4               0.020576   0.016930   1.215  0.22472
Xppi_diff_lag5               0.037523   0.017145   2.189  0.02901 *
Xppi_diff_lag6               0.006571   0.017562   0.374  0.70842
Xppi_diff_lag7              -0.028030   0.017407  -1.610  0.10785
Xppi_diff_lag8               0.006011   0.015155   0.397  0.69177
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.003 on 609 degrees of freedom
Multiple R-squared:  0.1628,    Adjusted R-squared:  0.1284
F-statistic: 4.735 on 25 and 609 DF,  p-value: 1.145e-12
```

Figure 12: WLS regression results