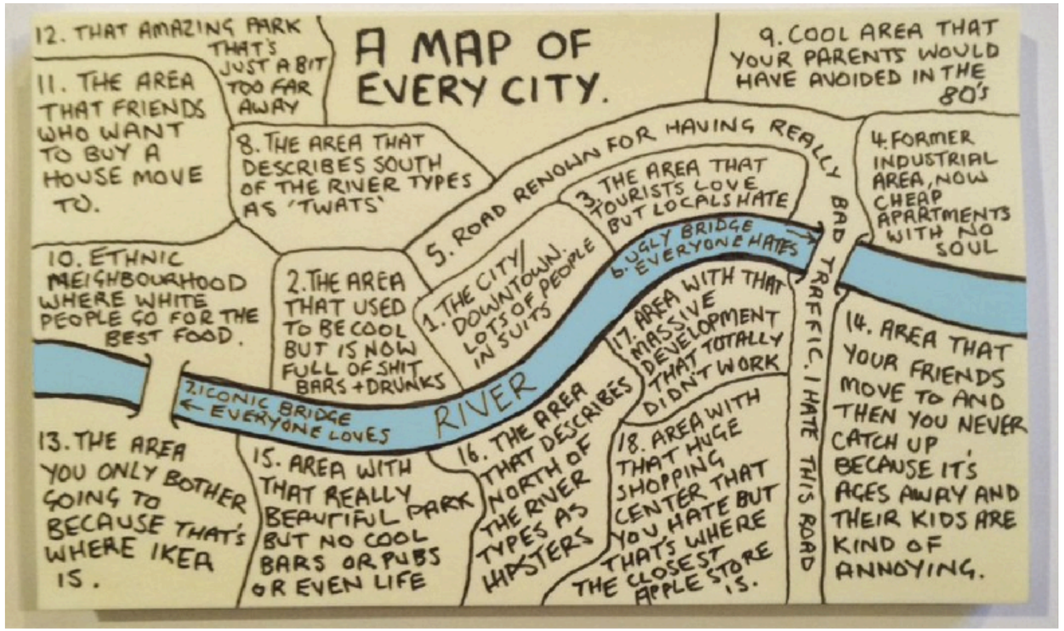
Lecture 13: Aggregate mobility and urban scaling

Instructor: Michael Szell

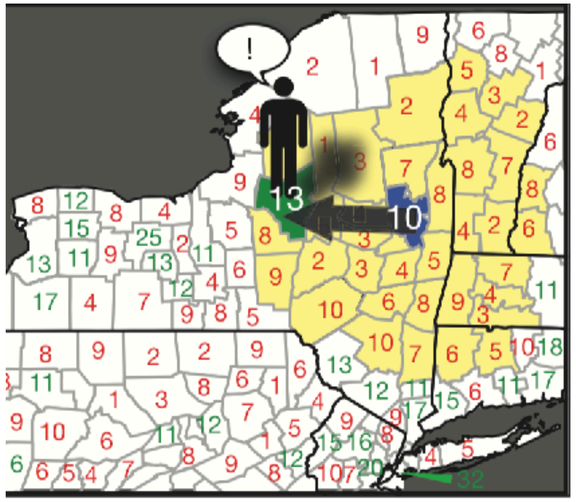
May 5, 2022



IT UNIVERSITY OF COPENHAGEN

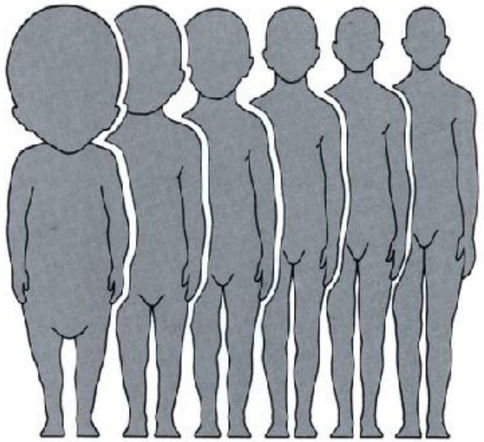
Today you will learn about aggregate mobility and urban scaling

Aggregate mobility models



Science of cities

Urban scaling



Aggregate mobility

You can study mobility=human movements at different scales

Individual

Single-scale

- Pedestrian movements
- Air transport
- Sea networks

Multi-scale

- Intra/Inter-urban mobility
- Epidemic spreading

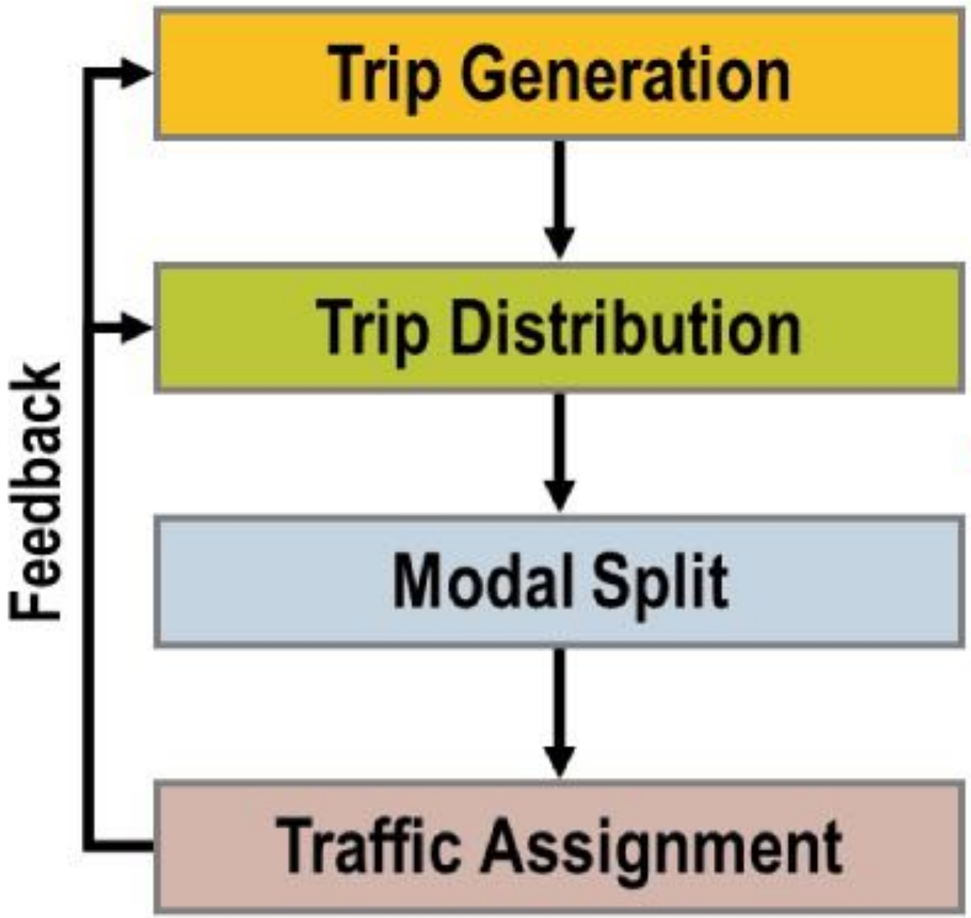
Virtual scale

Population / Aggregate

Migration

Spatial Interaction

The four-stage model of transport estimates flows between spatial units



Demand from/to each unit

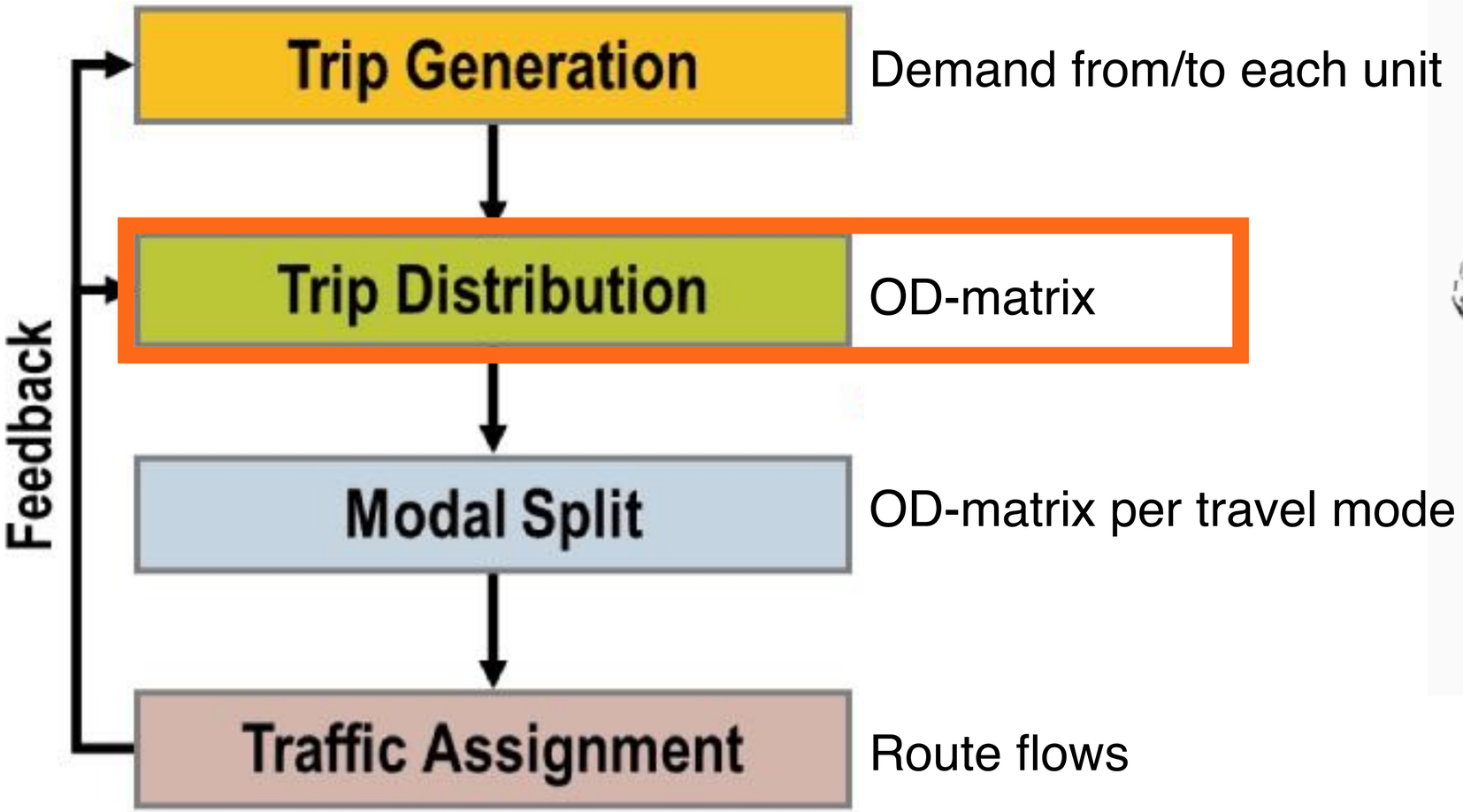
OD-matrix

OD-matrix per travel mode

Route flows

https://transportgeography.org/contents/methods/spatial-interactions-gravity-model/transportation-land-use-four-stages-model/

Spatial interaction models aim to predict the OD-matrix



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Spatial interaction models aim to predict the OD-matrix

Gravity model	1858, 1885
Intervening opportunities model	1940
Radiation model	2012
Hierarchy model	2017
Accounting for frequencies	2021

Mathematical travel laws have been noted since the 19th century

ZEITSCHRIFT
für
Eisenbahnen und Dampfschiffahrt
der
ÖSTERREICHISCH-UNGARISCHEN MONARCHIE.

HERAUSGEBER: PROF. DR. LORENZ v. STEIN.
REDACTEUR: SIGMUND SONNENSCHEIN.

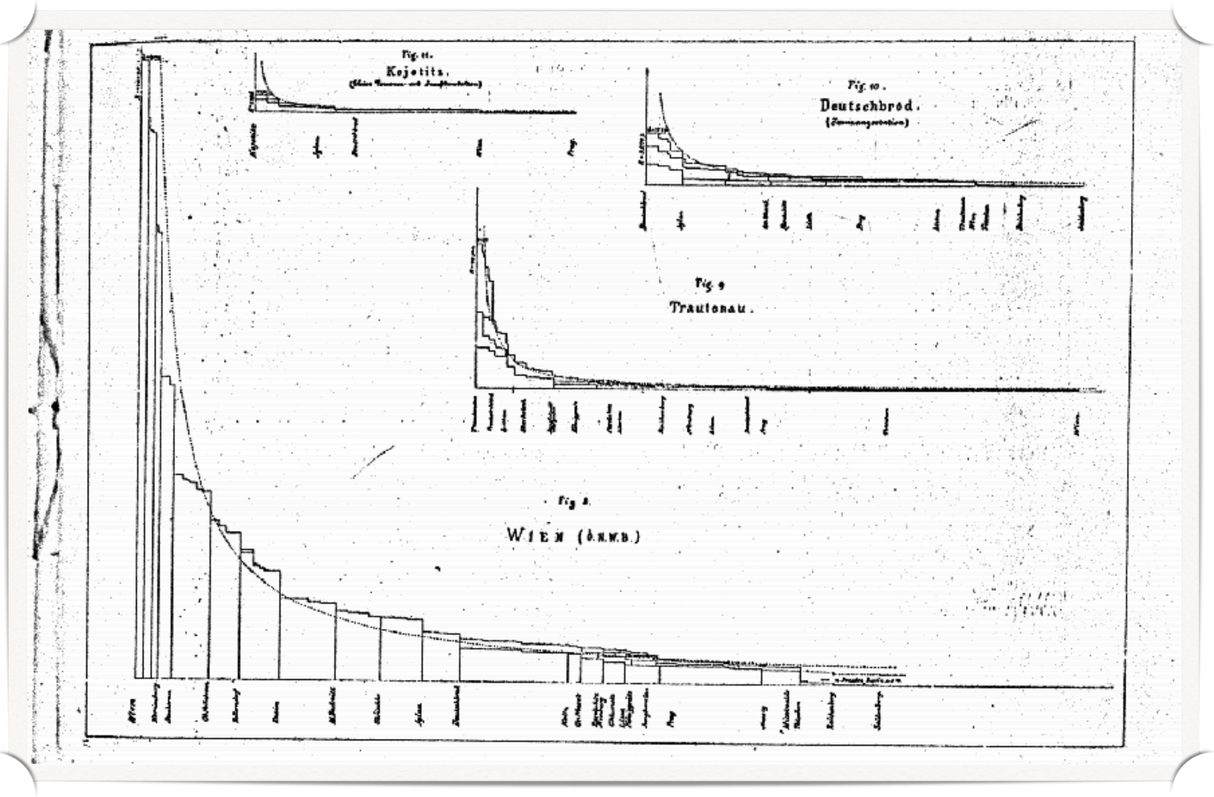
A. Hartleben's Verlag in Wien.

35. HEFT. 25. AUGUST 1889. II. JAHRGANG.

Die Grundgesetze des Personenverkehrs.

Eine Studie.

Von
Eduard Lill,
Oberinspector der k. k. priv. österreichischen Nordwestbahn.



Lill, 1889: The number of people traveling from a place decreases with distance, following a hyperbola

Gravity model

Based on Newton's law of gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

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The number of individuals T_{ij} that move between locations i and j is proportional to the populations m of the source and destination:

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$$T_{ij} = G\frac{m_i^\alpha m_j^\beta}{r_{ij}^\gamma} \qquad T_{ij} = G\frac{m_i^\alpha m_j^\beta}{e^{dr}} \qquad T_{ij} = \frac{m_i^\alpha m_j^\beta}{f(r_{ij})}$$

Generalizations due to disagreeing measurements

Gravity model

has been used to model:

- Population flows
- Access to health services
- International trade
- Traffic in transport networks
- Phone communications

The gravity model needs many parameters to calibrate

$$T_{ij} = G\frac{m_i^{\alpha} m_j^{\beta}}{r_{ij}^{\gamma}}$$

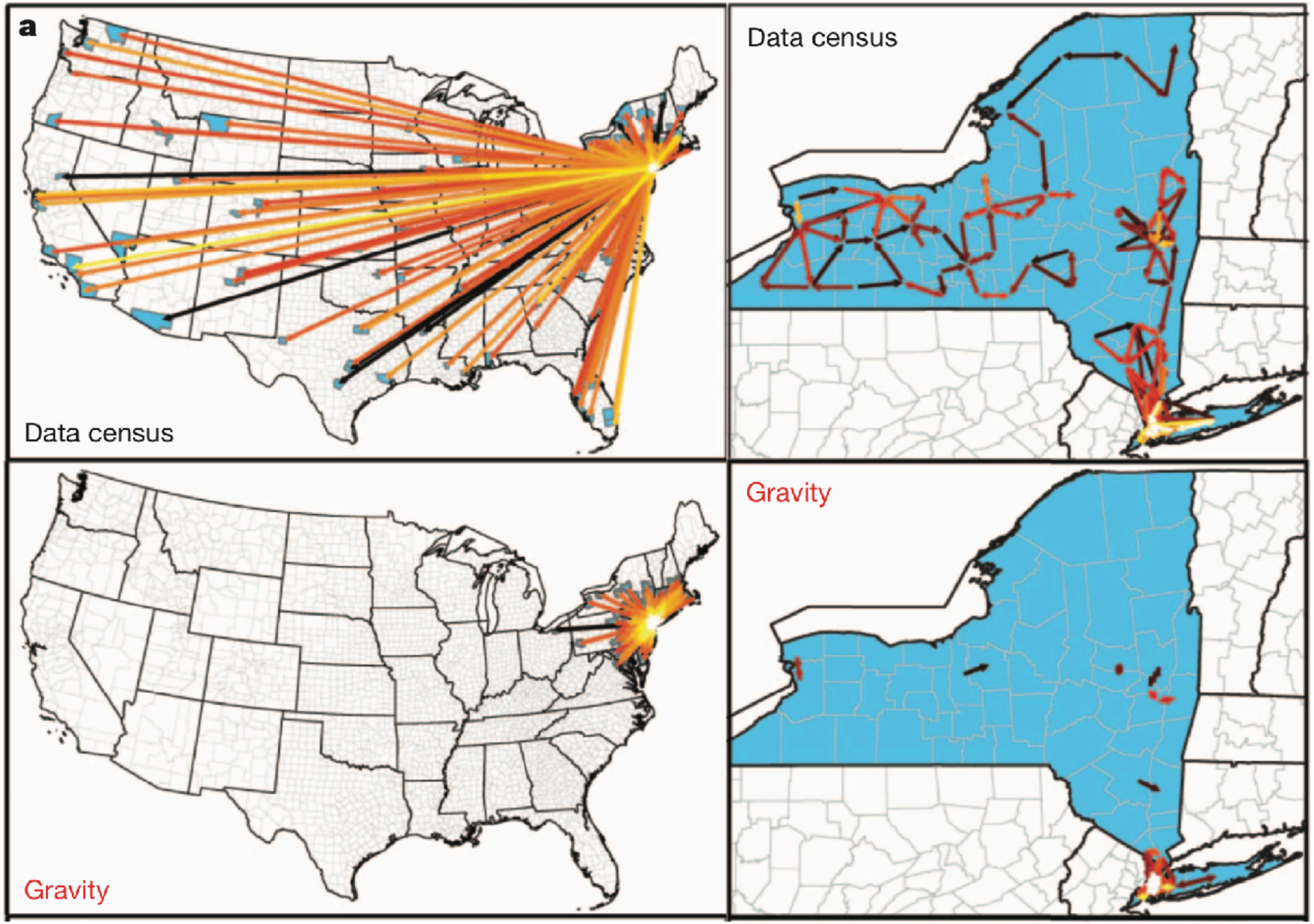
varies between 1.5 and 4.5

Systematic comparison of trip distribution laws and models

Maxime Lenormand,[1] Aleix Bassolas,[1] and José J. Ramasco[1]

[1] *Instituto de Física Interdisciplinar y Sistemas Complejos IFISC (CSIC-UIB), Campus UIB, 07122 Palma de Mallorca, Spain*

The gravity model underestimates long trips



Simini et al. *Nature* 484.7392 (2012)

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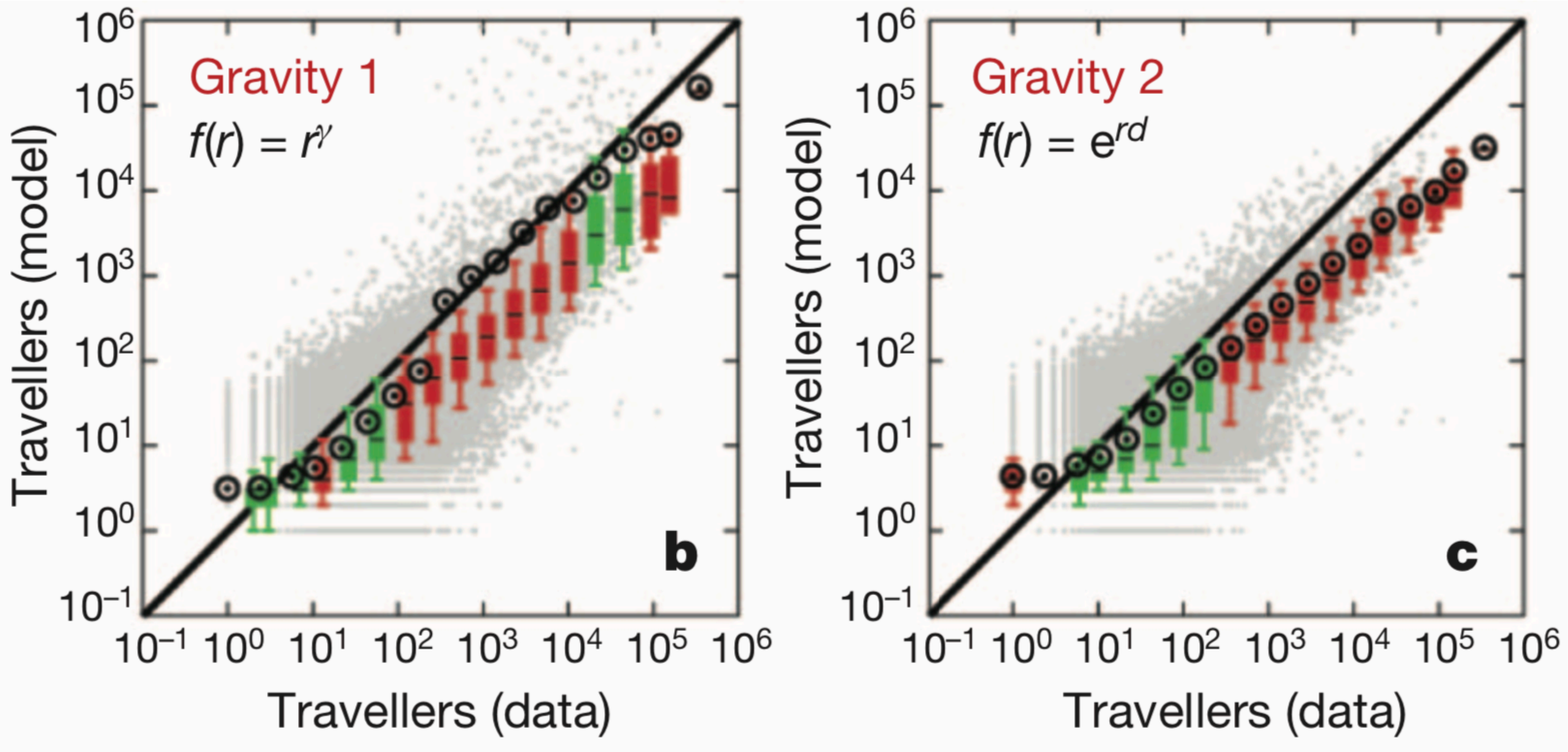

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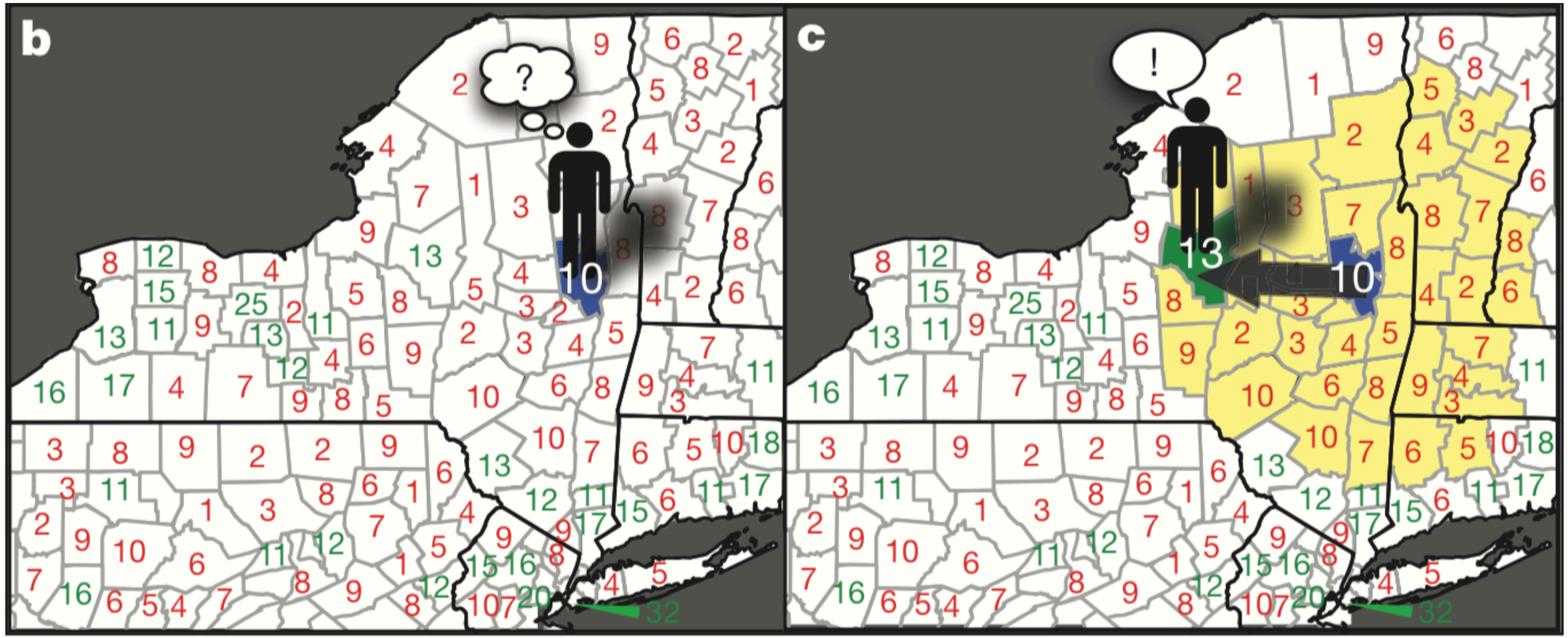
Intervening opportunities model

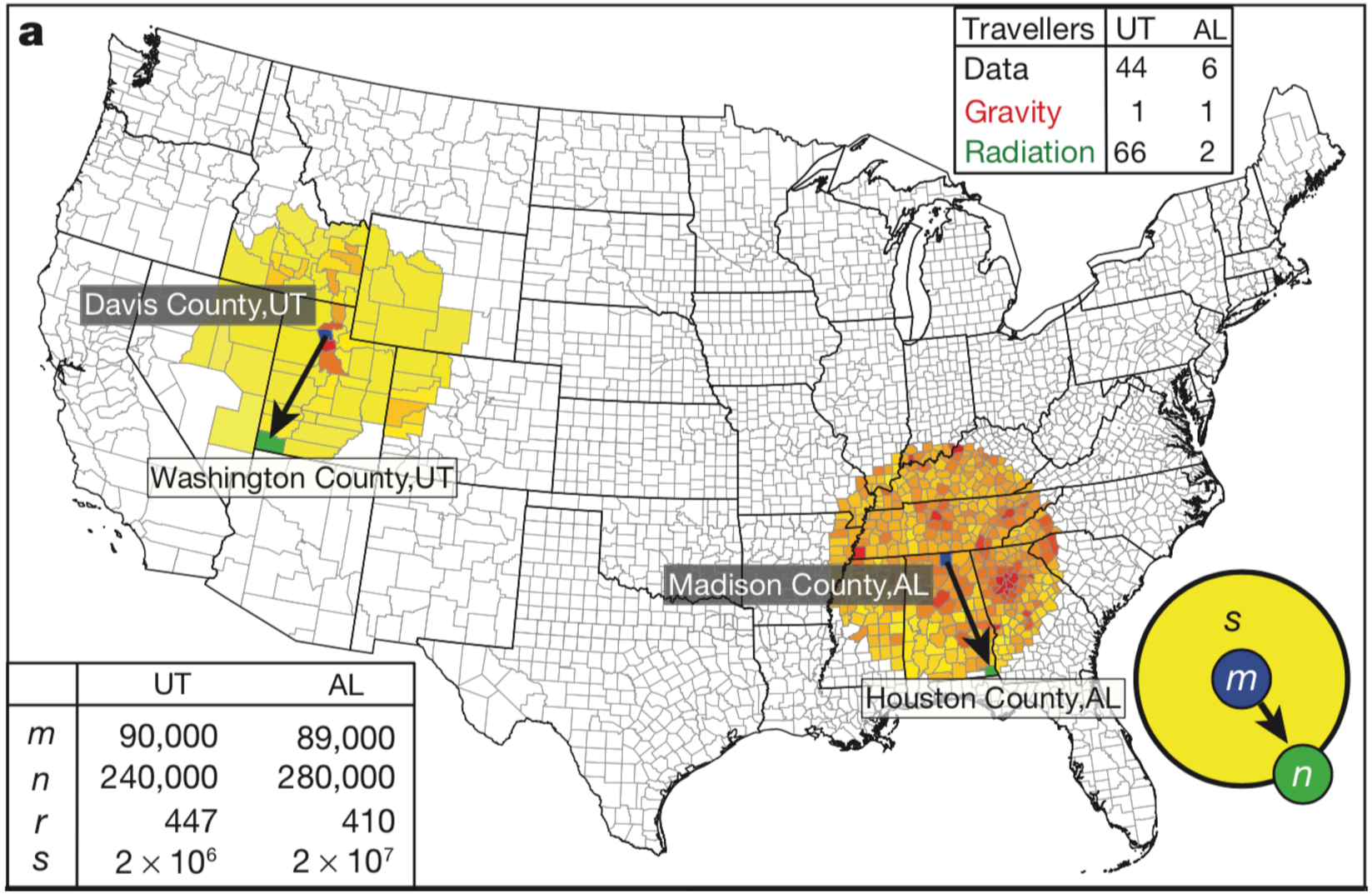
Stouffer (1940): The number of people going a given distance is directly proportional to the number of opportunities at that distance and inversely proportional to the number of intervening opportunities.



Simini et al. *Nature* 484.7392 (2012)

The radiation model aims to fix problems with gravity

based on intervening opportunities



Simini et al. *Nature* 484.7392 (2012)

The radiation model aims to fix problems with gravity

The average flows from i to j are predicted as:

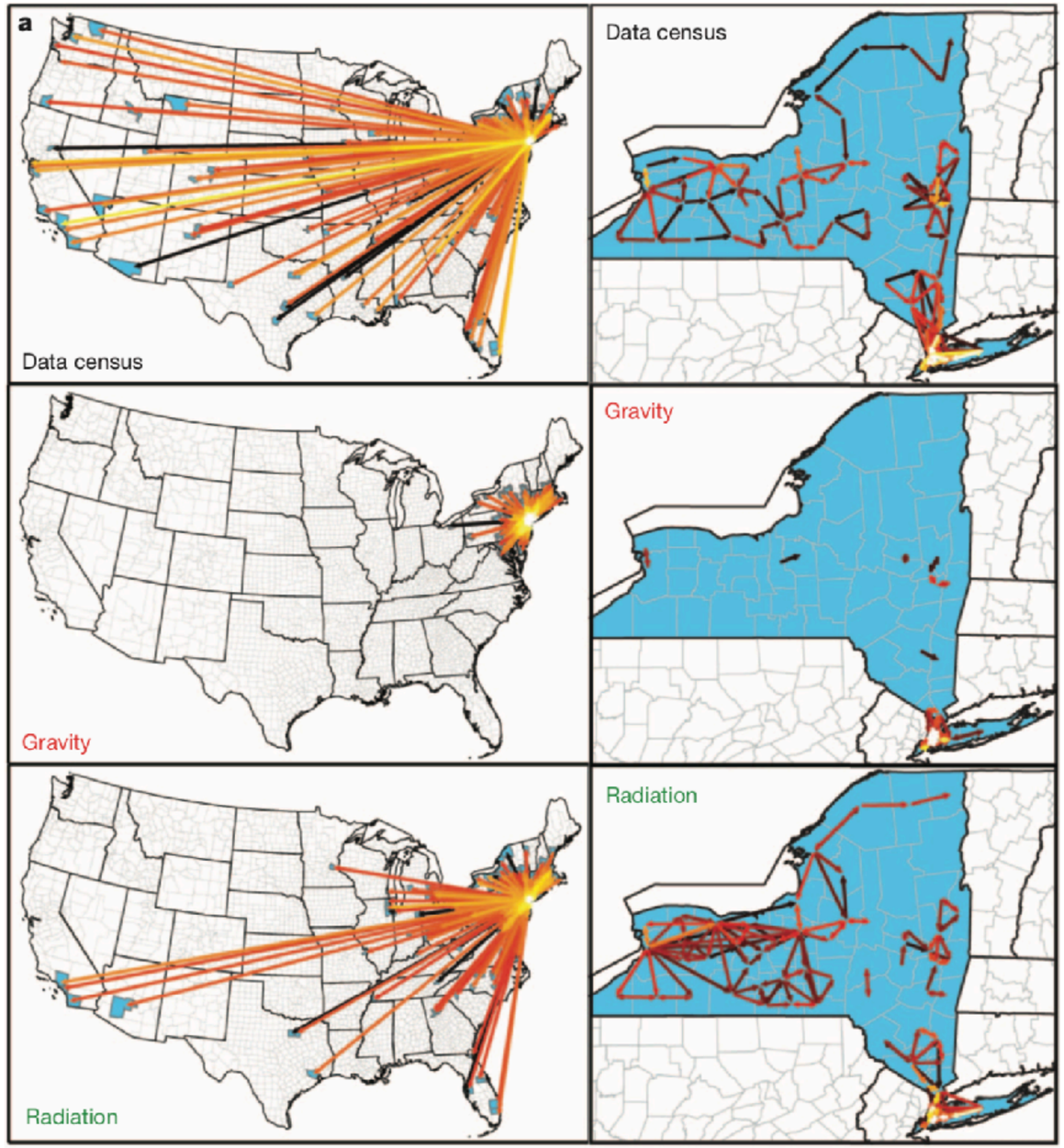
$$\langle T_{ij} \rangle = T_i \frac{m_i n_j}{(m_i + s_{ij})(m_i + n_j + s_{ij})}$$

Number of commuters starting at i

Total population in circle with radius r_{ij} centered in i

Simini et al. *Nature* 484.7392 (2012)

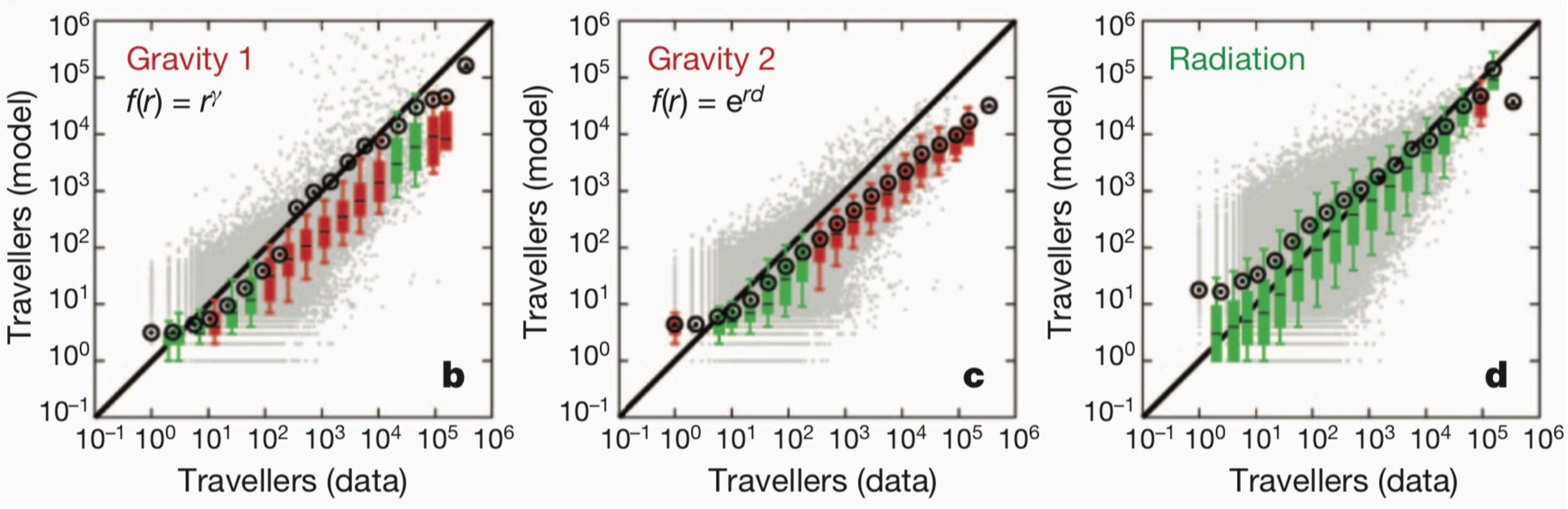
The radiation model aims to fix problems with gravity



Simini et al. *Nature* 484.7392 (2012)

The radiation model aims to fix problems with gravity

No underestimation of long trips
No parameters



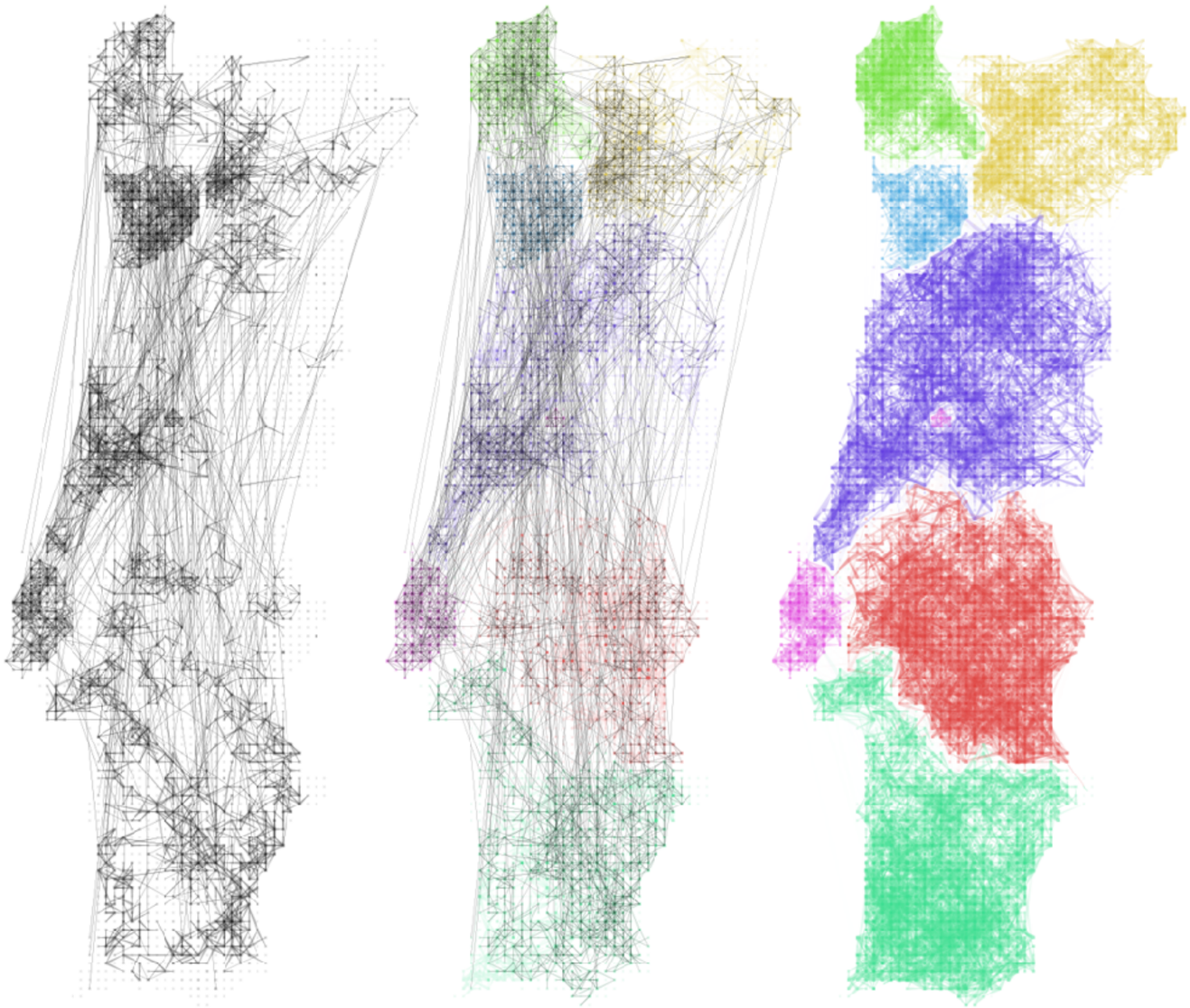
Simini et al. *Nature* 484.7392 (2012)

scikit mobility has implementations of mobility models

`gravity.Gravity` ([deterrence_func_type, ...])	Gravity model.
`radiation.Radiation` ([name])	Radiation model.
`geosim.GeoSim` ([name, rho, gamma, alpha, ...])	GeoSim model.
`sts_epr.STS_epr` ([name, rho, gamma, alpha])	STS-EPR model.

https://scikit-mobility.github.io/scikit-mobility/reference/models.html

There are structural discontinuities in spatial organization of society



Community detection of human interaction networks

Sobolevsky et al, PLOS ONE 8, e81707 (2013)
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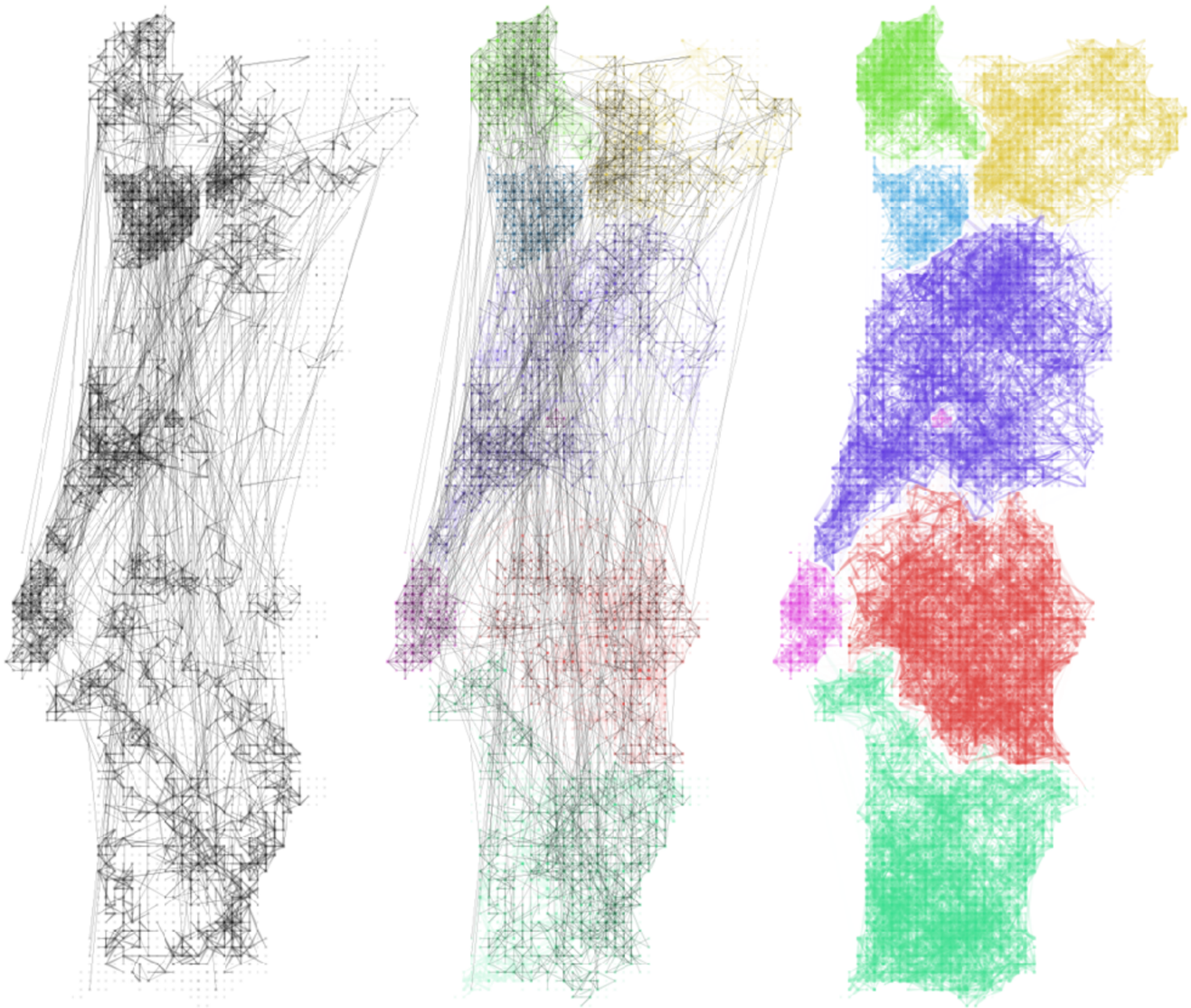
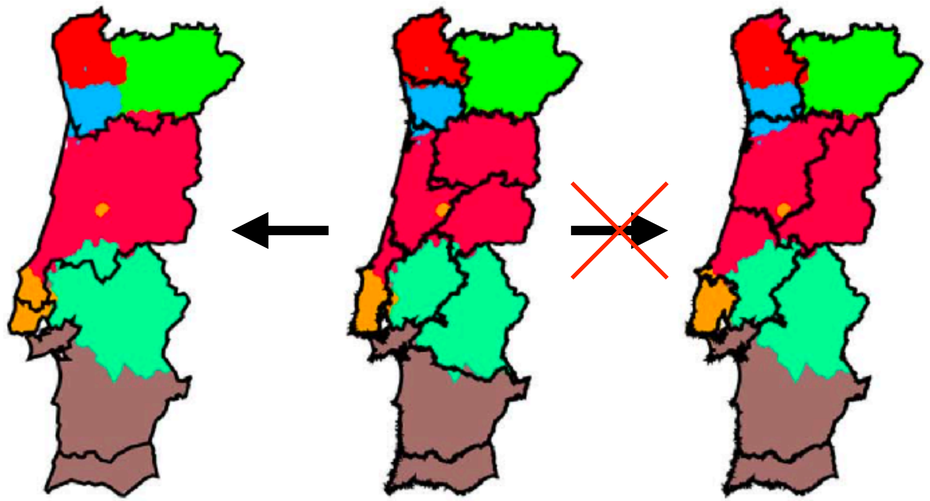
There are structural discontinuities in spatial organization of society



Community detection of human interaction networks

matches closely administrative borders!



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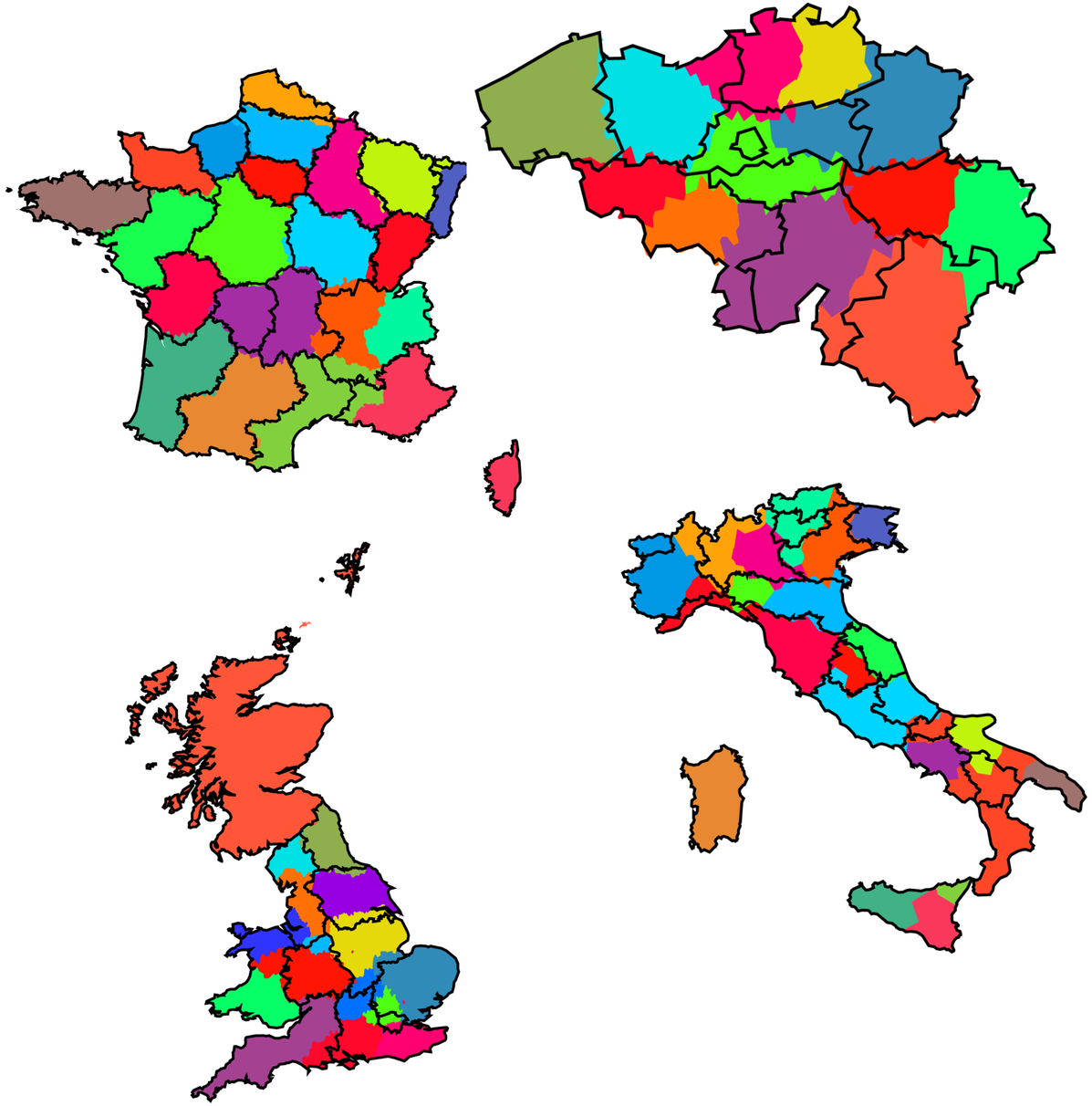

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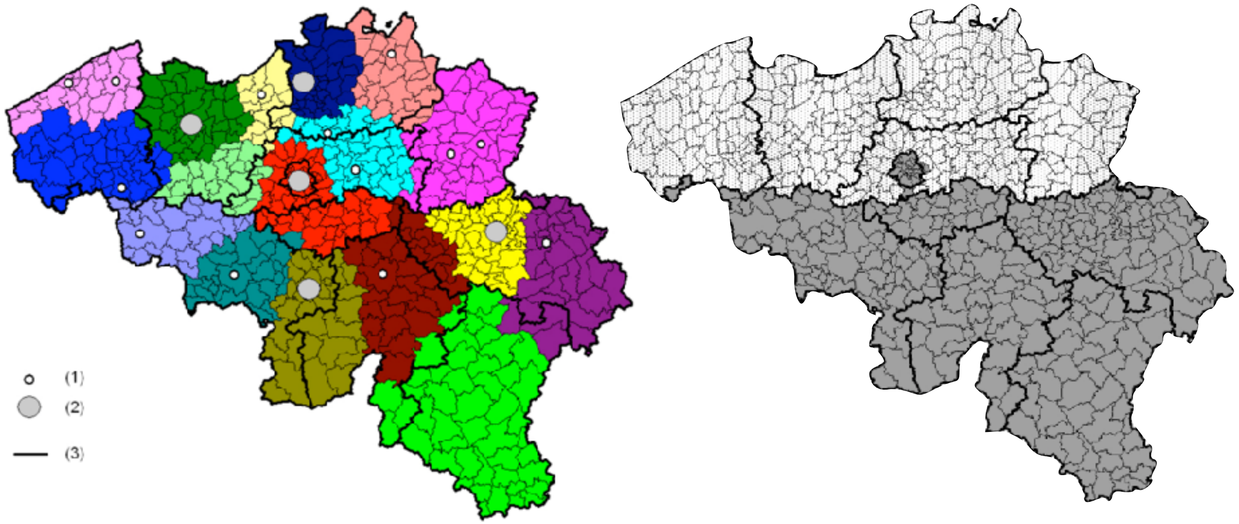
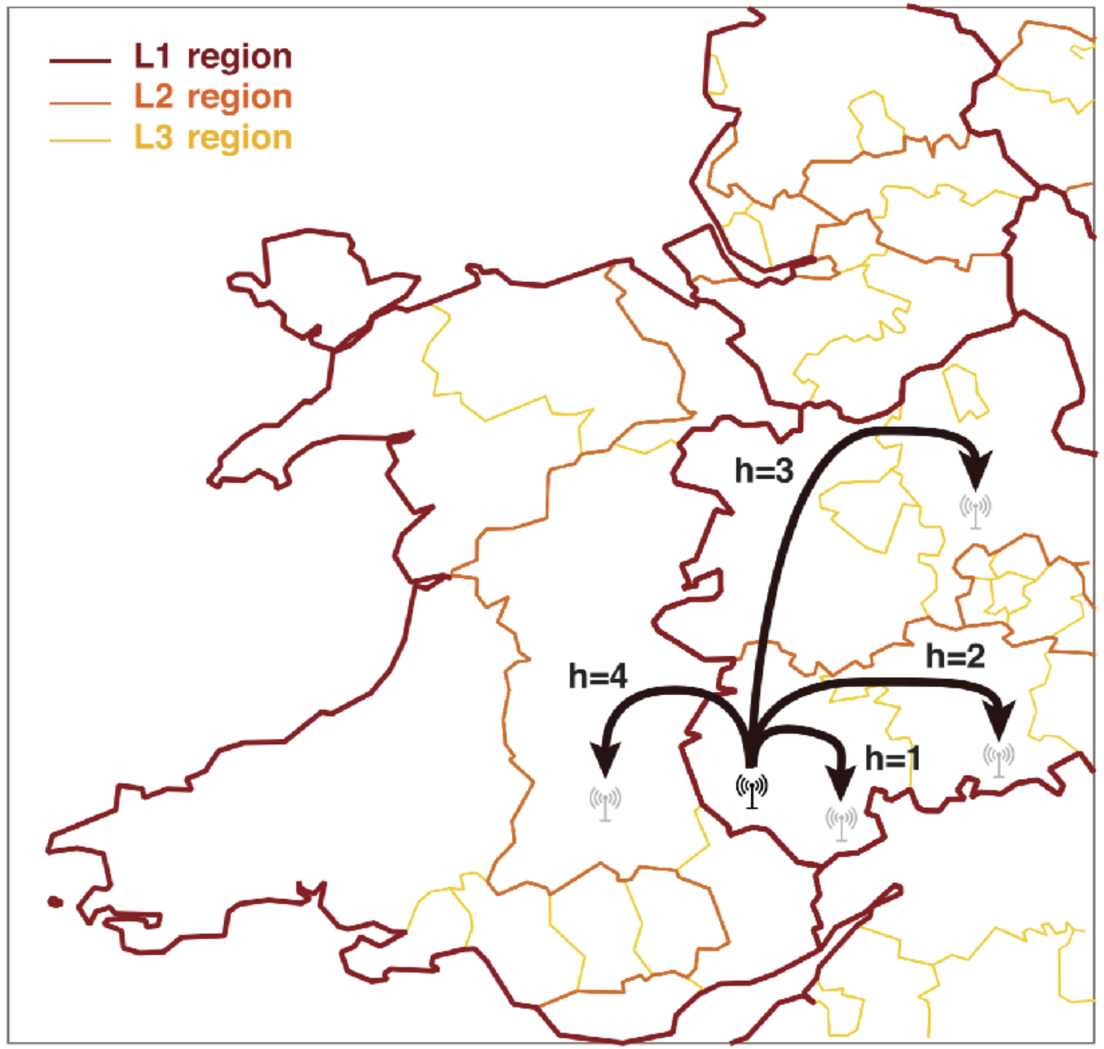
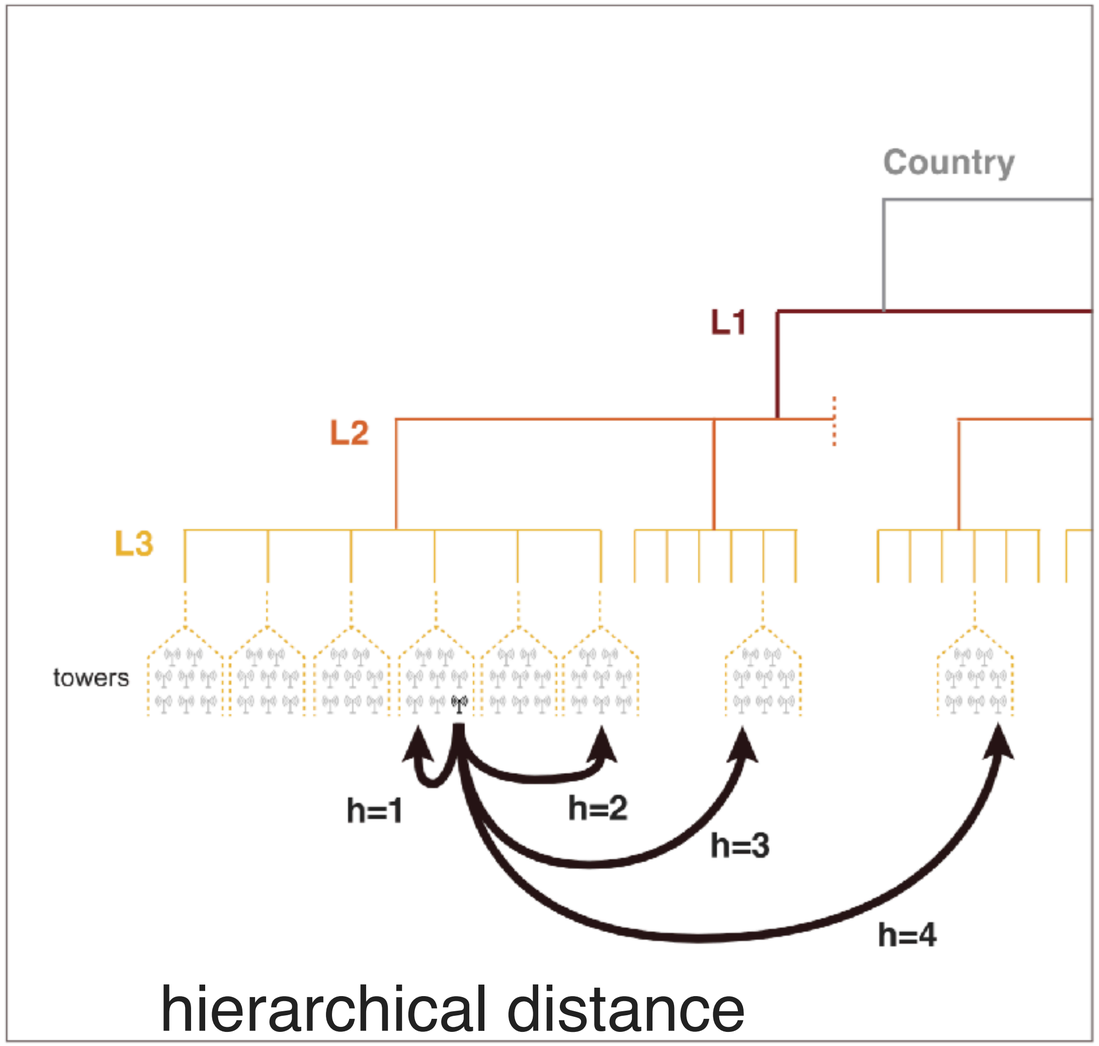


Blondel, Krings, and Thomas, Brussels Studies 42 (2010)

There are structural discontinuities in spatial organization of society

Idea for new model: Society is organized hierarchically in space

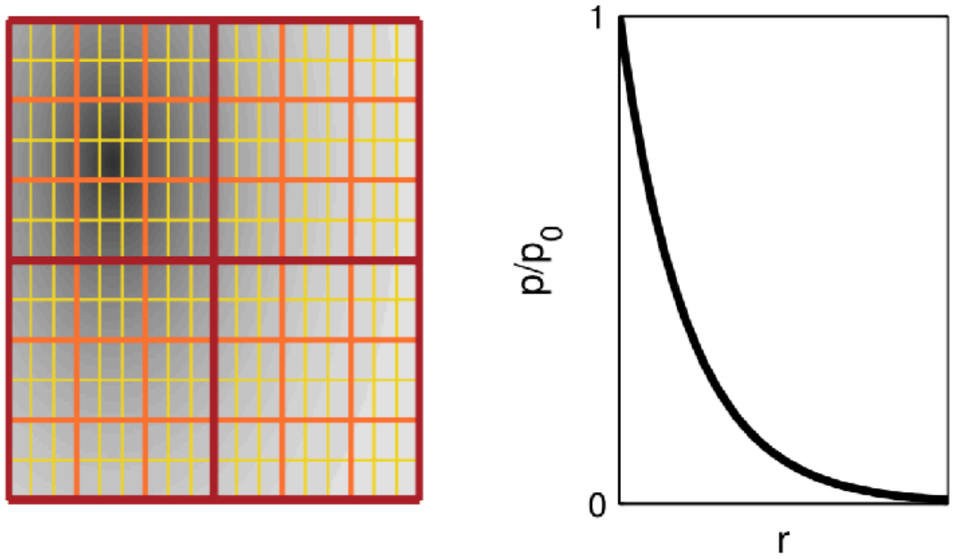




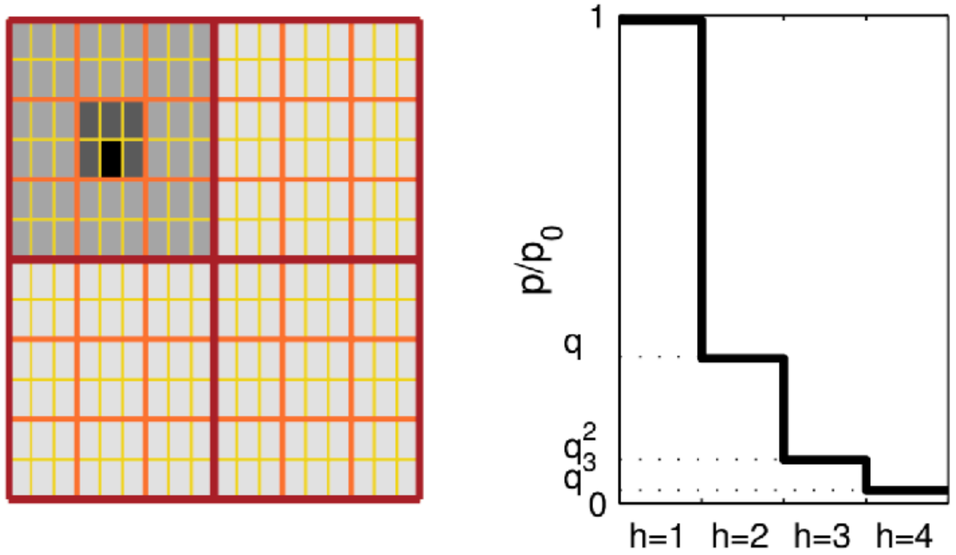
Grauwin et al, Sci Rep 7:46677 (2017)

There are structural discontinuities in spatial organization of society

Old models



Hierarchy model



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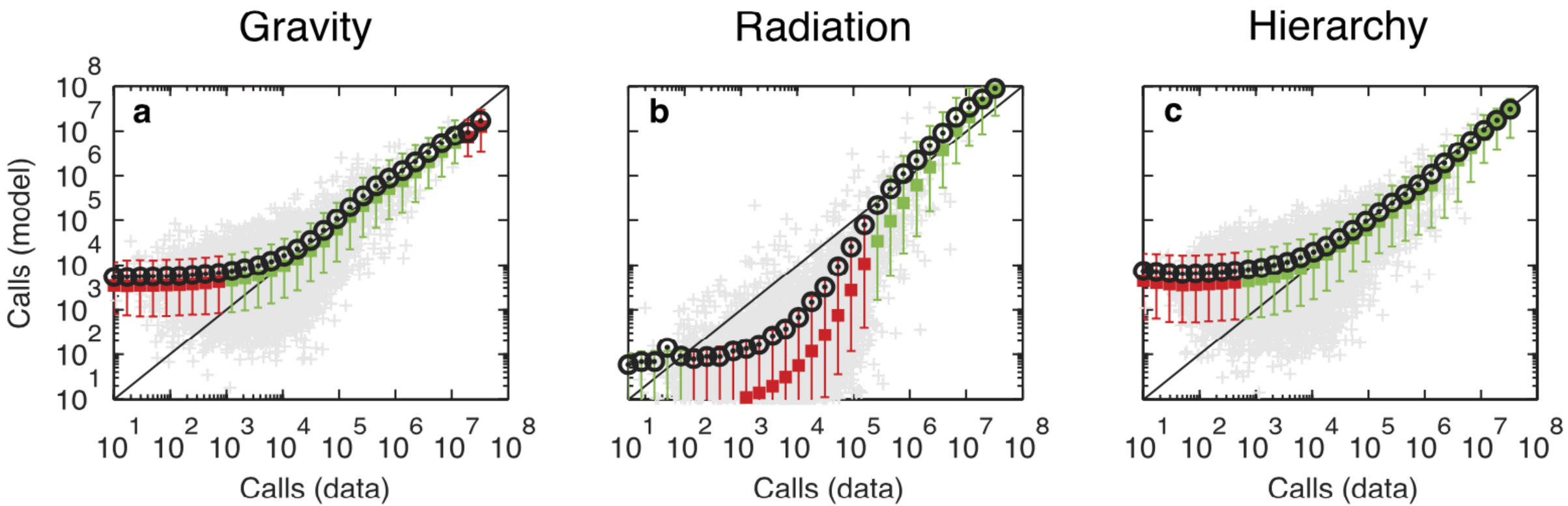
The hierarchy model describes better human interactions



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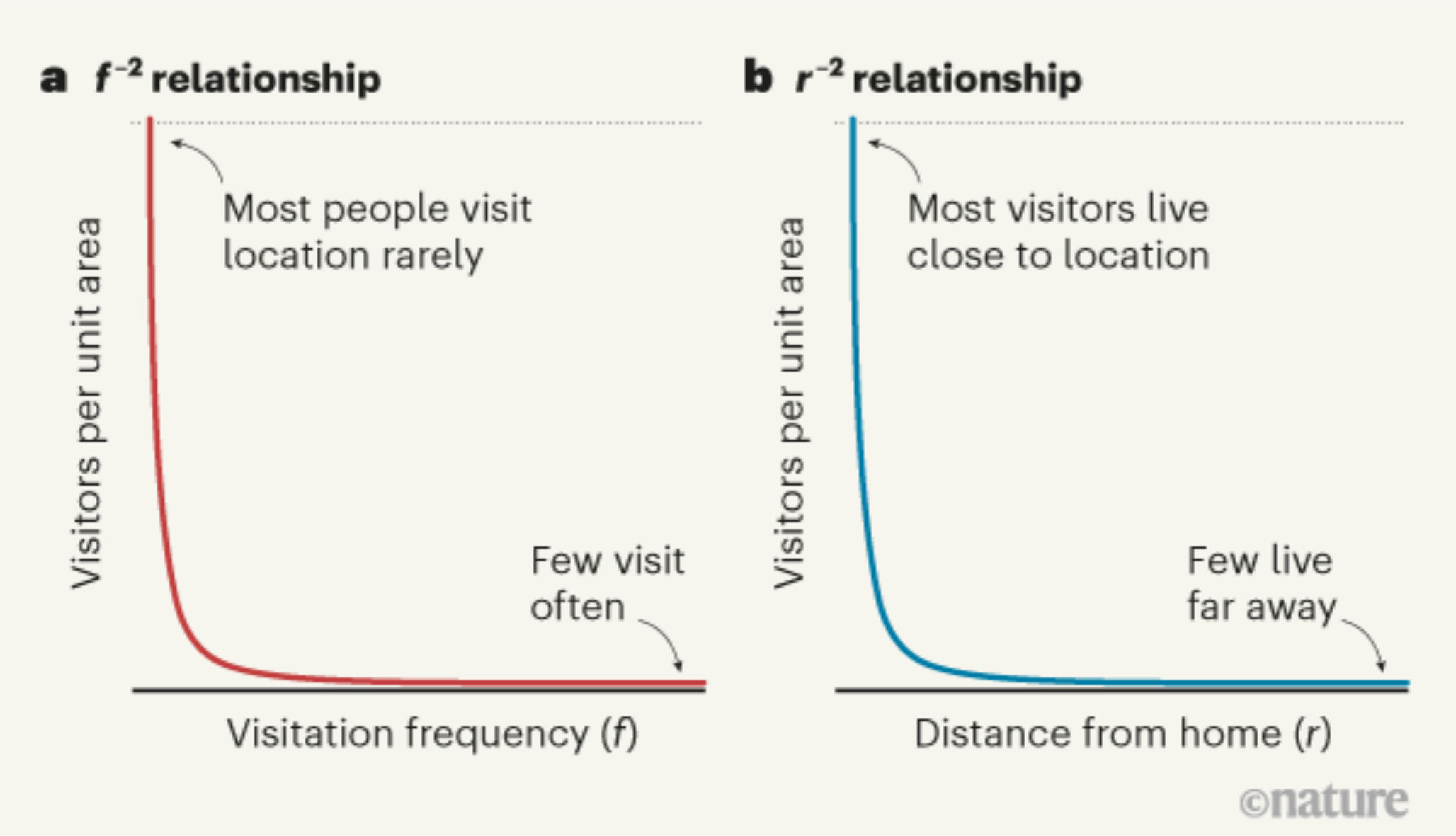
Accounting for frequency makes models more powerful

https://youtube.com/watch?v=RrfR2YG0g4Q

Accounting for frequency makes models more powerful



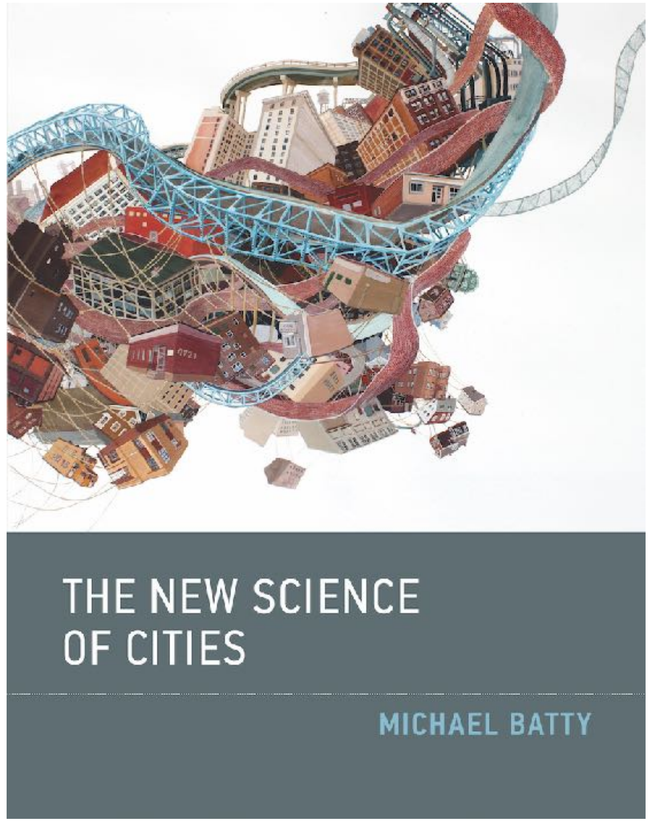
Science of Cities
Urban Science
Urban Data Science

Isn't Data Science just Statistics+BIG DATA?

Isn't Urban Data Science just Geography+BIG DATA?

We want to understand the fundamental processes behind cities

This approach originates from complex systems / physics, building on the rich history of the field



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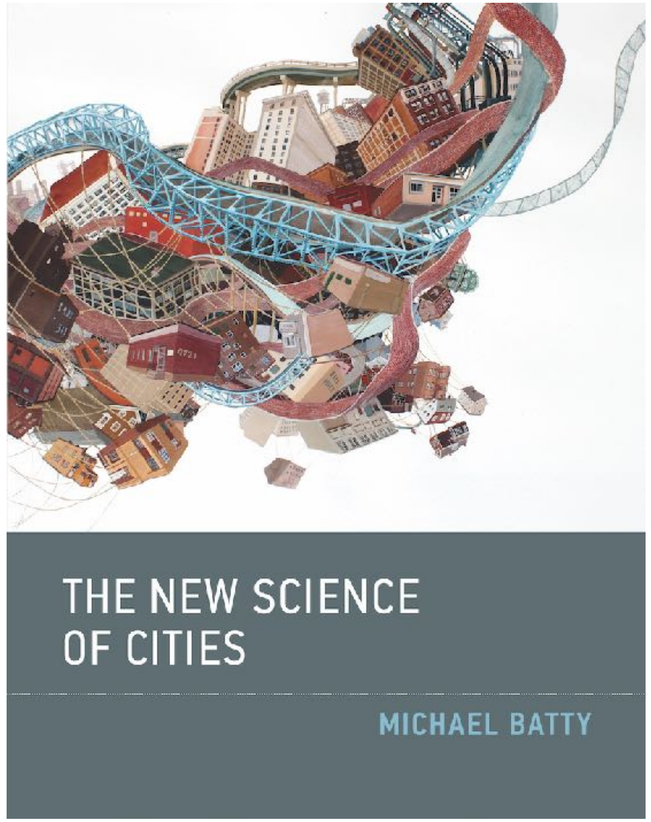


What is a city?

Why are there cities?

What do cities have in common? Why are some different?

How is a city built? How much are we in control?

How should we manage a city? Can we change a city?

What is a city?

What is a city?

A human settlement above some size

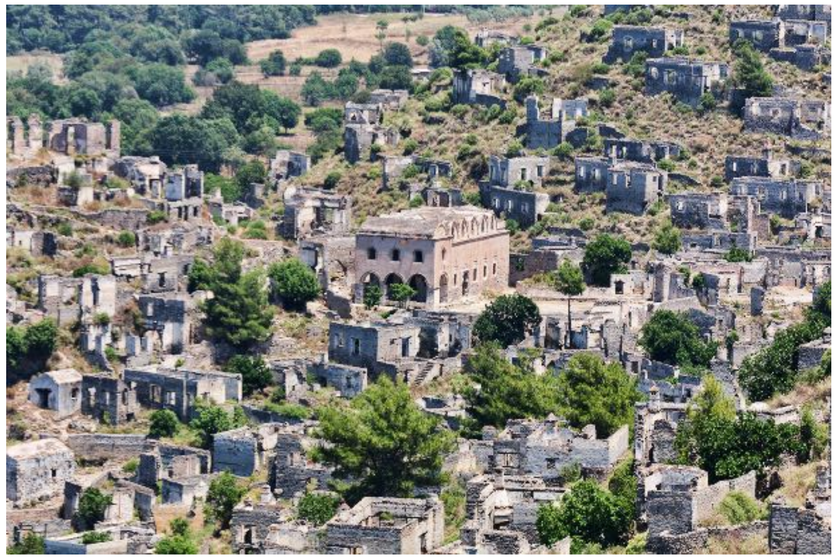
Size = population? buildings? streets?

US: at least between 1500 and 50000 inhabitants

What is a city? A human settlement with many streets/buildings?

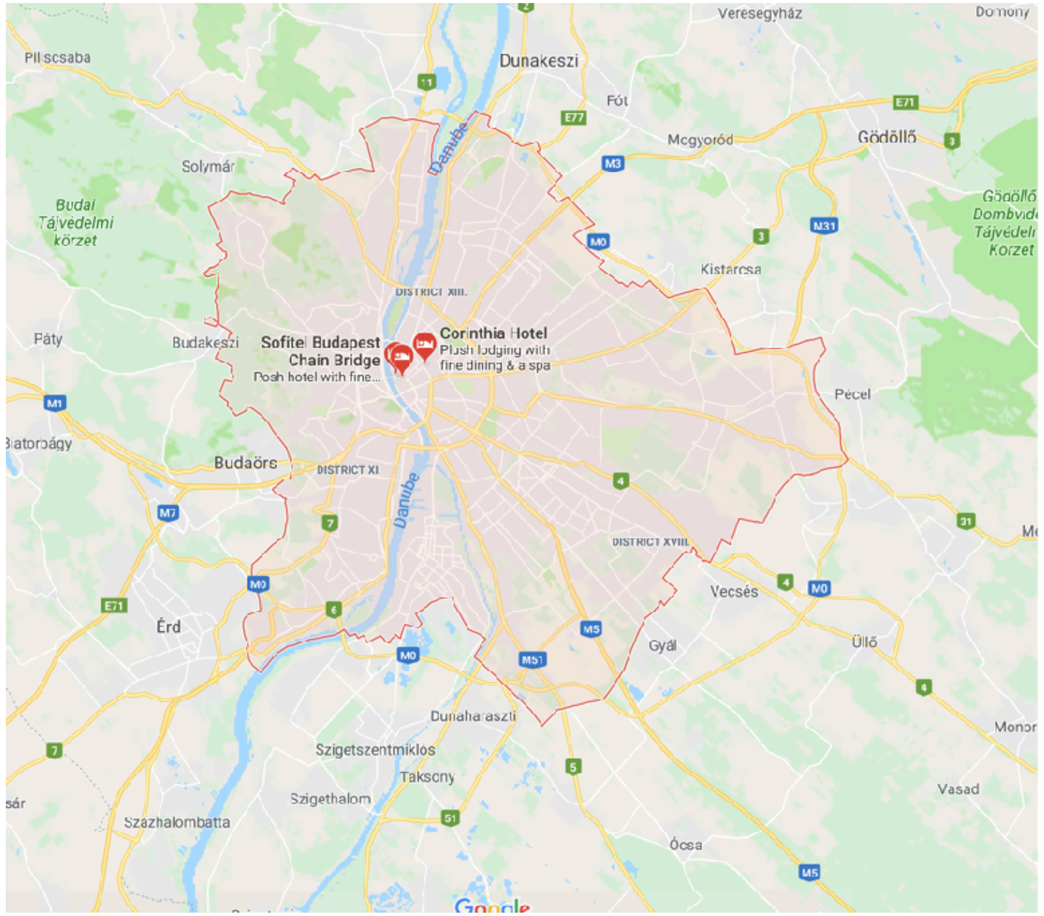
Varosha: military ghost town

Kangbashi: political/ economic ghost town



Kayaköy: religious ghost town

What is a city? Everything inside administrative boundaries?

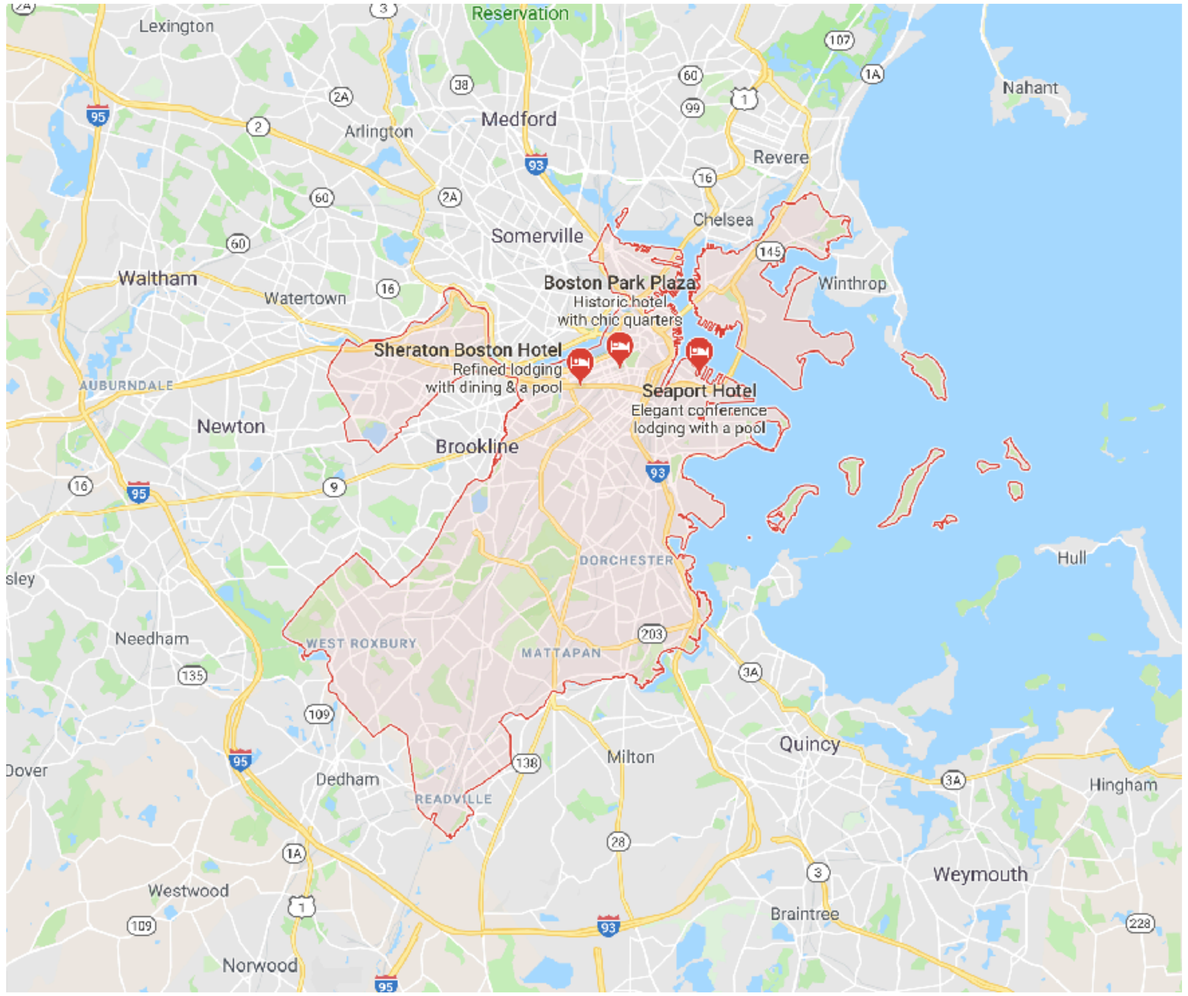


What is a city? Everything inside administrative boundaries?

Buda Pest

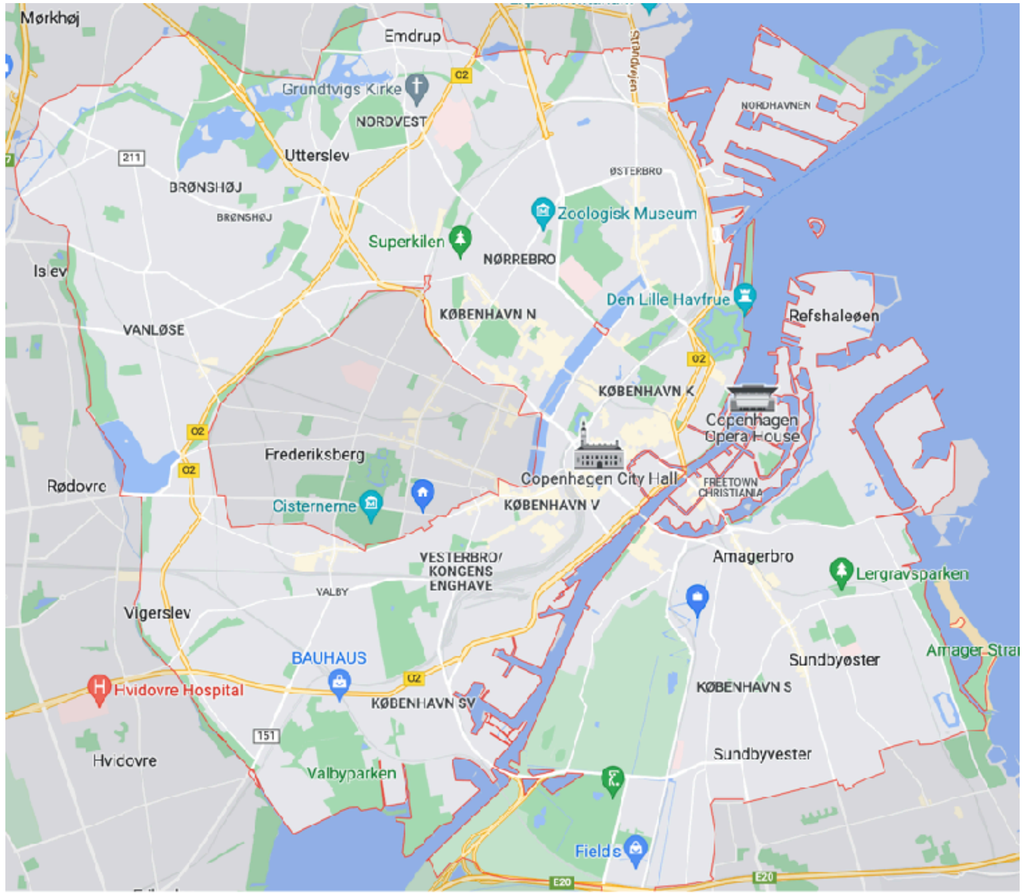
Merge 1870

What is a city? Everything inside administrative boundaries?



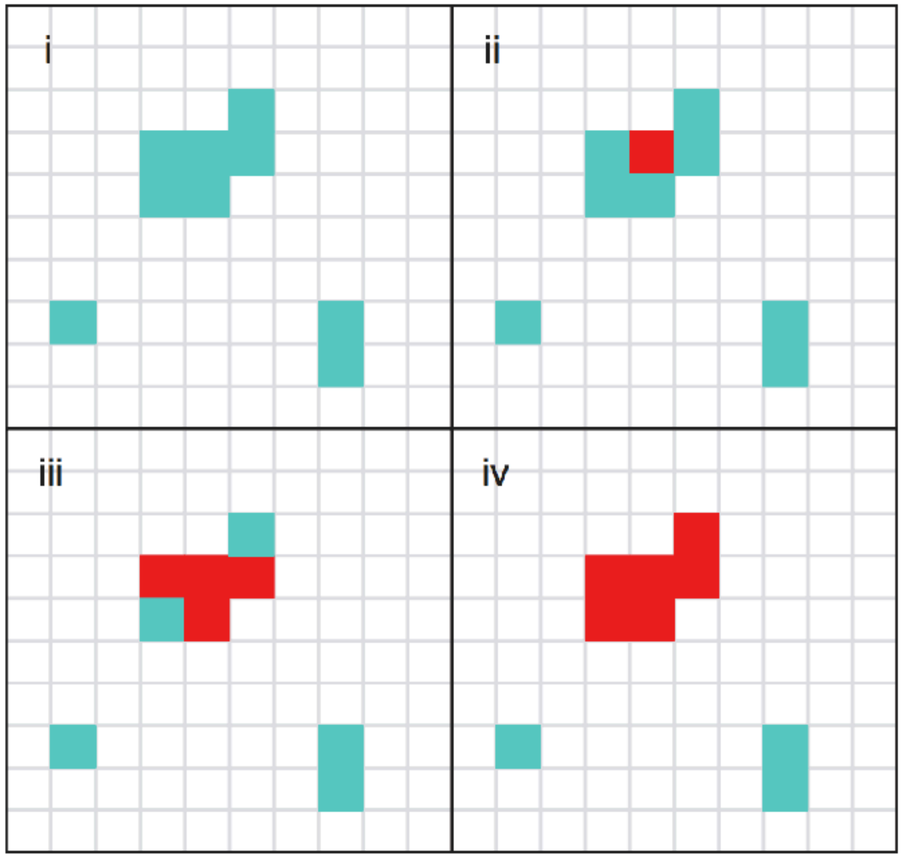
Boundaries are arbitrary.

They also change much slower than humans (e.g. sprawl).



What is a city? Data-driven attempt 1

City clustering algorithm



i) Consider populated cells only

ii) Select a random populated cell

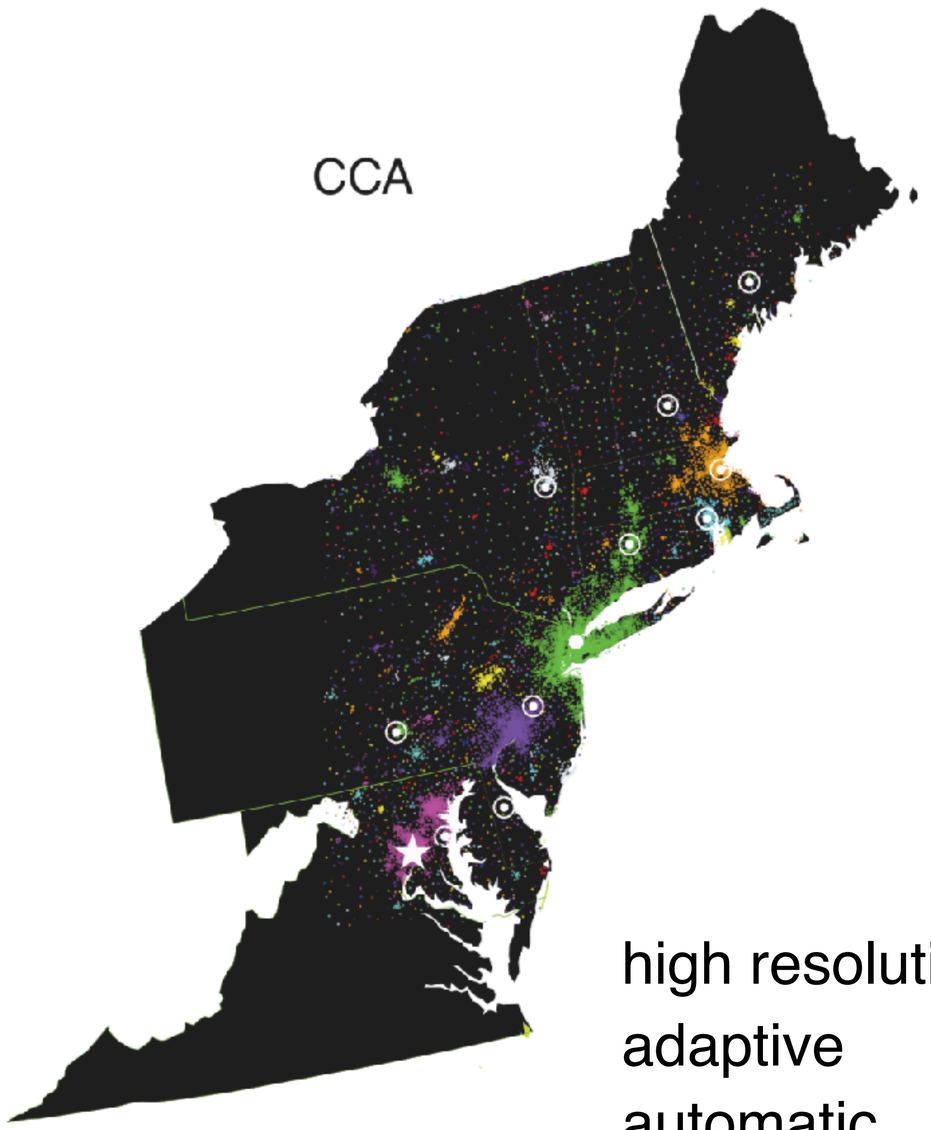
iii) Burn: Merge adjacent populated cells

iv) until all burnt

Bottom-up algorithm using high-resolution population data

Rozenfeld et al, *PNAS* 105.48 (2008)

What is a city? Data-driven attempt 1

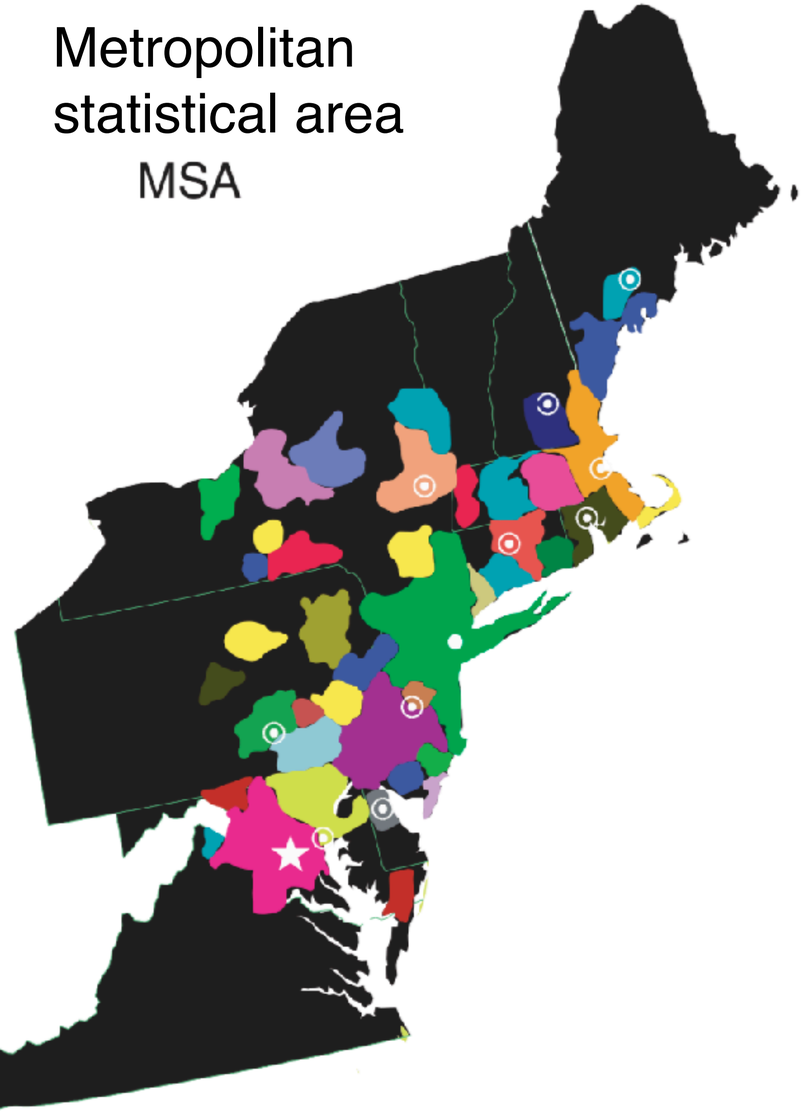


high resolution
adaptive
automatic
one parameter (can study its robustness)

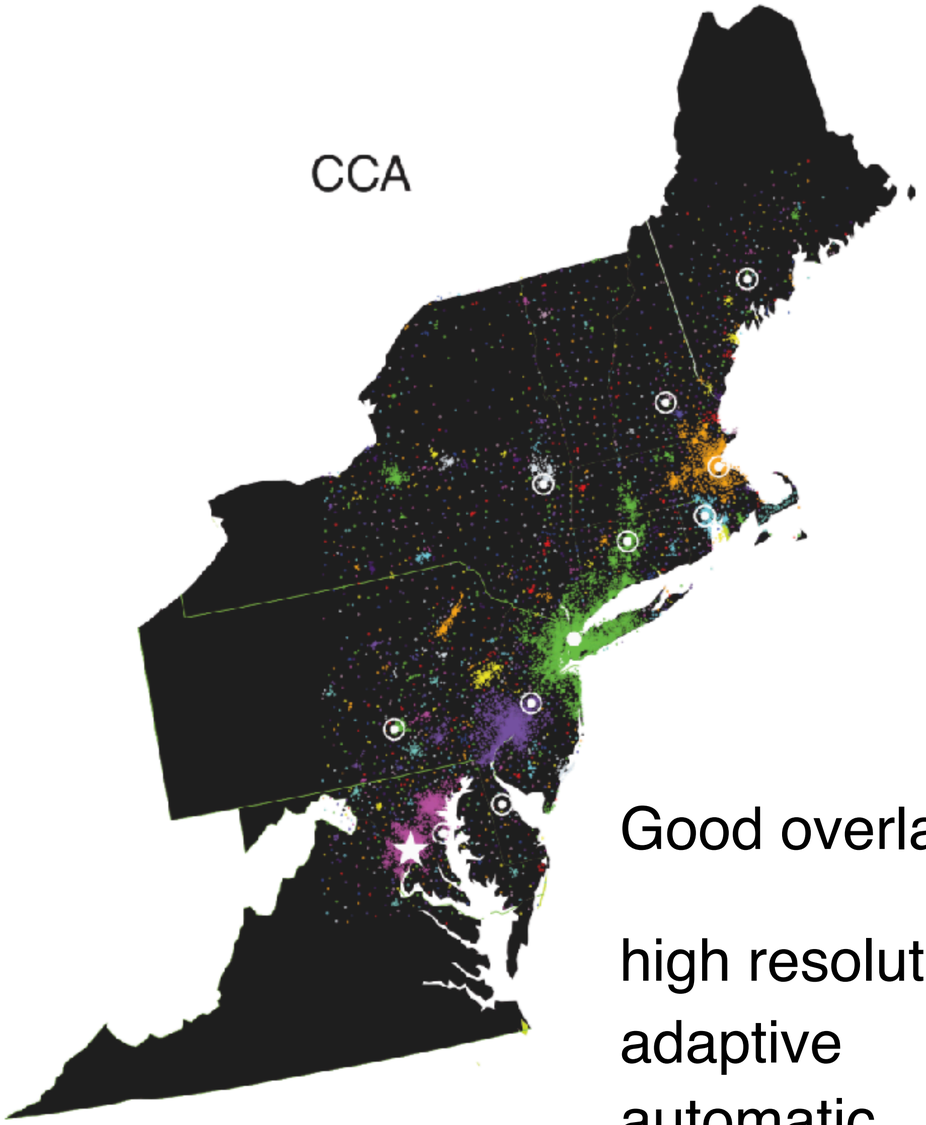
What is a city? Data-driven attempt 1

Metropolitan statistical area

MSA



CCA



Connect urban cores with high-commuter surroundings

Good overlap with some MSAs, plus:

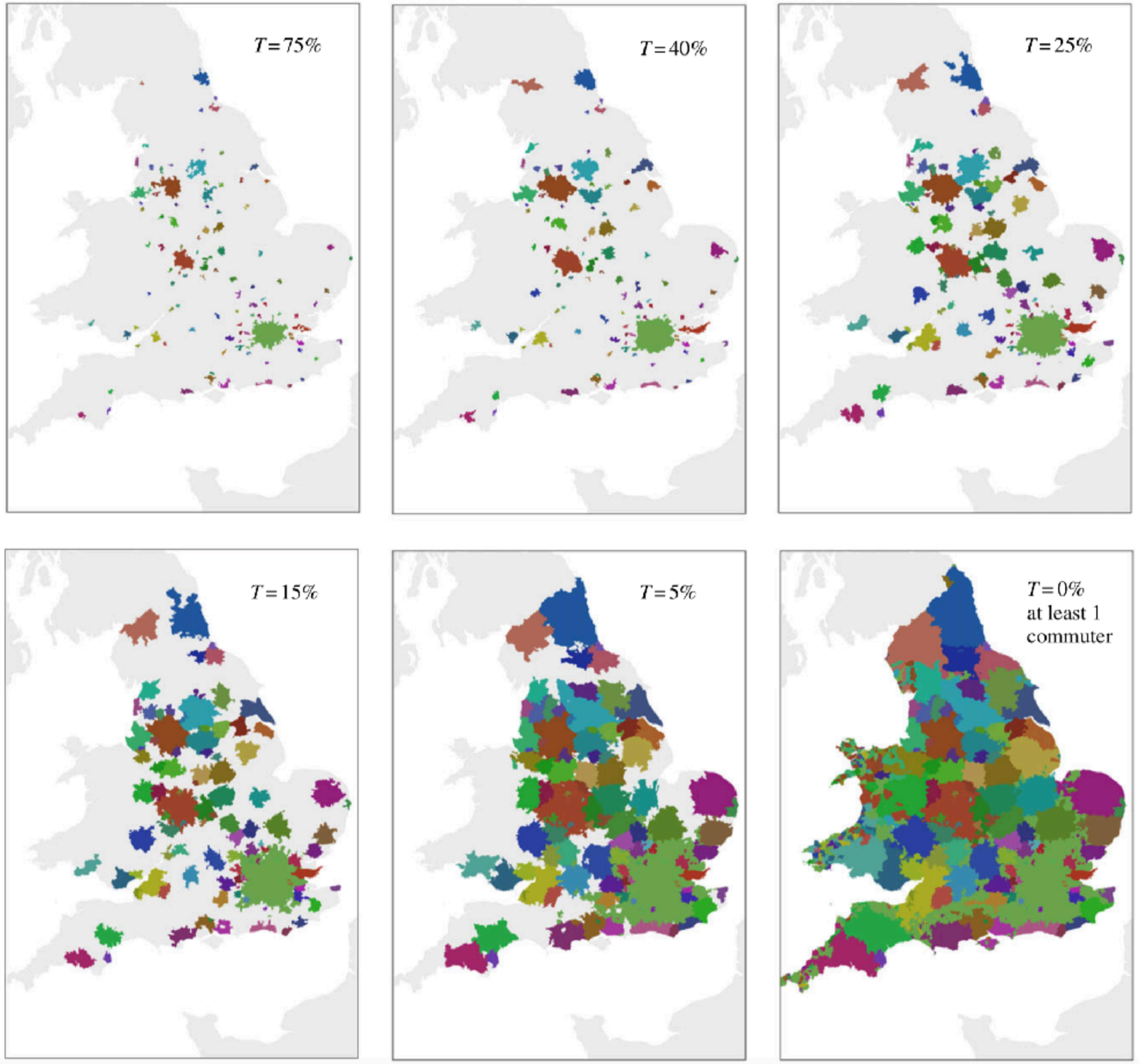
high resolution

adaptive

automatic

one parameter (can study its robustness)

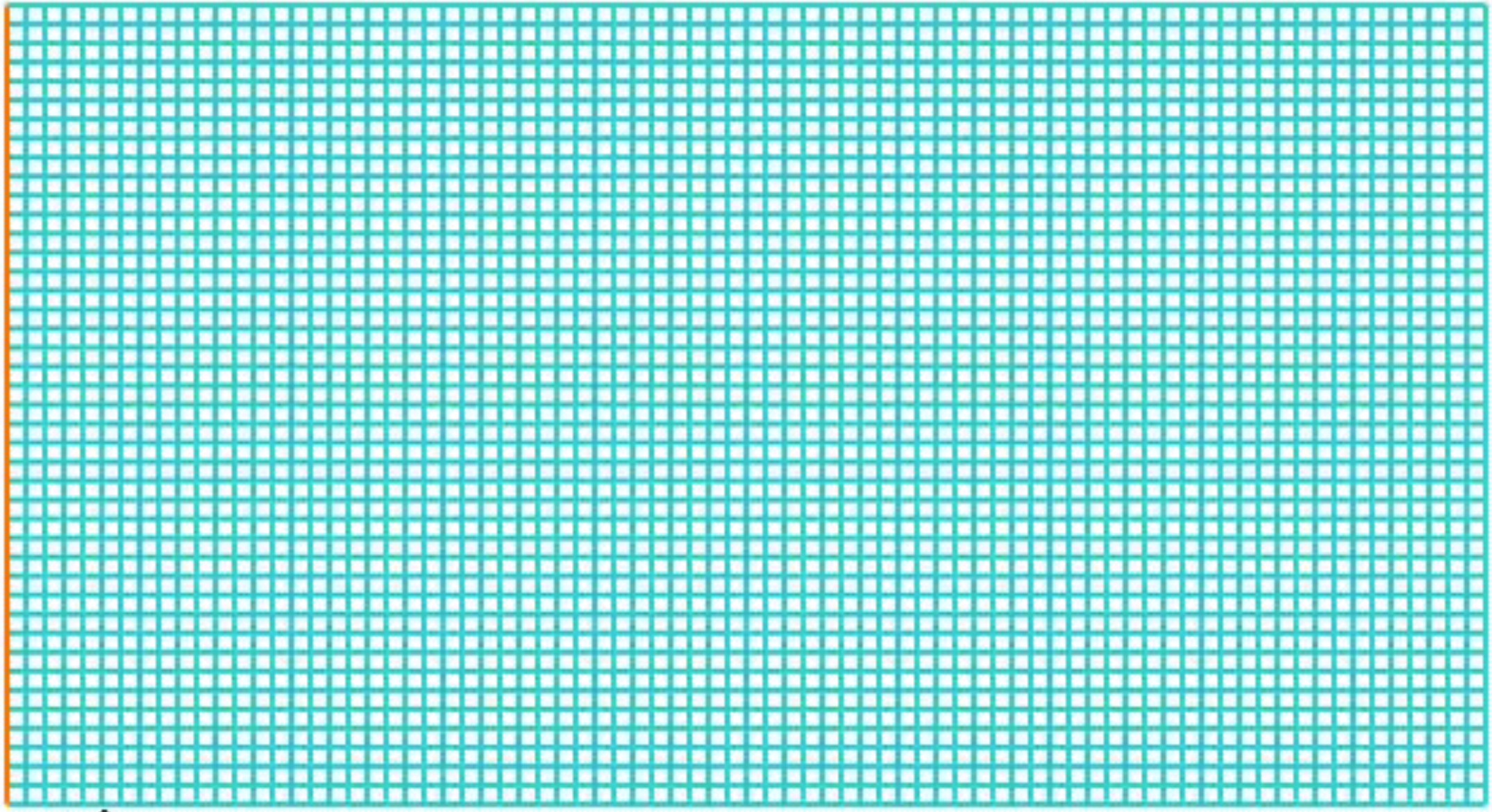
What is a city? Data-driven attempt 2



Building with commuter flows

Functional definition

Percolation transition: Emergence of a global property from piecewise growth



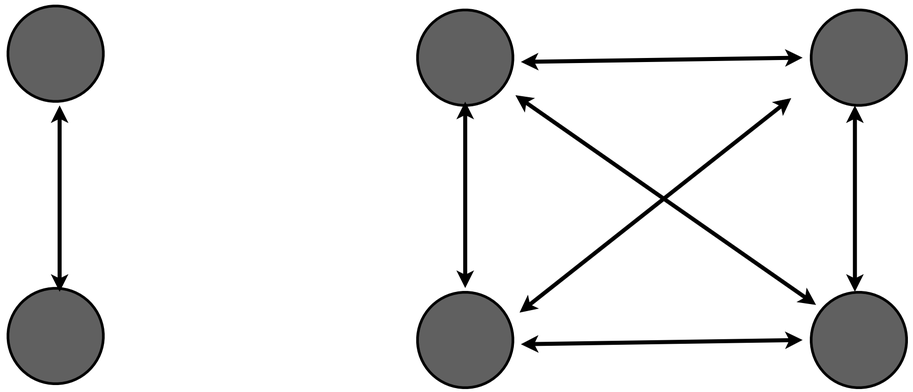
p = 0

https://www.youtube.com/watch?v=cI_B9iqsB9E

Why are there cities?

1) Protection, water, water transport

2) More opportunities

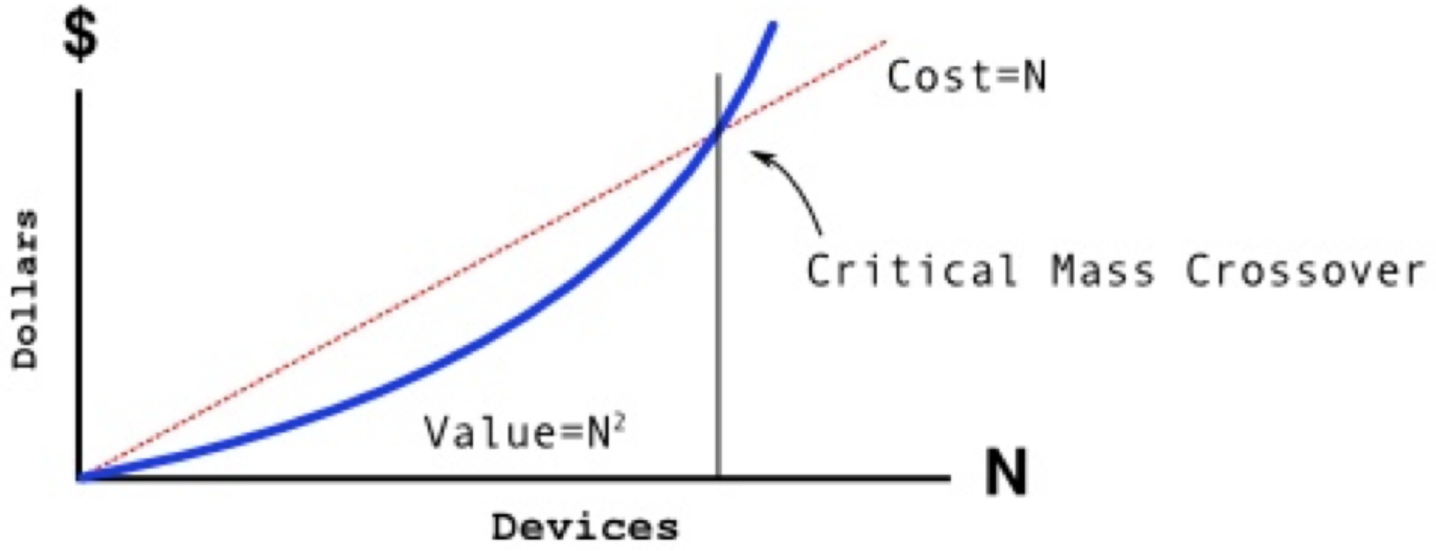


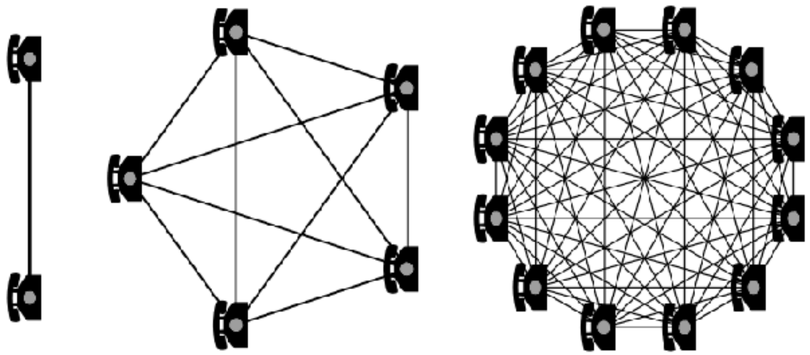
$$L = \mathcal{O}(N^2)$$

The person who bought the first fax,
what was he/she thinking???

Metcalfe's law

The value of a communication network increases with the square of users





$$L_{\max} = \frac{N(N-1)}{2} \sim N^2$$

More opportunities are not always good

Good things?

Bad things?

More opportunities are not always good

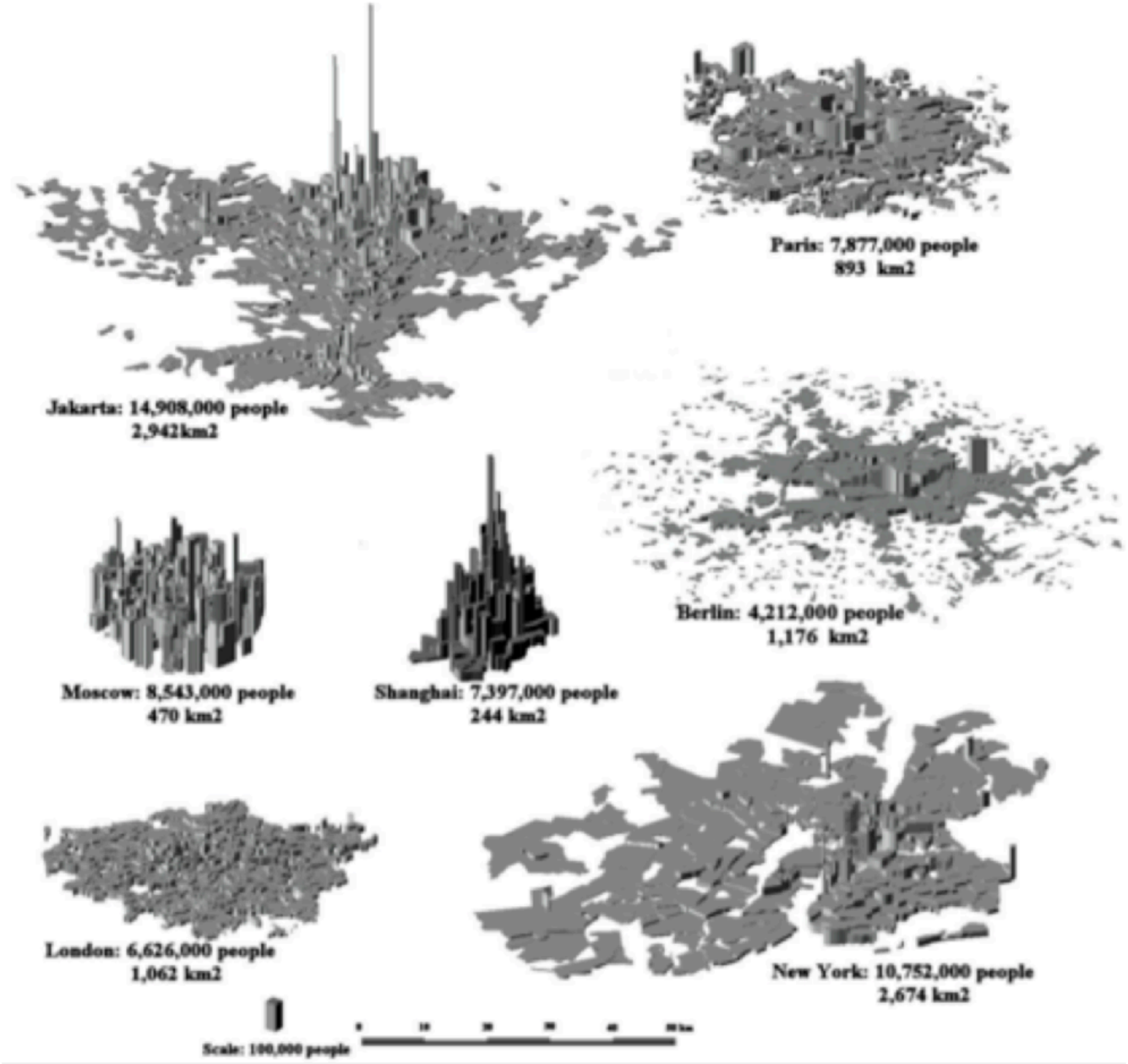
Trade, friendships, research,…

Crime, infectious diseases,…

Military: Cities are easy targets

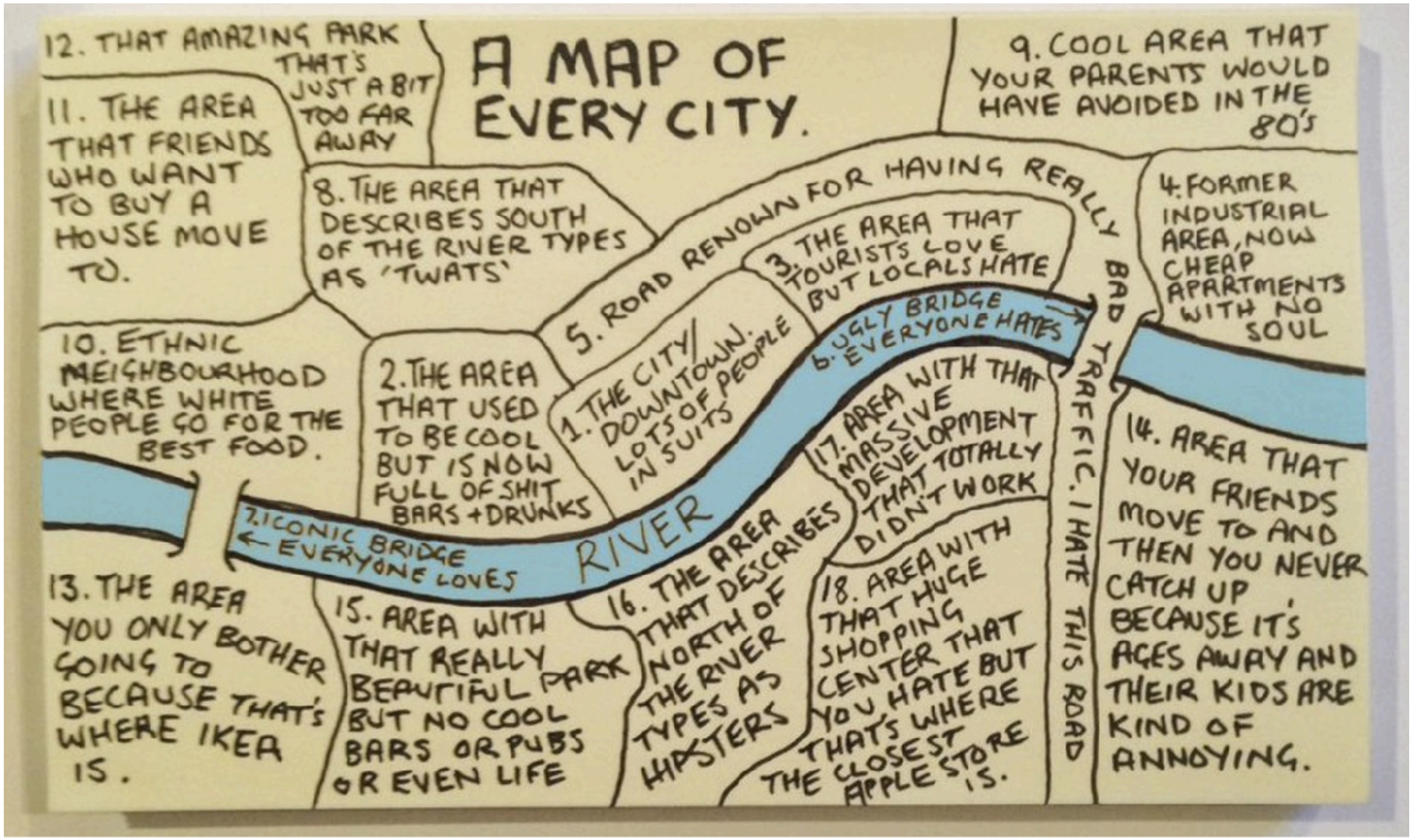
Cities only make sense if the benefits outweigh the downsides!

What do cities have in common?



Marc Barthelemy

What do cities have in common?



https://bigthink.com/strange-maps/familiarity-breeds-cartography-a-map-of-every-city

Is New York just a scaled up Venice?

= 8 x

?

Is New York just a scaled up Venice?

= 8 x

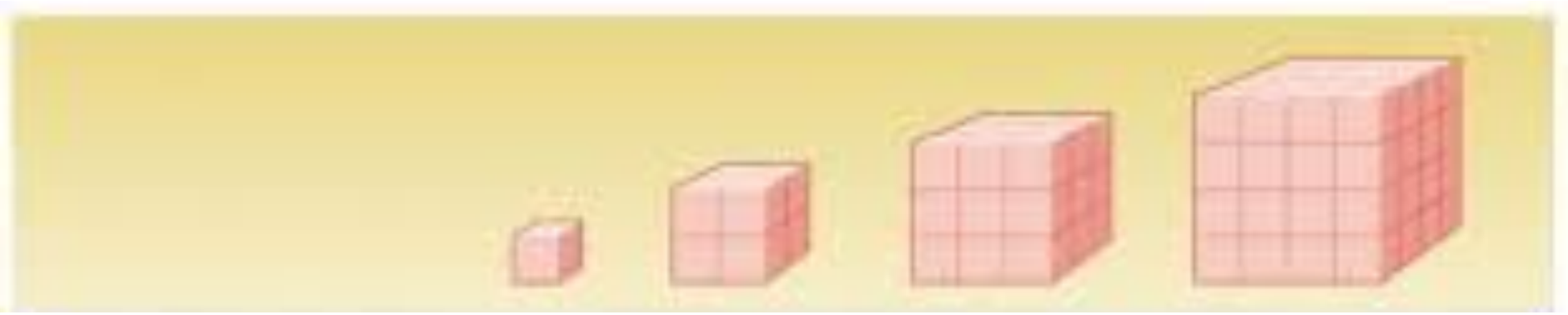
?

Once we control for place-specific peculiarities, can we construct a theory that focuses only on physical features?

Scaling

In isometric scaling proportions are preserved

"The big thing is the small thing blown up evenly"



Length L	1	2	3	4
Volume V	1	8	27	64

https://en.wikipedia.org/wiki/Allometry

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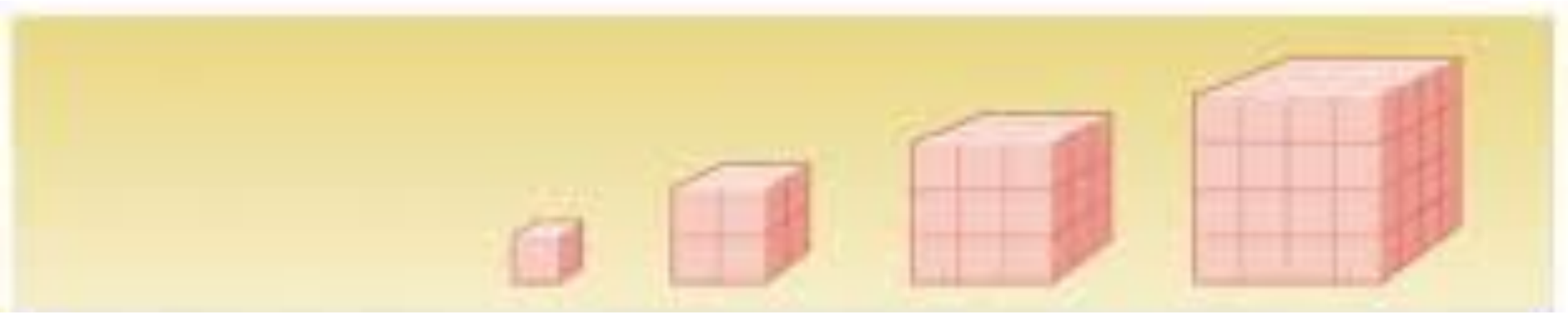

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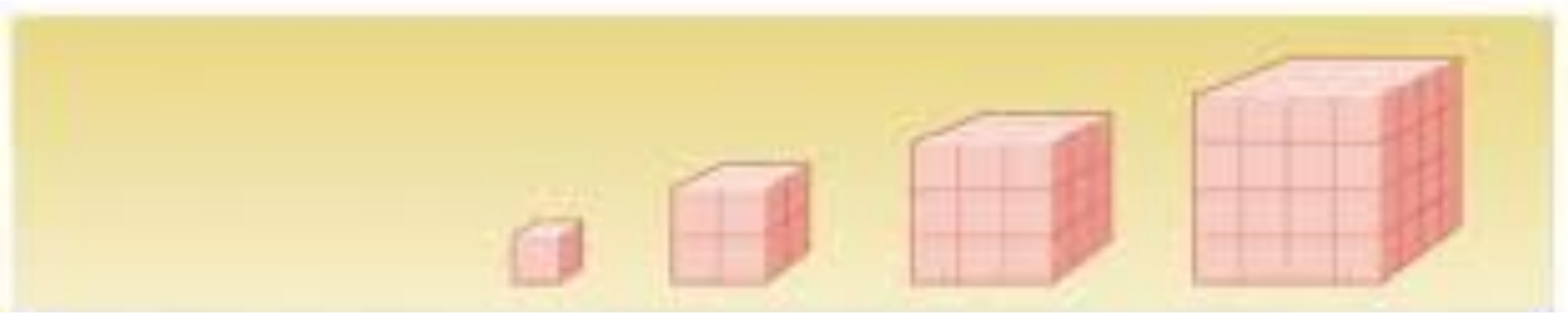

Length L	1	2	3	4
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$$V = L^3$$

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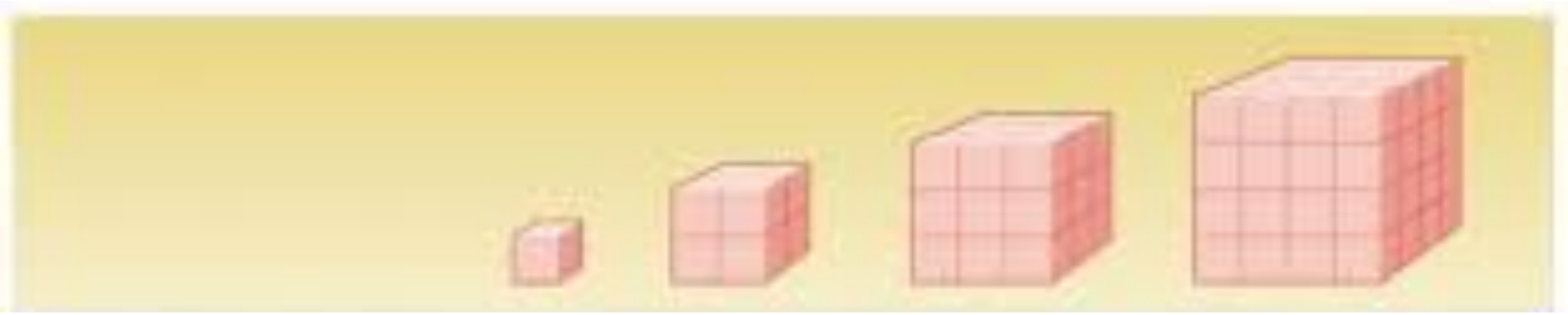


Length L	1	2	3	4
Volume V	1	8	27	64
Surface Area A	6	24	54	96

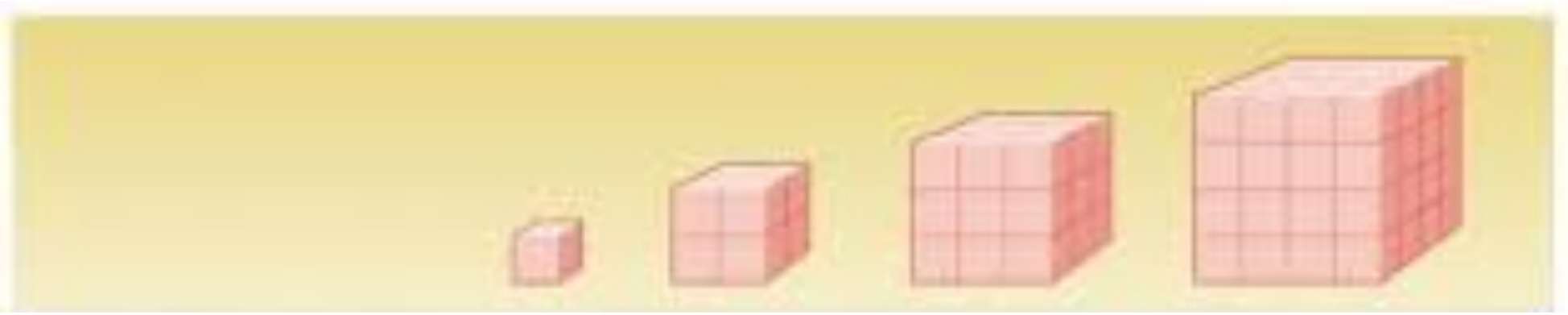
$V = L^3$

?

https://en.wikipedia.org/wiki/Allometry

In isometric scaling proportions are preserved

"The big thing is the small thing blown up evenly"



Length L	1	2	3	4
Volume V	1	8	27	64
Surface Area A	6	24	54	96

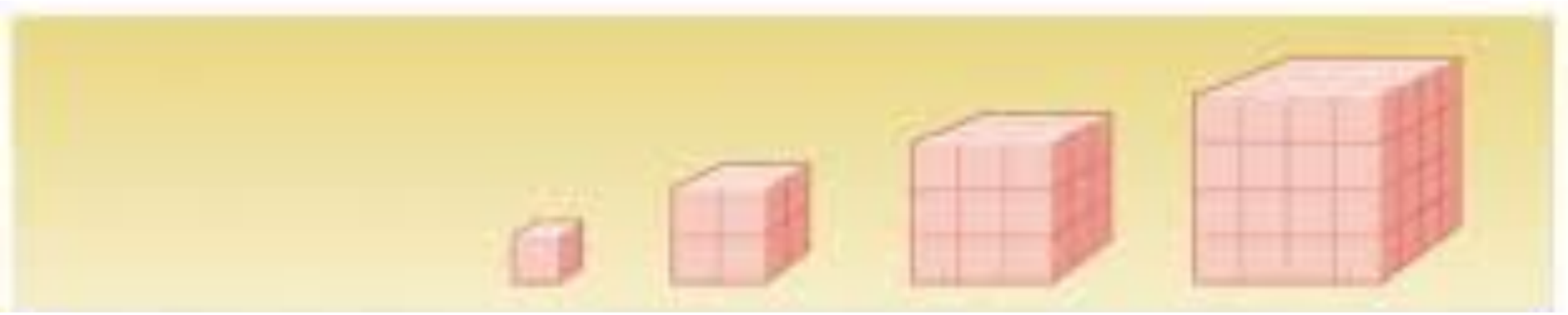
$V = L^3$

$A = 6L^2$

https://en.wikipedia.org/wiki/Allometry

In isometric scaling proportions are preserved

"The big thing is the small thing blown up evenly"



Length L	1	2	3	4
Volume V	1	8	27	64
Surface Area A	6	24	54	96

$V = L^3$

$A = 6L^2$

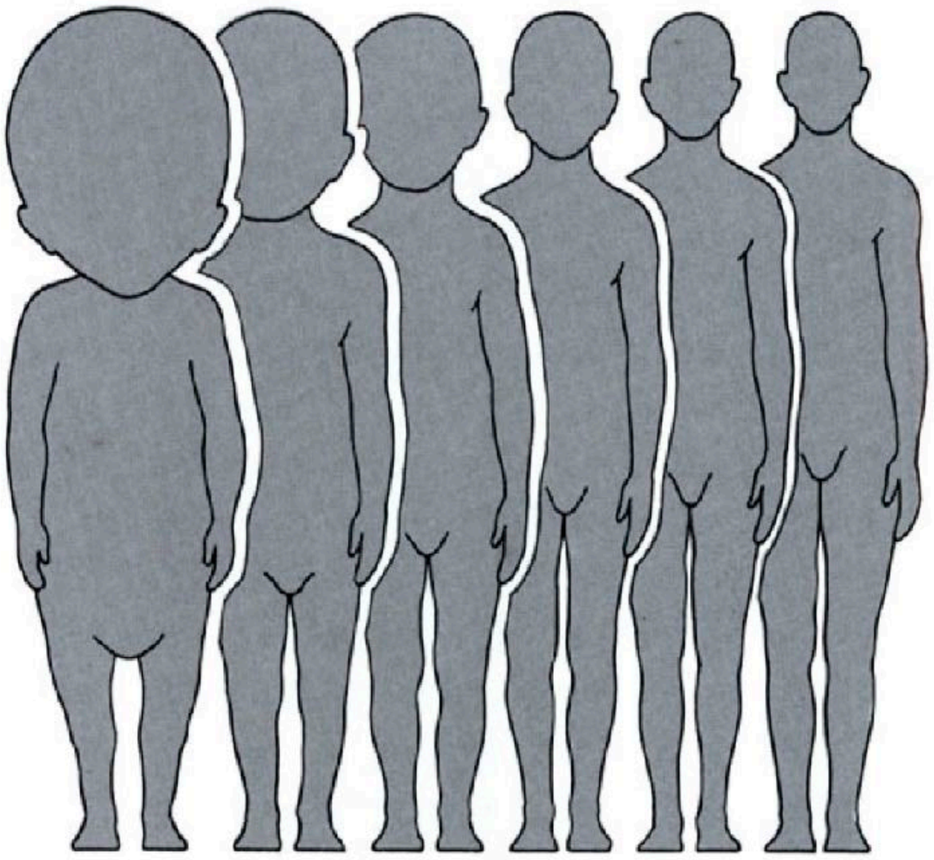
The mismatch between square and cube growth causes problems for an organism's growth, for example tissue to support vs surface area of the respiratory organs

$$A \propto V^{\frac{2}{3}}$$

https://en.wikipedia.org/wiki/Allometry

In allometric scaling proportions are not preserved

"The big thing is the small thing blown up unevenly"



https://en.wikipedia.org/wiki/Allometry

Allometry

Allometry is the study of the relationship of body size to shape, anatomy, physiology and finally behavior.

(Snell, 1892)

Can be within one species
Can be between different species

https://en.wikipedia.org/wiki/Allometry

Allometry

$$y \propto x^a$$

$$\ln y = a \ln x$$

a scaling exponent

allometric coefficient

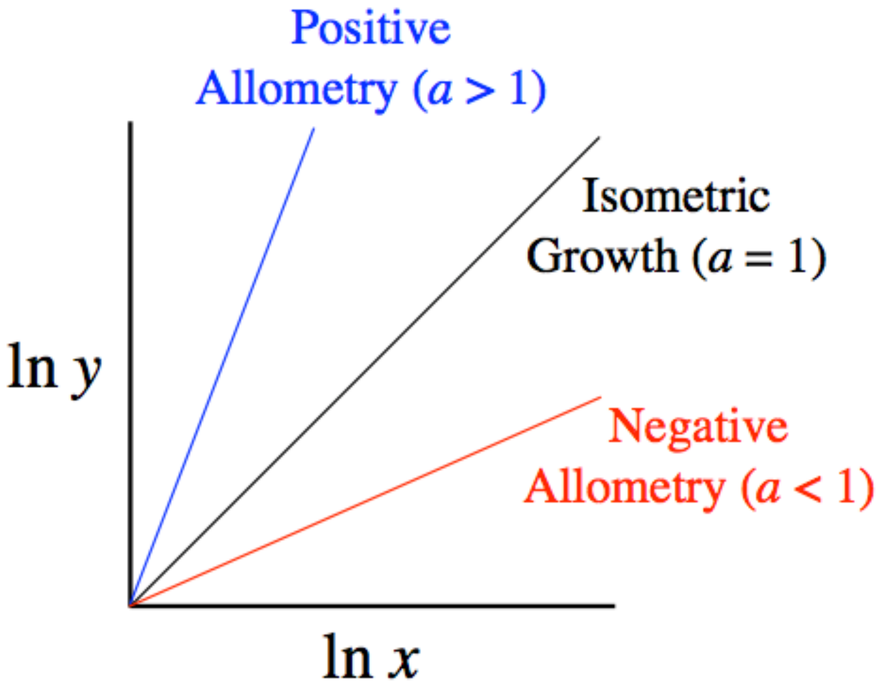
Allometry

$$y \propto x^a$$

$$\ln y = a \ln x$$

a scaling exponent

allometric coefficient



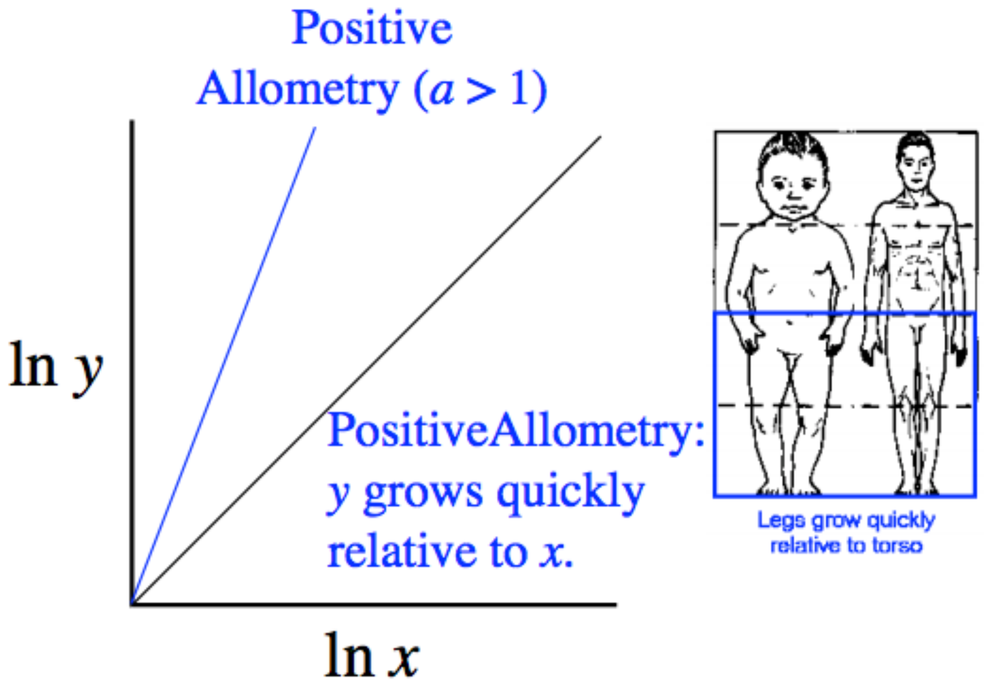
Positive allometry

$$y \propto x^a$$

$$\ln y = a \ln x$$

a scaling exponent

allometric coefficient



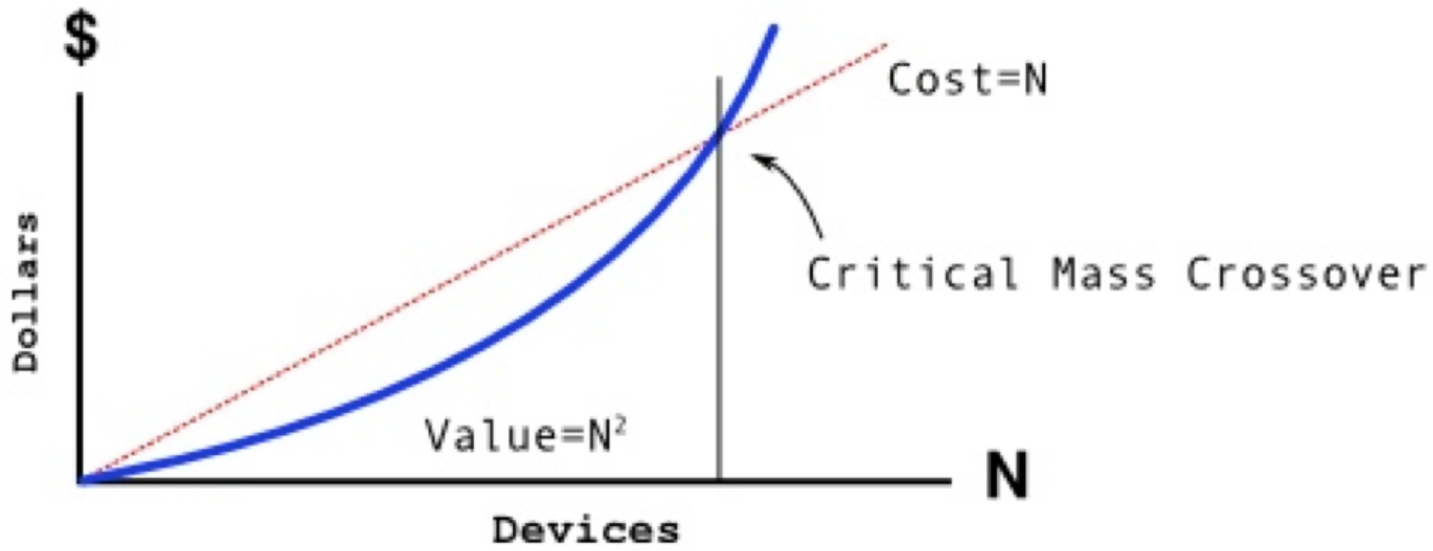
Positive allometry: Network effects

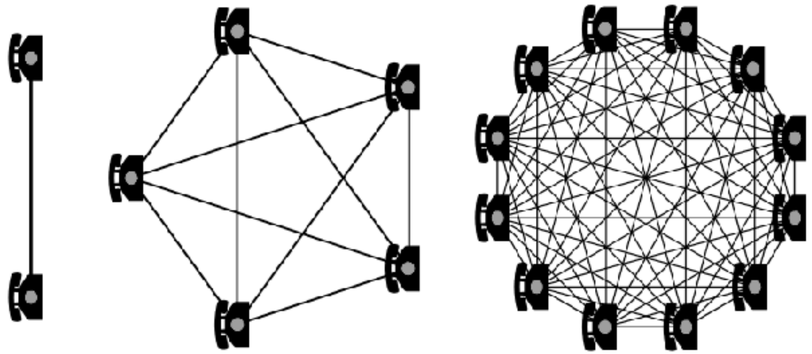
$$y \propto x^a$$

$$\ln y = a \ln x$$

a scaling exponent

allometric coefficient





$a = 2$

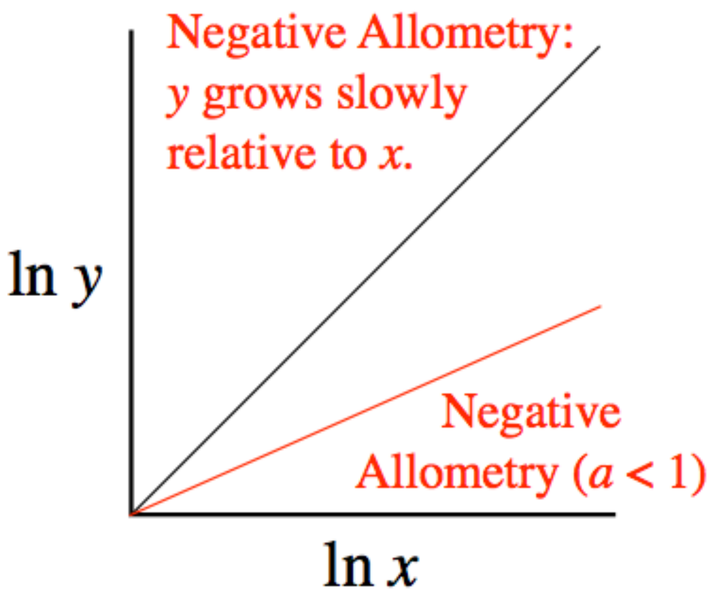
Negative allometry

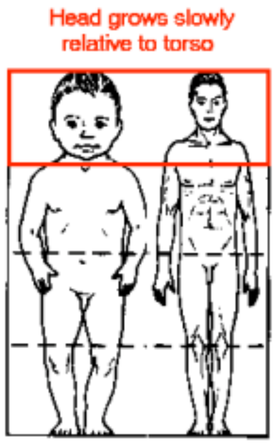
$$y \propto x^a$$

$$\ln y = a \ln x$$

a scaling exponent

allometric coefficient





Negative allometry: Economies of scale

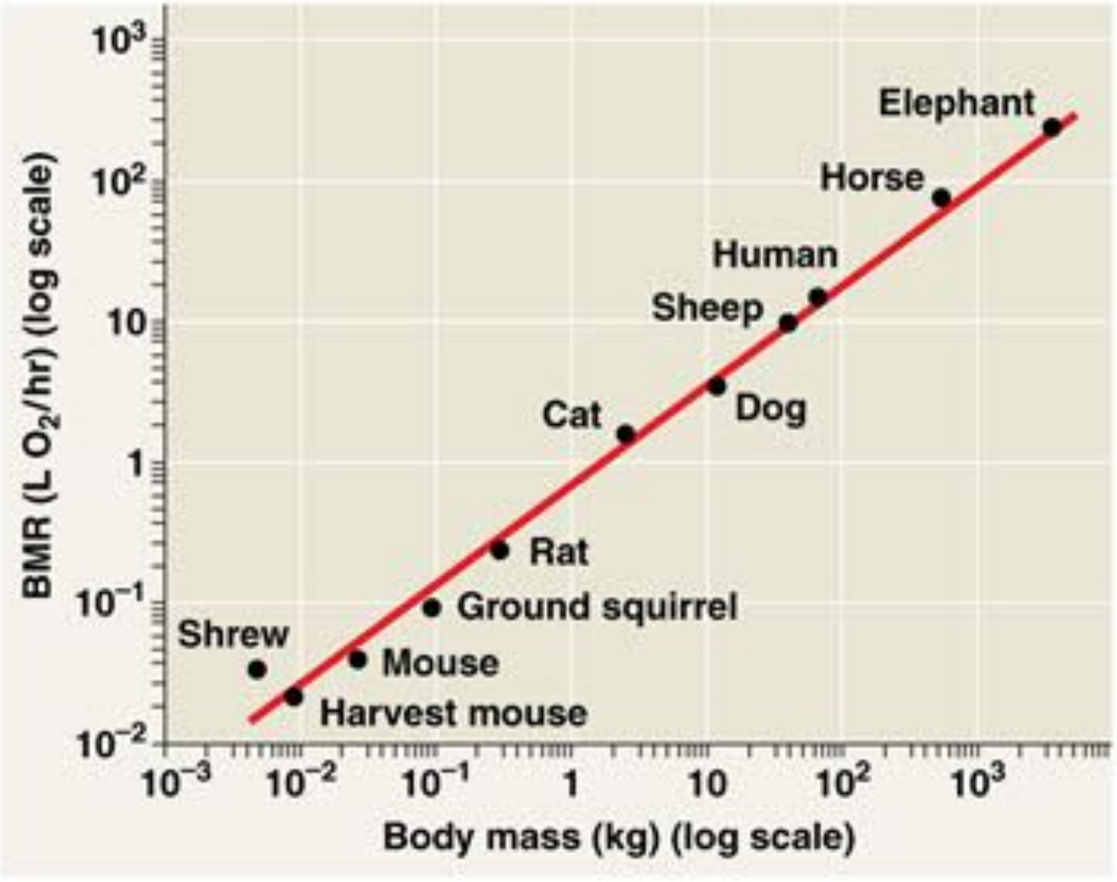
$$y \propto x^a$$

$$\ln y = a \ln x$$

a scaling exponent

allometric coefficient

Negative allometry: Bigger animals are more efficient



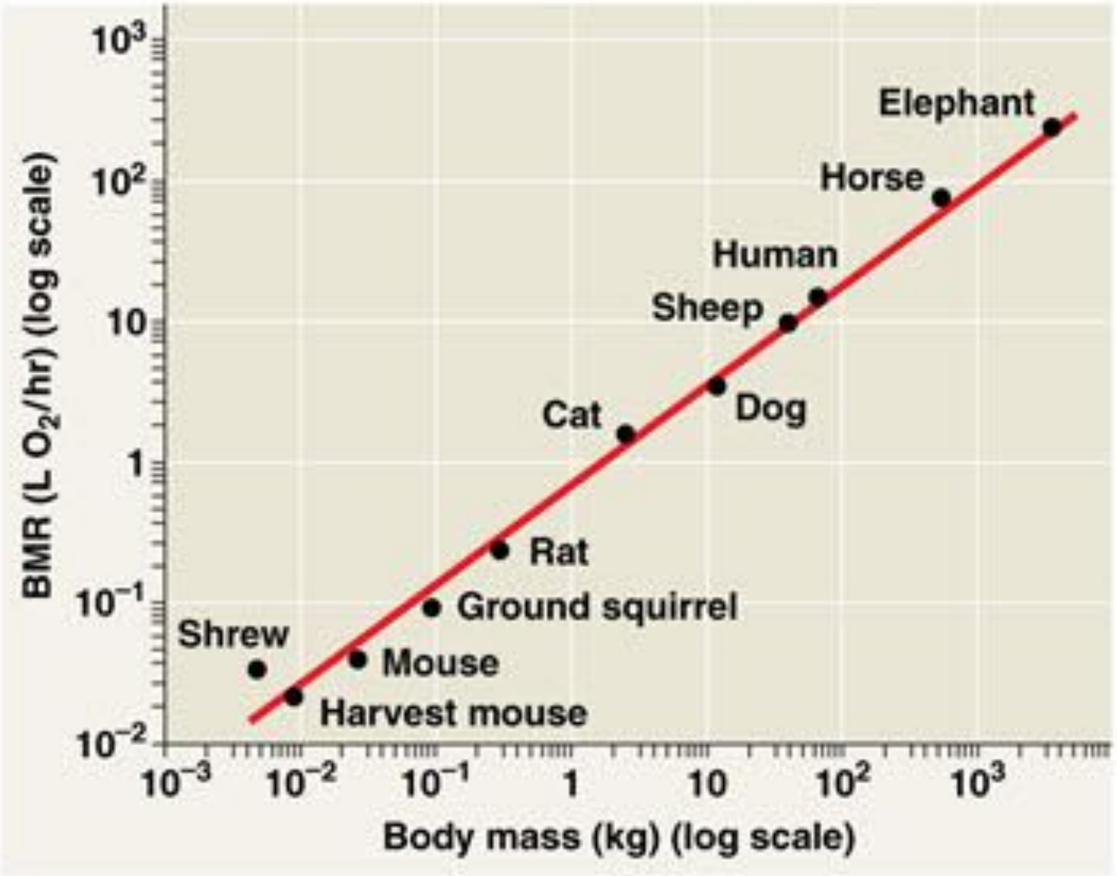
(a) Relationship of basal metabolic rate (BMR) to body size for various mammals

© 2011 Pearson Education, Inc.

Mouse to elephant curve

$$\mathrm{BMR} \propto \mathrm{BM}^{\frac{3}{4}}$$

Negative allometry: Bigger animals are more efficient



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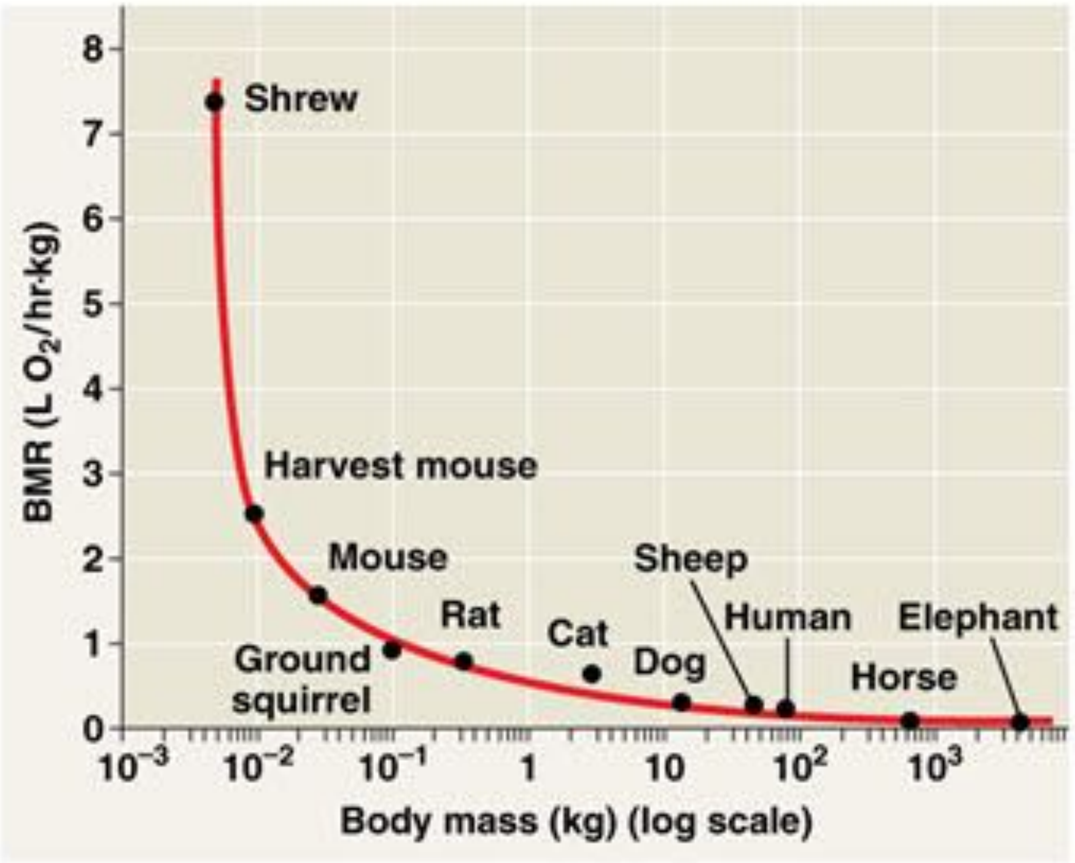


(b) Relationship of BMR per kilogram of body mass to body size

Mouse to elephant curve

$$\mathrm{BMR} \propto \mathrm{BM}^{\frac{3}{4}}$$

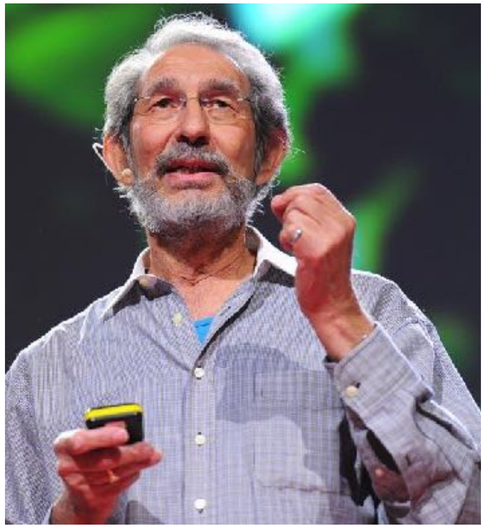
Bigger animals
are more efficient

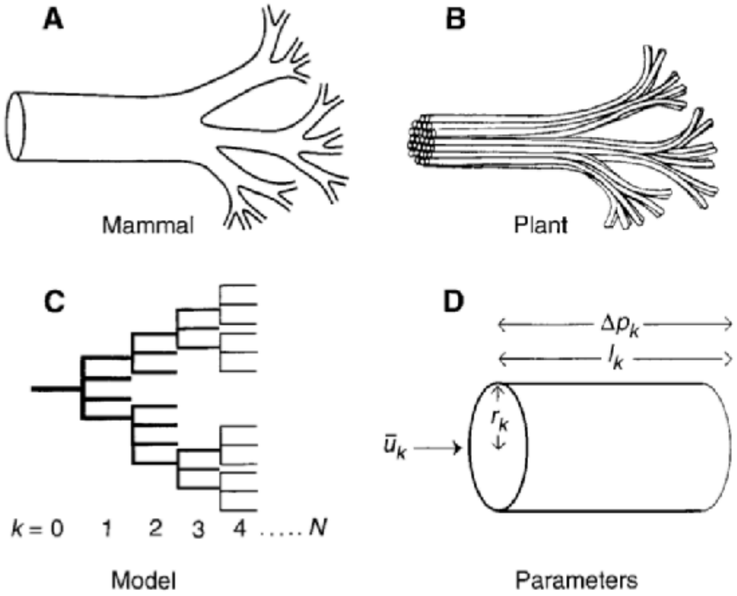
Negative allometry: Bigger animals are more efficient

A General Model for the Origin of Allometric Scaling Laws in Biology

Geoffrey B. West, James H. Brown,* Brian J. Enquist

1997





Cardiovascular			Respiratory		
Variable	Exponent		Variable	Exponent	
	Predicted	Observed		Predicted	Observed
Aorta radius r_0	3/8 = 0.375	0.36	Tracheal radius	3/8 = 0.375	0.39
Aorta pressure Δp_0	0 = 0.00	0.032	Interpleural pressure	0 = 0.00	0.004
Aorta blood velocity u_0	0 = 0.00	0.07	Air velocity in trachea	0 = 0.00	0.02
Blood volume V_b	1 = 1.00	1.00	Lung volume	1 = 1.00	1.05
Circulation time	1/4 = 0.25	0.25	Volume flow to lung	3/4 = 0.75	0.80
Circulation distance l	1/4 = 0.25	ND	Volume of alveolus V_A	1/4 = 0.25	ND
Cardiac stroke volume	1 = 1.00	1.03	Tidal volume	1 = 1.00	1.041
Cardiac frequency ω	−1/4 = −0.25	−0.25	Respiratory frequency	−1/4 = −0.25	−0.26
Cardiac output $\dot{E}$	3/4 = 0.75	0.74	Power dissipated	3/4 = 0.75	0.78
Number of capillaries N_c	3/4 = 0.75	ND	Number of alveoli N_A	3/4 = 0.75	ND
Service volume radius	1/12 = 0.083	ND	Radius of alveolus r_A	1/12 = 0.083	0.13
Womersley number α	1/4 = 0.25	0.25	Area of alveolus A_A	1/6 = 0.083	ND
Density of capillaries	−1/12 = −0.083	−0.095	Area of lung A_L	11/12 = 0.92	0.95
O_2 affinity of blood P_{50}	−1/12 = −0.083	−0.089	O_2 diffusing capacity	1 = 1.00	0.99
Total resistance Z	−3/4 = −0.75	−0.76	Total resistance	−3/4 = −0.75	−0.70
Metabolic rate B	3/4 = 0.75	0.75	O_2 consumption rate	3/4 = 0.75	0.76

Urban scaling

Allometric scaling in cities

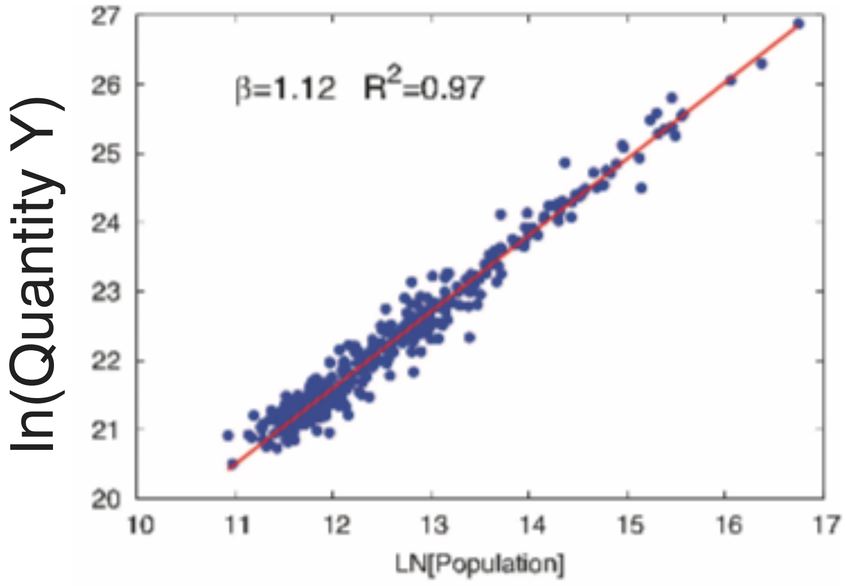
Growth, innovation, scaling, and the pace of life in cities

Luís M. A. Bettencourt*†, José Lobo‡, Dirk Helbing§, Christian Kühnert§, and Geoffrey B. West*¶

Idea: 1) Plot quantity Y versus population P for many cities

2) Measure the slope β in $Y \sim P^{\beta}$

3) Do we have isometry or negative/positive allometry?



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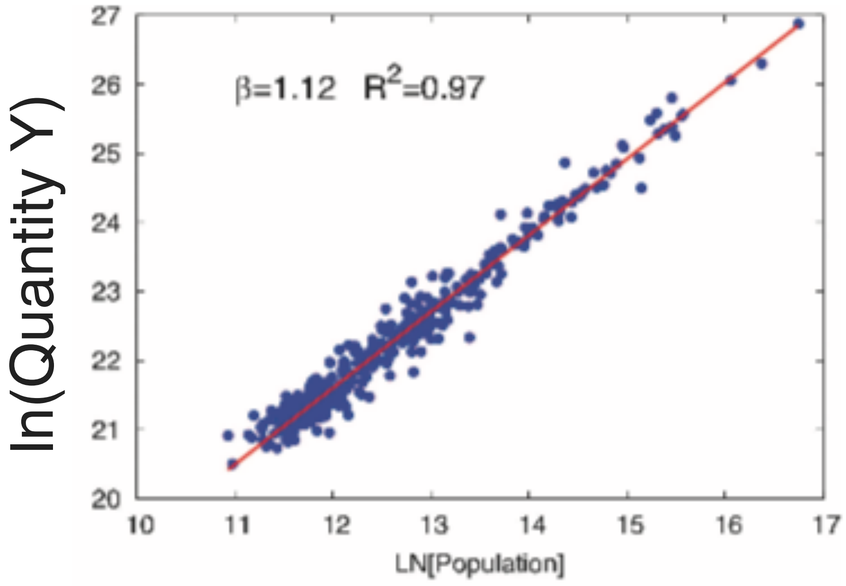
4) Why?



? scaling example: Streets

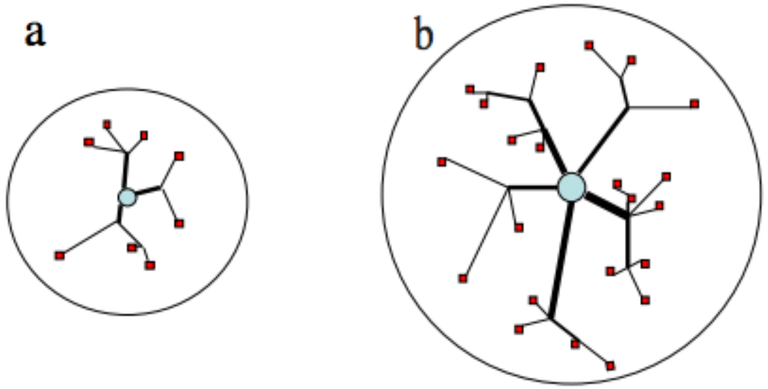
Journal of Transport and Land Use 1:1 (Summer 2008) pp. 21–39
Available at http://jtlu.org

JTLU

Cities as organisms: Allometric scaling of urban road networks

Horacio Samaniego
Instituto de Silvicultura, Universidad Austral de Chile [a]

Melanie E. Moses
University of New Mexico [b]



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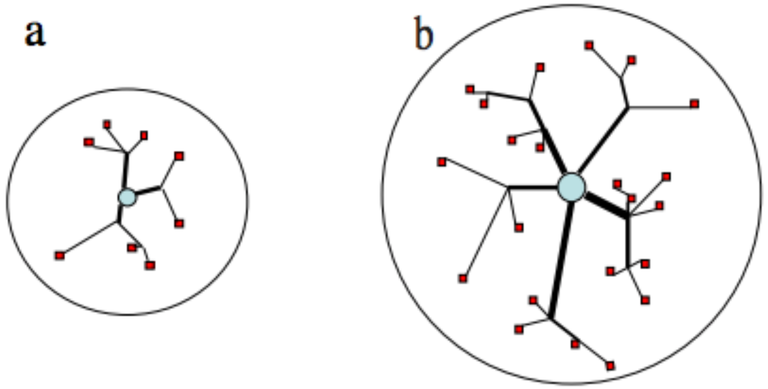


? scaling examples

Housing

Consumption

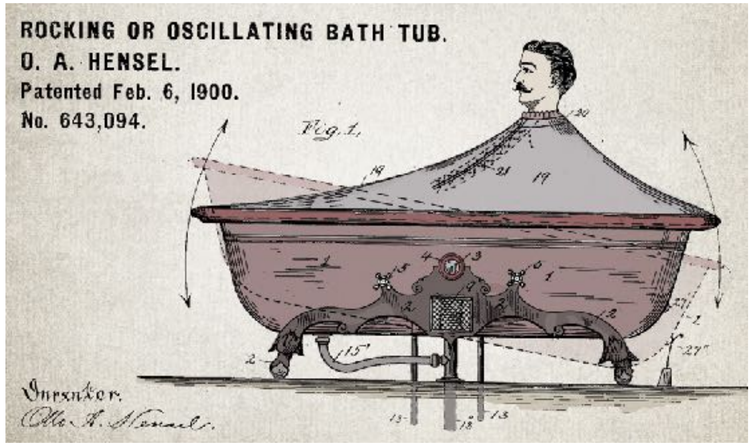
Isometric scaling examples

Housing

Consumption

? scaling examples

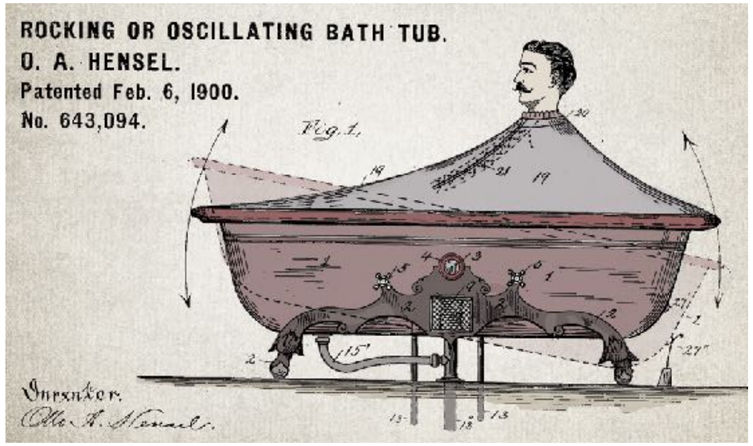
Patents / Research



Wages / GDP

Positive allometric scaling examples

Patents / Research



Wages / GDP

Urban quantities have different allometric scaling

	Y	β
superlinear	New patents	1.27
	Inventors	1.25
	Private R&D employment	1.34
	"Supercreative" employment	1.15
	R&D establishments	1.19
	R&D employment	1.26
	Total wages	1.12
	Total bank deposits	1.08
	GDP	1.15
	GDP	1.26
	GDP	1.13
	Total electrical consumption	1.07
	New AIDS cases	1.23
	Serious crimes	1.16
linear	Total housing	1.00
	Total employment	1.01
	Household electrical consumption	1.00
	Household electrical consumption	1.05
	Household water consumption	1.01
sublinear	Gasoline stations	0.77
	Gasoline sales	0.79
	Length of electrical cables	0.87
	Road surface	0.83

Urban quantities have different allometric scaling

	Y	β	
superlinear	New patents	1.27	good stuff
	Inventors	1.25	
	Private R&D employment	1.34	
	"Supercreative" employment	1.15	
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linear	Total housing	1.00	
	Total employment	1.01	
	Household electrical consumption	1.00	
	Household electrical consumption	1.05	
	Household water consumption	1.01	
sublinear	Gasoline stations	0.77	
	Gasoline sales	0.79	
	Length of electrical cables	0.87	
	Road surface	0.83	

Human interactions have positive allometric scaling

	Y	β
superlinear	New patents	1.27
	Inventors	1.25
	Private R&D employment	1.34
	"Supercreative" employment	1.15
	R&D establishments	1.19
	R&D employment	1.26
	Total wages	1.12
	Total bank deposits	1.08
	GDP	1.15
	GDP	1.26
	GDP	1.13
	Total electrical consumption	1.07
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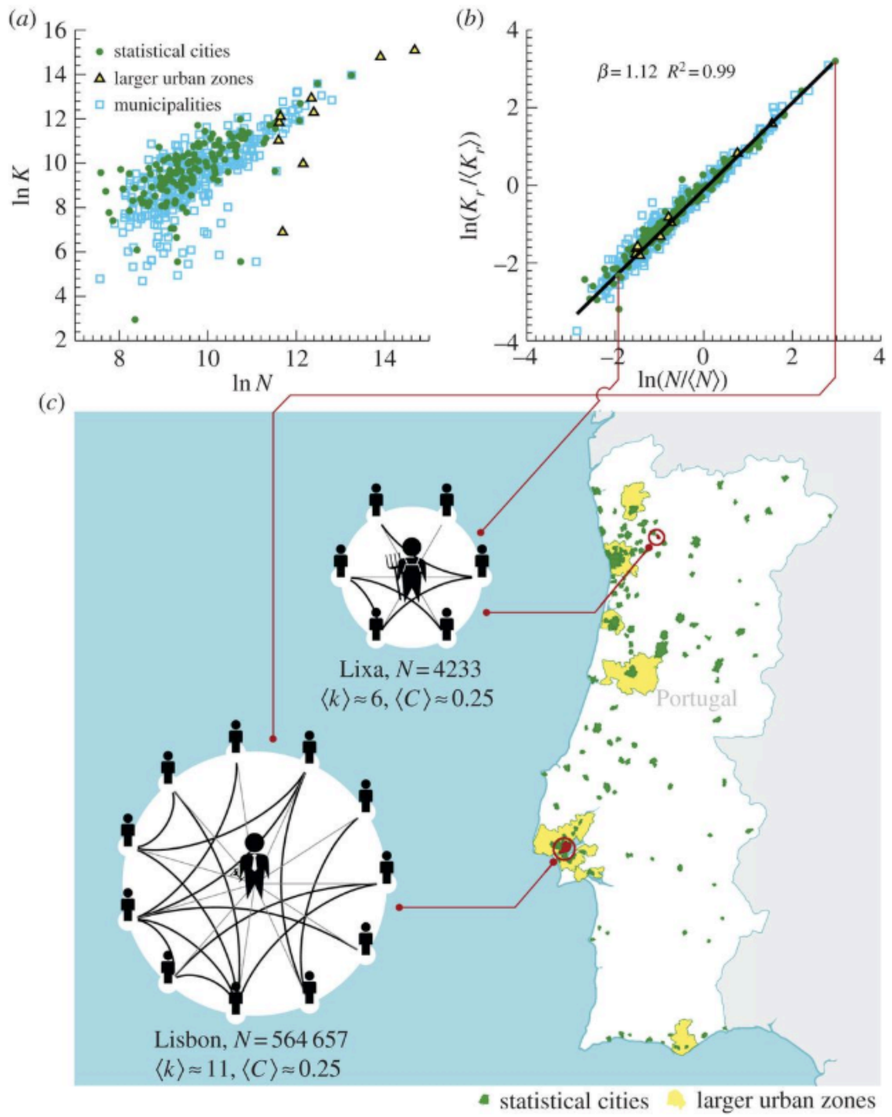
	Serious crimes	1.16
linear	Total housing	1.00
	Total employment	1.01
	Household electrical consumption	1.00
	Household electrical consumption	1.05
	Household water consumption	1.01
sublinear	Gasoline stations	0.77
	Gasoline sales	0.79
	Length of electrical cables	0.87
	Road surface	0.83

good stuff

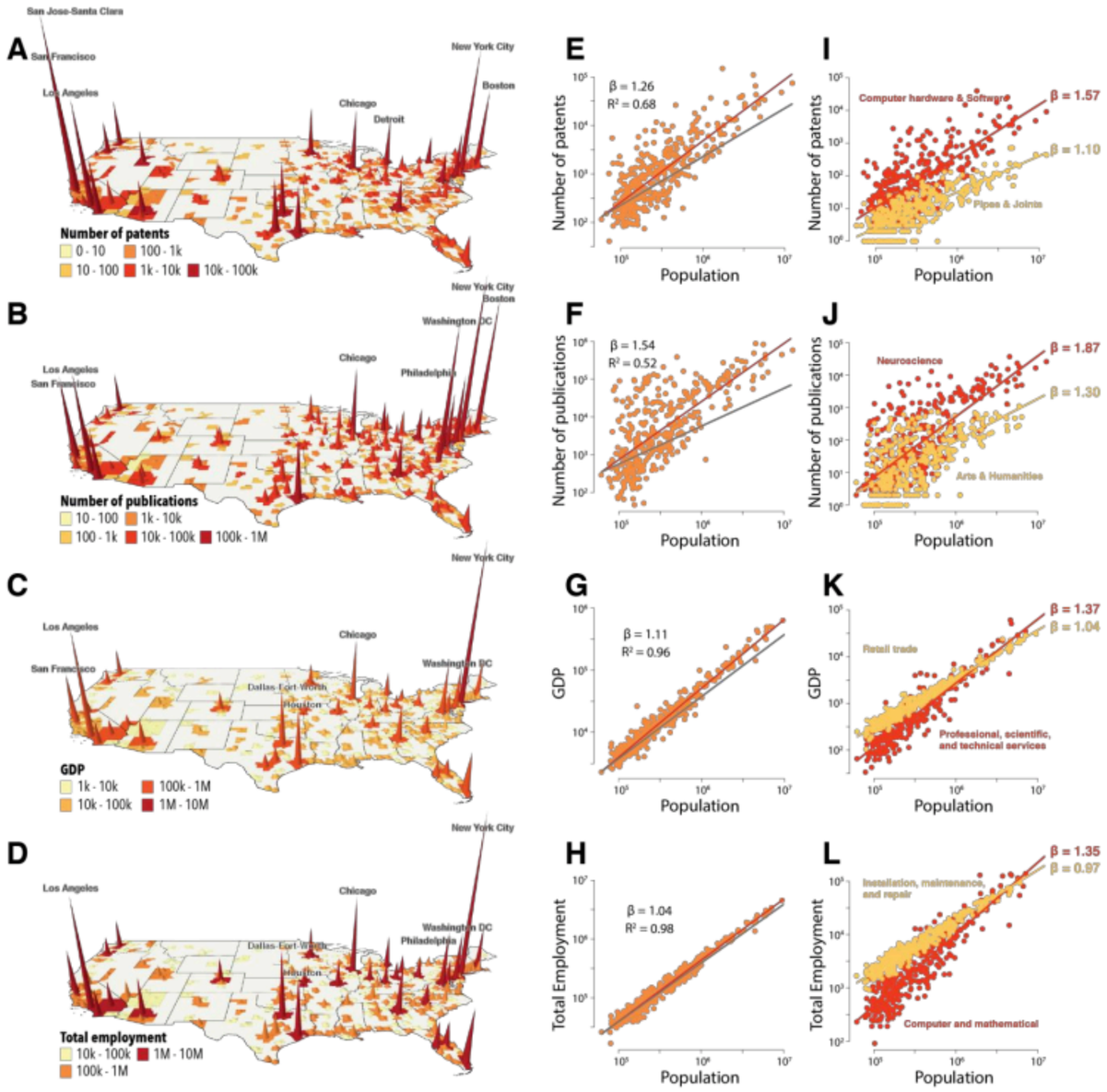
bad stuff



Schläpfer et al. *Journal of the Royal Society Interface* 11.98 (2014)

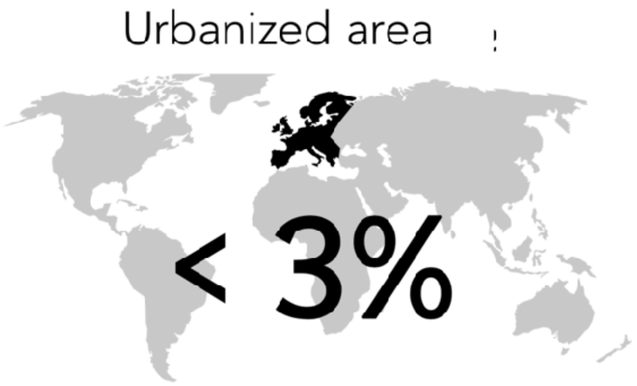
Economic activities have positive allometric scaling

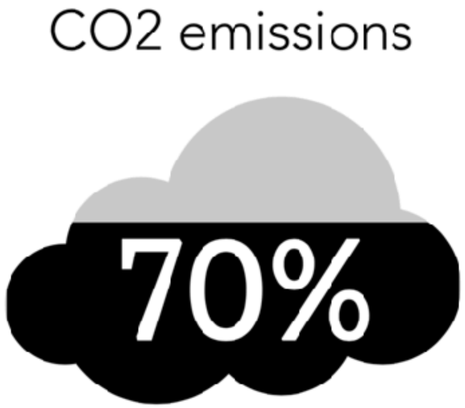
Also they cluster



Balland et al. *Nature human behaviour* 4.3 (2020)

Cities are the socio-economic "reactors" of society





ONU-HABITAT 2011

ONU 2011

Marc Barthelemy

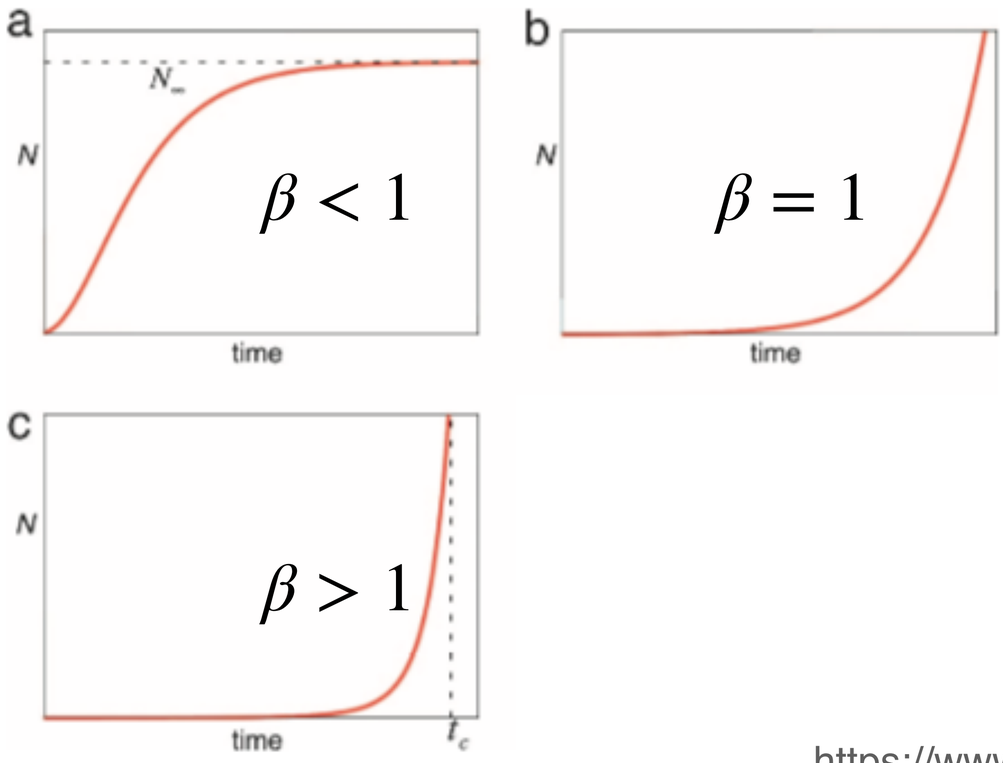
The dynamical implications of scaling in cities

How can cities keep growing and scaling superlinearly?

The dynamical implications of scaling in cities

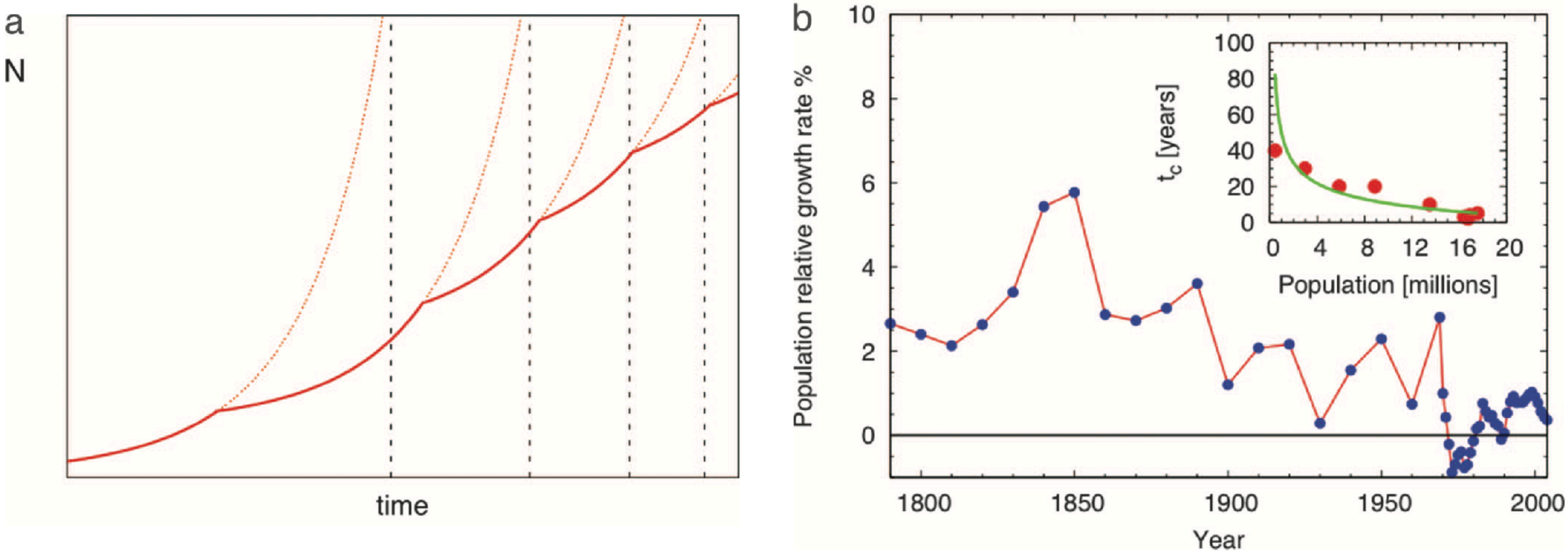
Table 2. Classification of scaling exponents for urban properties and their implications for growth

Scaling exponent	Driving force	Organization	Growth
$\beta < 1$	Optimization, efficiency	Biological	Sigmoidal: long-term population limit
$\beta > 1$	Creation of information, wealth and resources	Sociological	Boom/collapse: finite-time singularity/unbounded growth; accelerating growth rates/discontinuities
$\beta = 1$	Individual maintenance	Individual	Exponential



https://www.pnas.org/doi/abs/10.1073/pnas.0610172104

Cycles of innovation postpone societal collapse

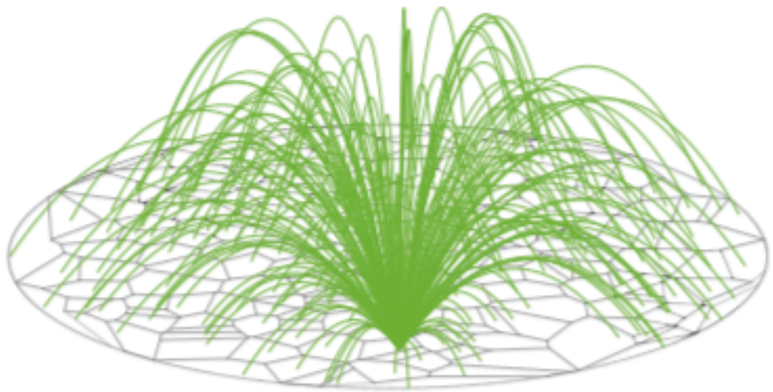


https://www.pnas.org/doi/abs/10.1073/pnas.0610172104

Agglomeration economies vesus congestion

Can a city grow indefinitely or will its transport system collapse?

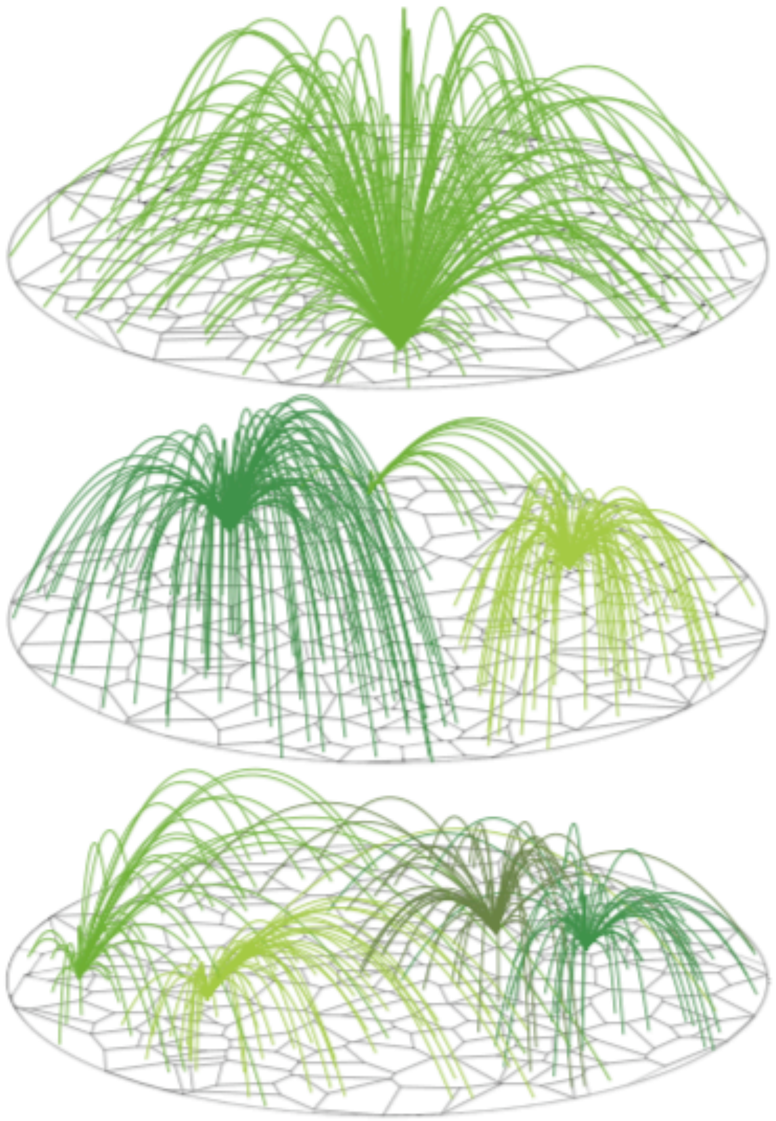
Agglomeration economies vesus congestion



Model: Commuting cost versus scaling of hotspots

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.111.198702

Agglomeration economies vesus congestion

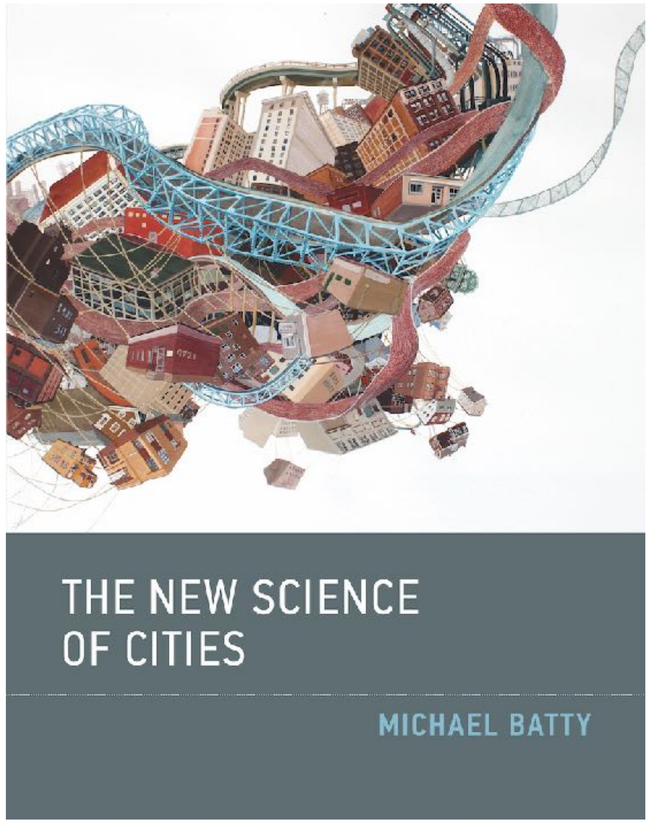
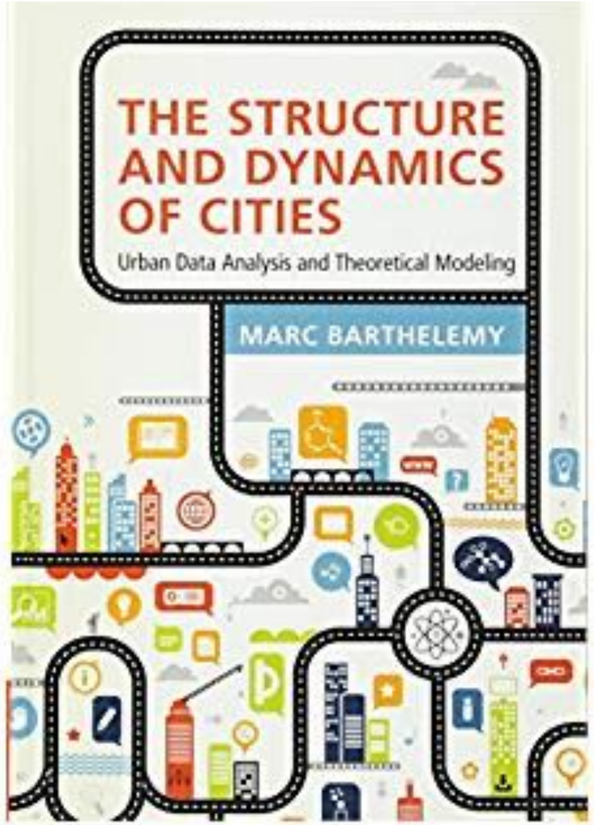


Model: Commuting cost versus scaling of hotspots

At a certain size or congestion level, monocentricity becomes unstable

As cities grow, congestion delays dominate the time spent in traffic, increasing economic losses per capita

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.111.198702

With Urban Data Science we can quantify cities, identify their vulnerabilities, and show how to make them better

Sources and further materials for today's class