CSCI 4350/5350

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Homework 8

Due: Tue. Nov. 20, 11:00 PM

1. (2 points each) Given the joint probability distribution table, calculate the following probabilities:

	play		¬ play	
	hot	¬ hot	hot	¬hot
rain	0.05	0.1	0.2	0.03
¬rain	0.15	0.3	0.11	0.06

$$P(X \vee Y) = P(X) + P(Y) - P(X \wedge Y)$$

$$P(X \mid Y) = P(X \wedge Y) / P(Y)$$

a. P(hot) $= 0.05 + 0.15 + 0.2 + 0.11 = \boxed{0.51}$

b. P(rain ∨ hot) $= 0.51 + 0.1 + 0.03 = \boxed{0.64}$

c. P(play | hot) $= (0.05+0.15) \div 0.51 = \frac{20}{100} \cdot \frac{100}{51} = \boxed{\frac{20}{51}}$

d. P(play | rain ∧ hot) $= (0.05) \div (0.05+0.2) = \frac{5}{100} \cdot \frac{100}{25} = \boxed{\frac{1}{5}}$

2. (2 points each) Given the following probabilities, use Bayes' Rule to answer the following:

P(cold) = 0.2

P(fever | cold) = 0.8

P(fever | ¬cold) = 0.1

$$P(\neg cold|fever) = \frac{P(fever|\neg cold)\,P(\neg cold)}{P(fever)}$$

$$P(fever|\neg cold) = \frac{P(\neg cold|fever)\,P(fever)}{P(\neg cold)}$$

Bayes' Rule – P(H | E) = [P(E | H) * P(H)] / P(E) = α[P(E | H) * P(H)]

Reminder: P(H | E) + P(¬H | E) = 1

a. P(cold | fever)

$$P(cold|fever) = \frac{P(fever|cold)\,P(cold)}{P(fever)}$$

So:

$$1 - P(cold|fever) = 1 - \frac{P(fever|cold)\,P(cold)}{P(fever)} = \frac{P(fever|\neg cold)\,P(\neg cold)}{P(fever)}$$

$$\Rightarrow P(fever) = P(fever|\neg cold)P(\neg cold) + P(fever|cold)P(cold) = (0.1)(0.8)+(0.8)(0.2) = \frac{3}{10}\cdot\frac{8}{10} = 0.24$$

So: $P(cold|fever) = \left(\frac{8}{10}\cdot\frac{2}{10}\right)\left(\frac{100}{24}\right) = \boxed{\frac{2}{3}}$

b. P(cold | ¬fever)

$$P(cold|\neg fever) = \frac{P(\neg fever|cold)\,P(cold)}{P(\neg fever)} = \frac{(1-P(fever|cold))\,P(cold)}{1-P(fever)}$$

$$= \frac{(0.2)(0.2)}{(0.76)} = \left(\frac{4}{100}\right)\left(\frac{100}{76}\right) = \frac{4}{76} = \boxed{\frac{1}{19}}$$

3. (3 points each) Explain the meaning of P(cold | fever) and P(cold | ¬fever), relative to P(cold).

- P(cold|fever) is the probability that a person has a cold given that the person has a fever, which is precisely all the circumstances under which a person has a cold if the probability that the person has a fever is assumed to be 100%. This is given by the probability that the person has a cold and a fever and normalized by the probability that the person has a fever.
- The explanation of P(cold|¬fever) is analogous, however with the assumption that the probability has a fever is instead 0%, and thus the probability the person doesn't have a fever is 100%.