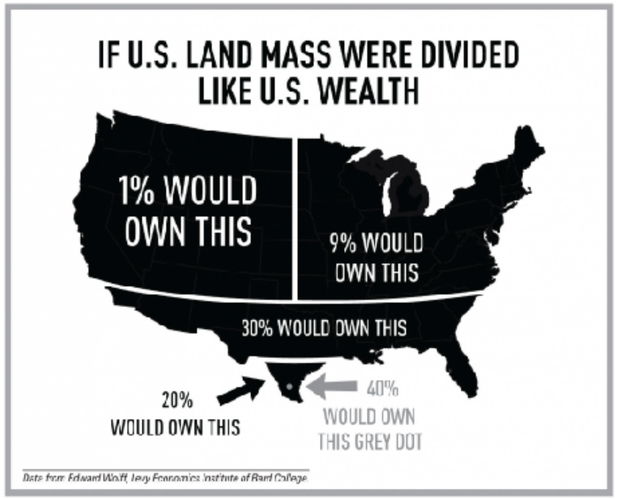
Class 22: Skewed data

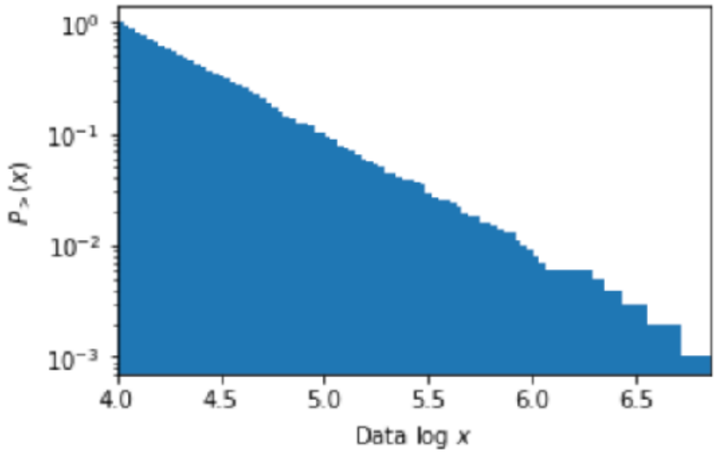
Instructor: Michael Szell
Nov 15, 2019

Today you will learn about skewed data distributions

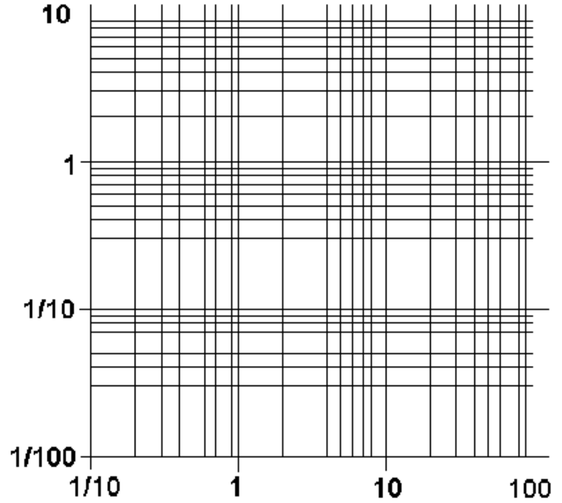
Abundance and importance of skewed data



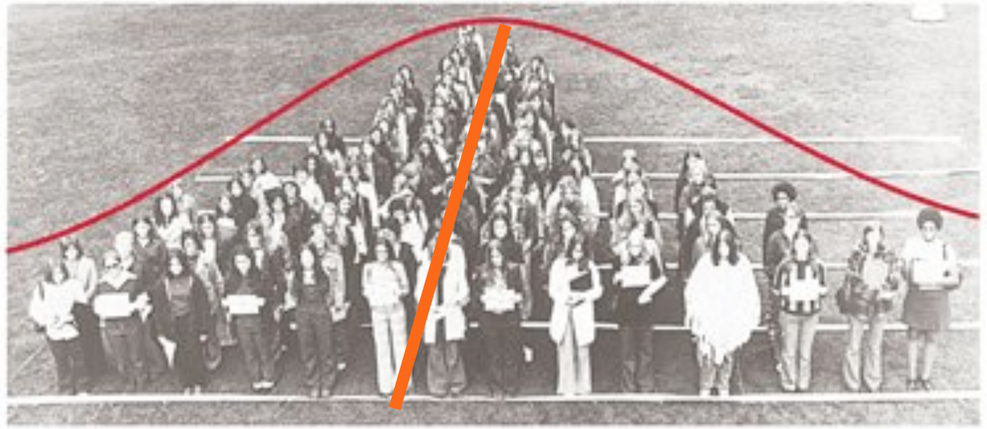
Inspecting distribution tails



Log-transformation



Mediocristan

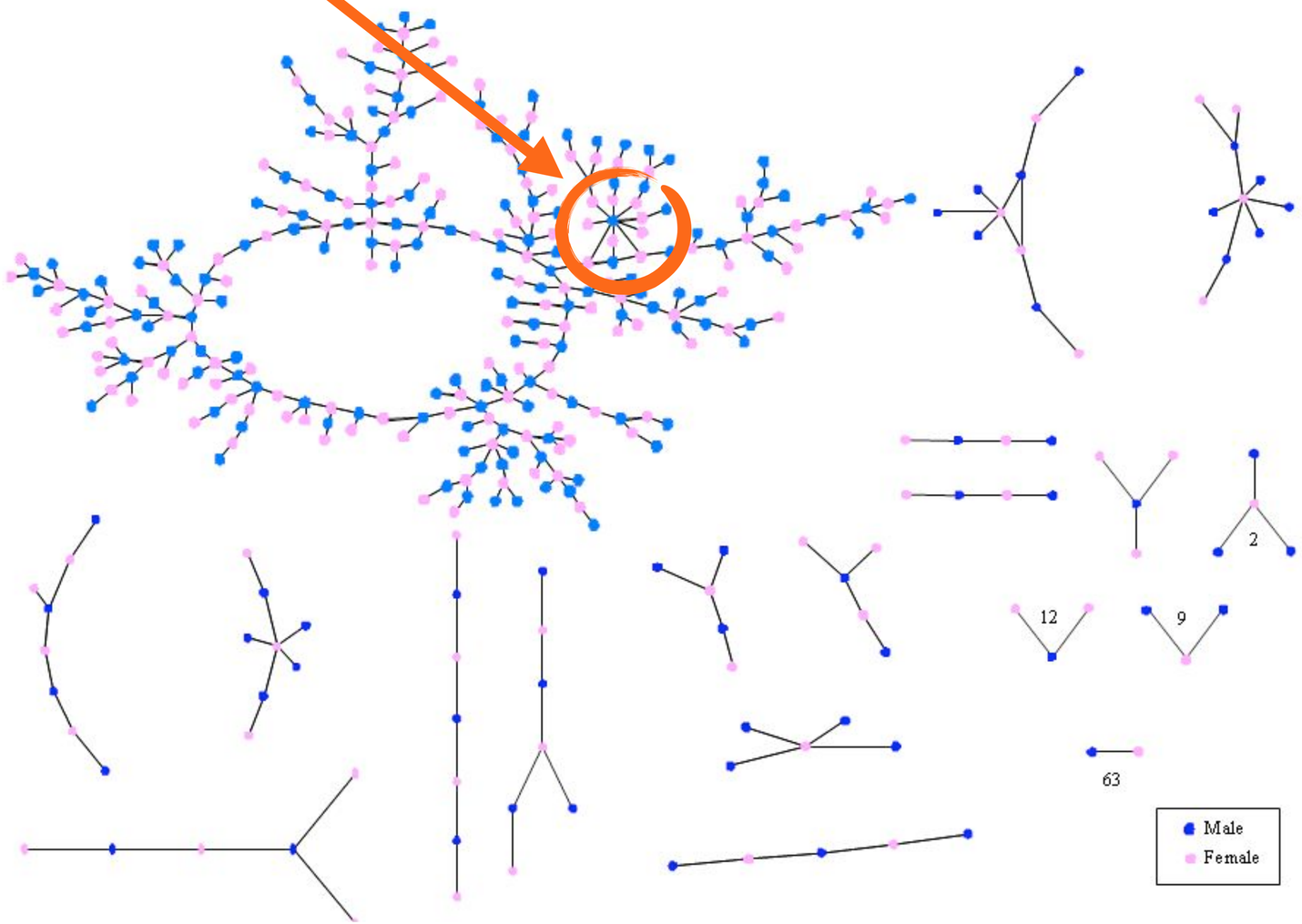


There is a typical scale

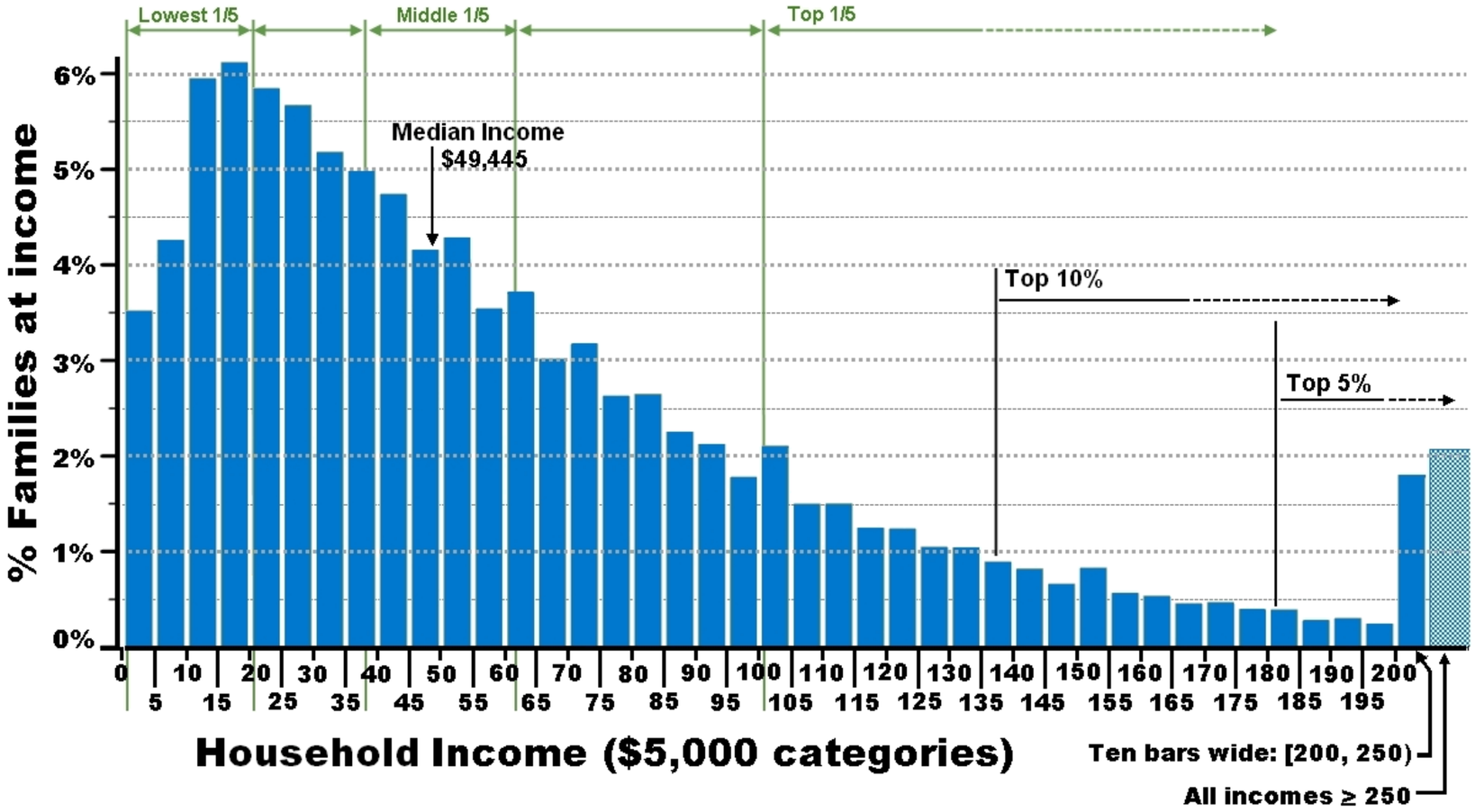
Extremistan

There is no typical scale

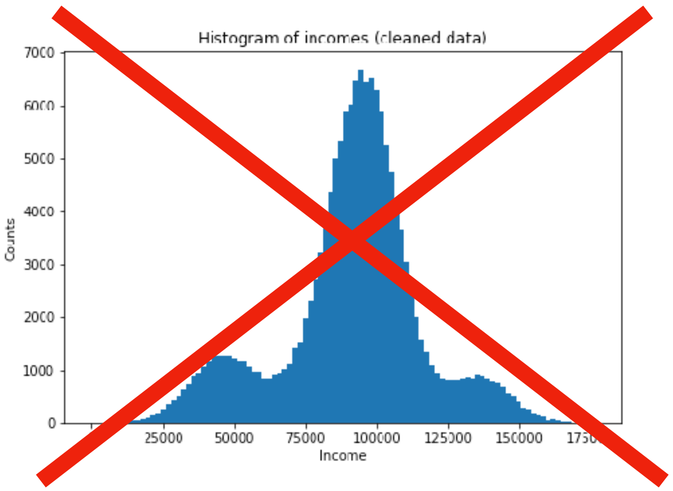
We live in Extremistan:
Some people have sex with MANY more people than others



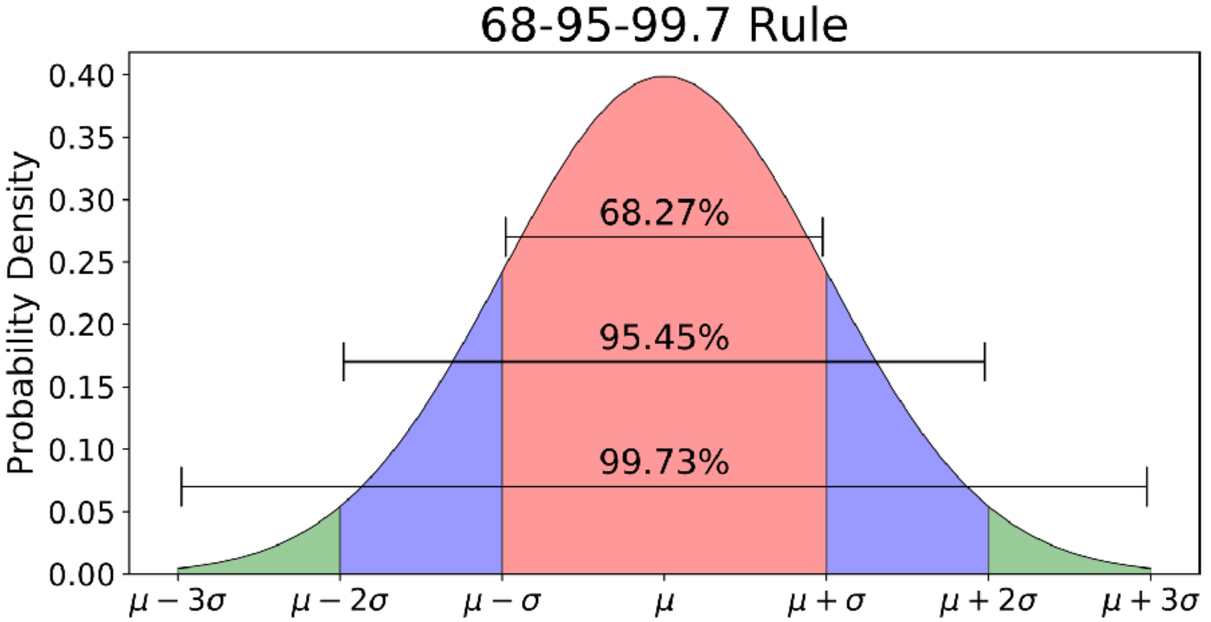
We live in Extremistan:
Some people earn MUCH more than others



Data source: http://www.census.gov/hhes/www/cpstables/032011/hhinc/new06_000.htm



We live in Extremistan:
Financial collapse is MUCH more likely than expected



"10σ events" happen regularly

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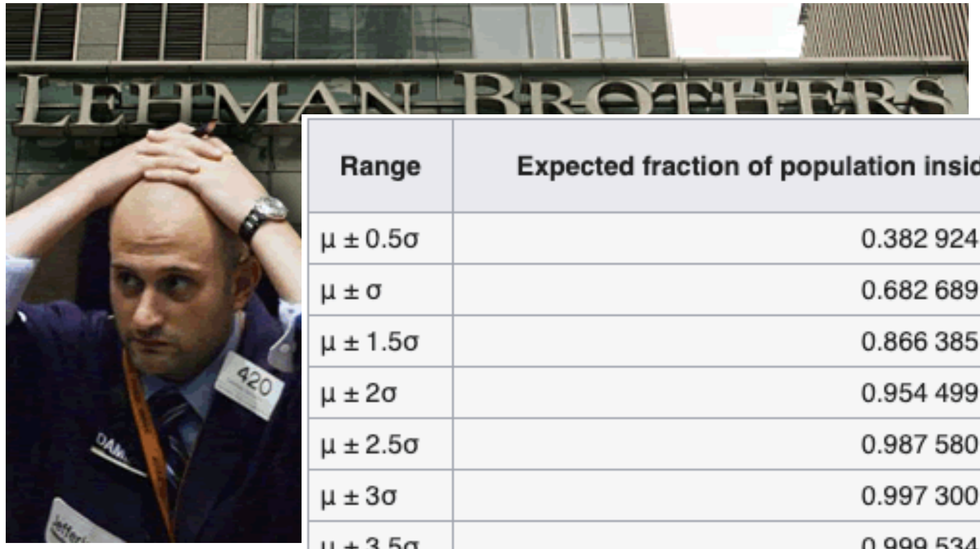


Range	Expected fraction of population inside range	Approximate expected frequency outside range	Approximate frequency for daily event
μ ± 0.5σ	0.382 924 922 548 026	2 in 3	Four or five times a week
μ ± σ	0.682 689 492 137 086	1 in 3	Twice a week
μ ± 1.5σ	0.866 385 597 462 284	1 in 7	Weekly
μ ± 2σ	0.954 499 736 103 642	1 in 22	Every three weeks
μ ± 2.5σ	0.987 580 669 348 448	1 in 81	Quarterly
μ ± 3σ	0.997 300 203 936 740	1 in 370	Yearly
μ ± 3.5σ	0.999 534 741 841 929	1 in 2149	Every 6 years
μ ± 4σ	0.999 936 657 516 334	1 in 15 787	Every 43 years (twice in a lifetime)
μ ± 4.5σ	0.999 993 204 653 751	1 in 147 160	Every 403 years (once in the modern era)
μ ± 5σ	0.999 999 426 696 856	1 in 1 744 278	Every 4776 years (once in recorded history)
μ ± 5.5σ	0.999 999 962 020 875	1 in 26 330 254	Every 72 090 years (thrice in history of modern humankind)
μ ± 6σ	0.999 999 998 026 825	1 in 506 797 346	Every 1.38 million years (twice in history of humankind)
μ ± 6.5σ	0.999 999 999 919 680	1 in 12 450 197 393	Every 34 million years (twice since the extinction of dinosaurs)
μ ± 7σ	0.999 999 999 997 440	1 in 390 682 215 445	Every 1.07 billion years (four times in history of Earth)

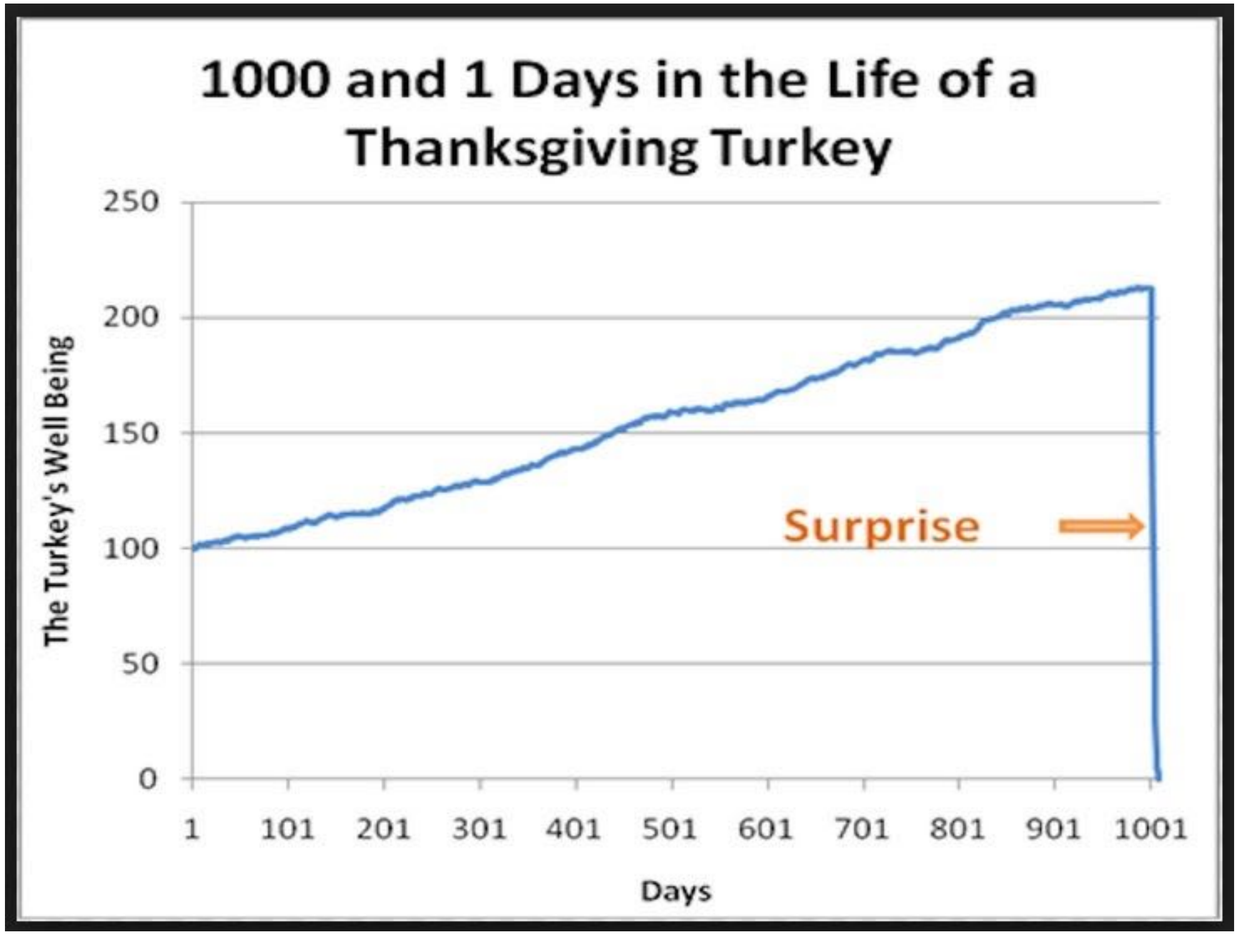
"10σ events" happen regularly

https://en.wikipedia.org/wiki/68%E2%80%9395%E2%80%9399.7_rule

We live in Extremistan:
Black swans are MUCH more likely than expected

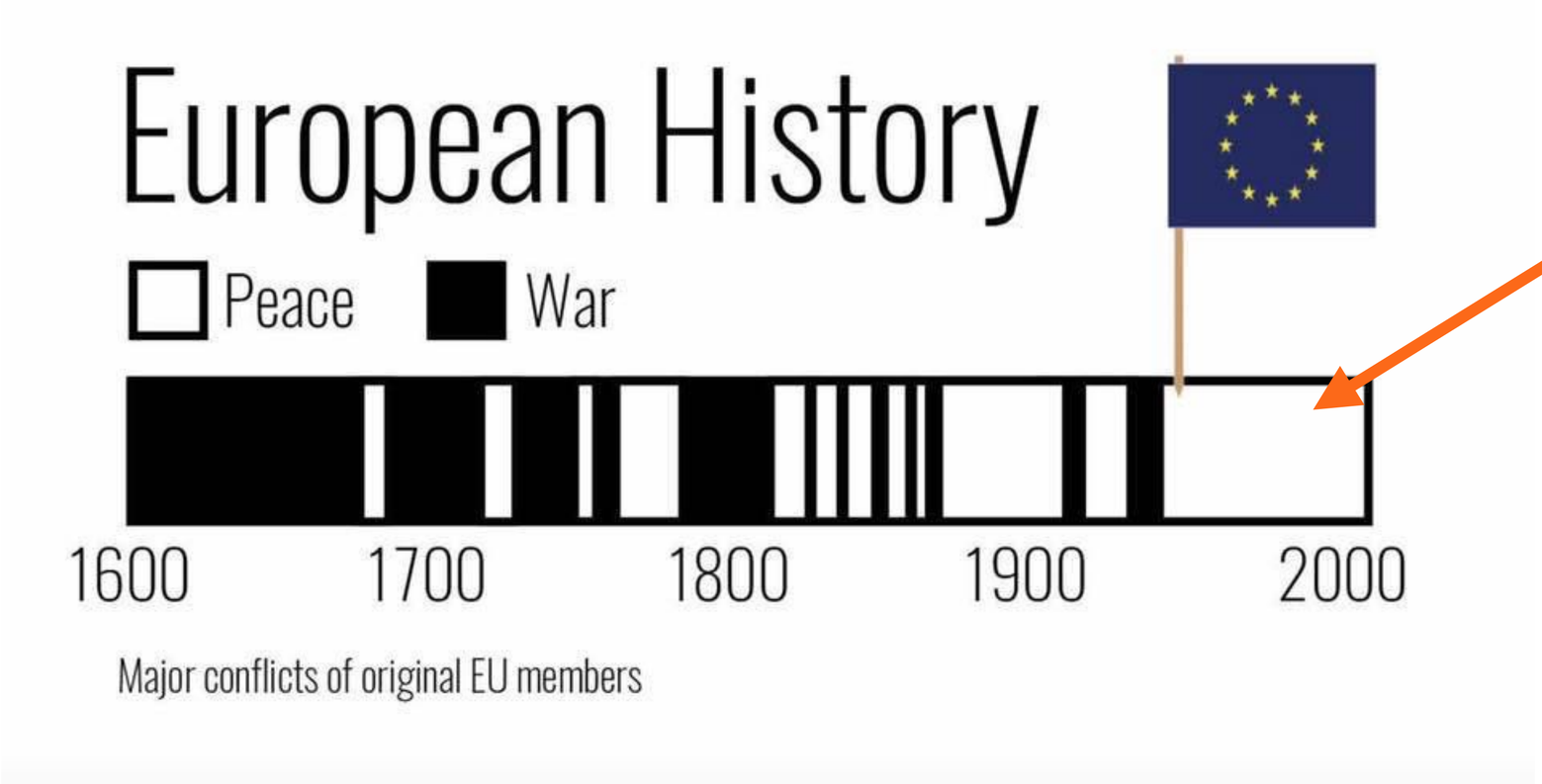
N.N.Taleb: The Black Swan

Black swans are rare, major impacts, rationalized by hindsight. We do not believe they happen until they do.



N.N.Taleb: The Black Swan

We live in Extremistan:
Wars are MUCH more likely than expected



Could be a statistical fluke

http://oefresearch.org/publications/enduring-threat-large-interstate-war

Extremistan produces skewed data:
It covers a VERY large range of values

Mediocristan

Tallest person: 272 cm

Shortest person: 56 cm

ratio: 4.8

Extremistan

Casanova: 20000

Monk: 1

ratio: 20,000

Mediocristan

Tallest person: 272 cm

Shortest person: 56 cm

ratio: 4.8

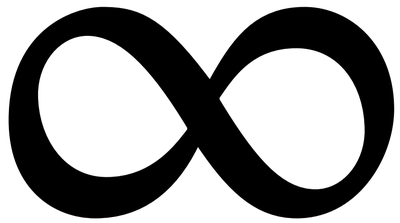
Extremistan

Biggest city: 8,000,000

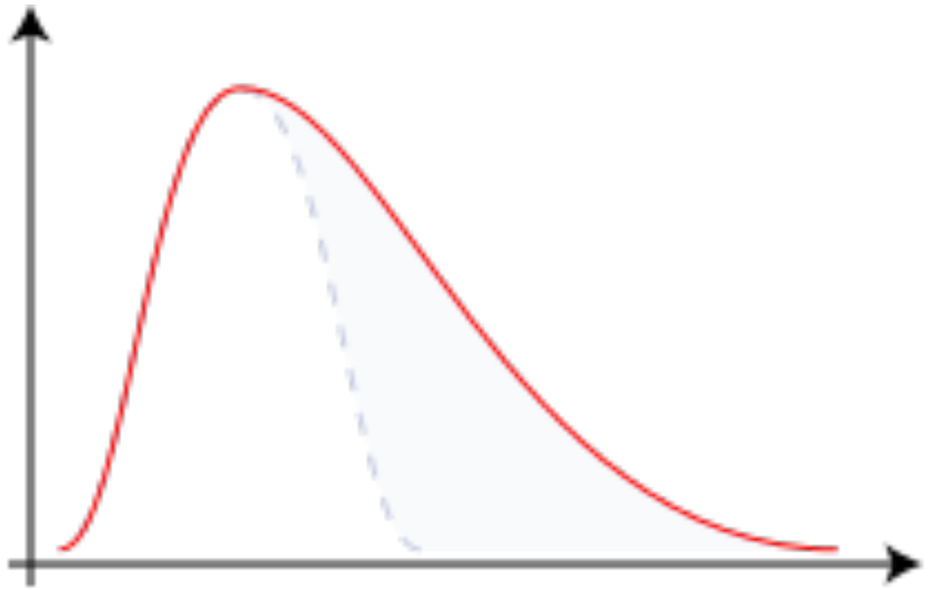
Smallest city: 52

ratio: 150,000

The theoretical spread and mean in skewed data is often:



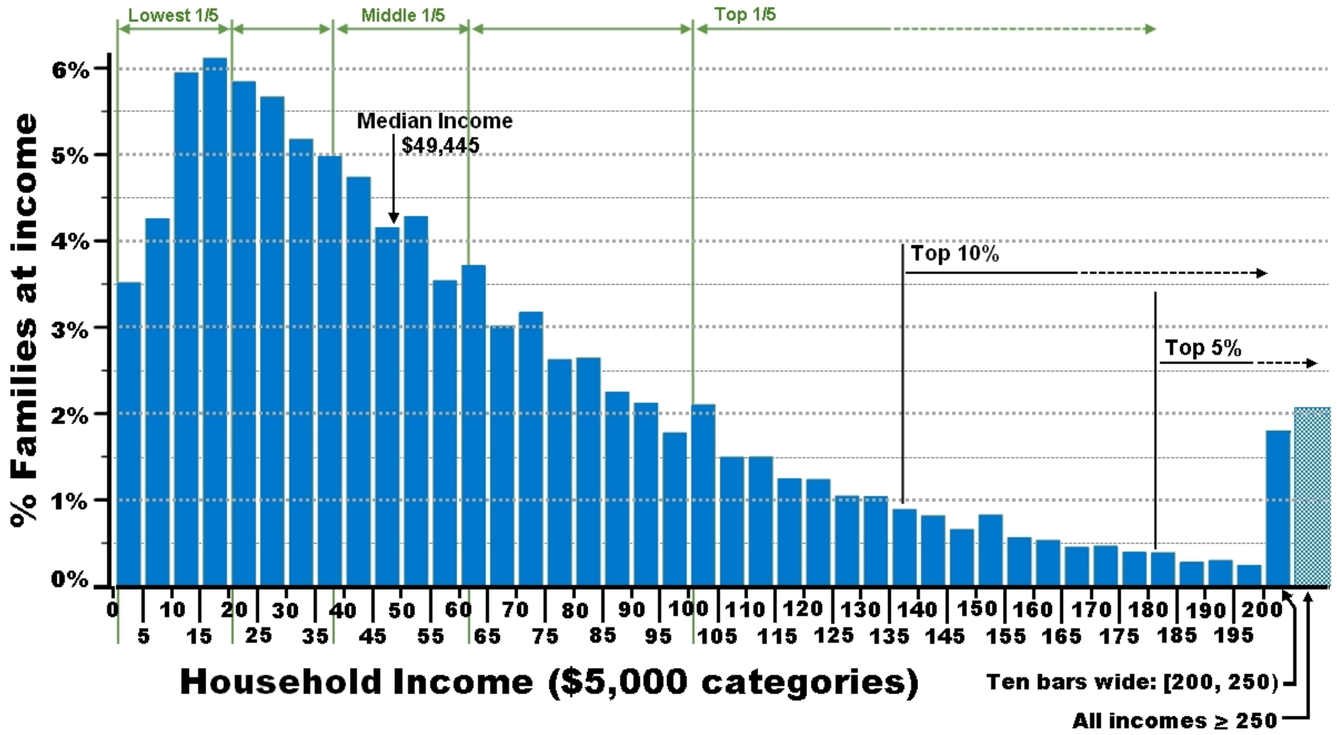
Skewed data means: Mean and median are very different.



Positive Skew

Right skew

Long tail to the right



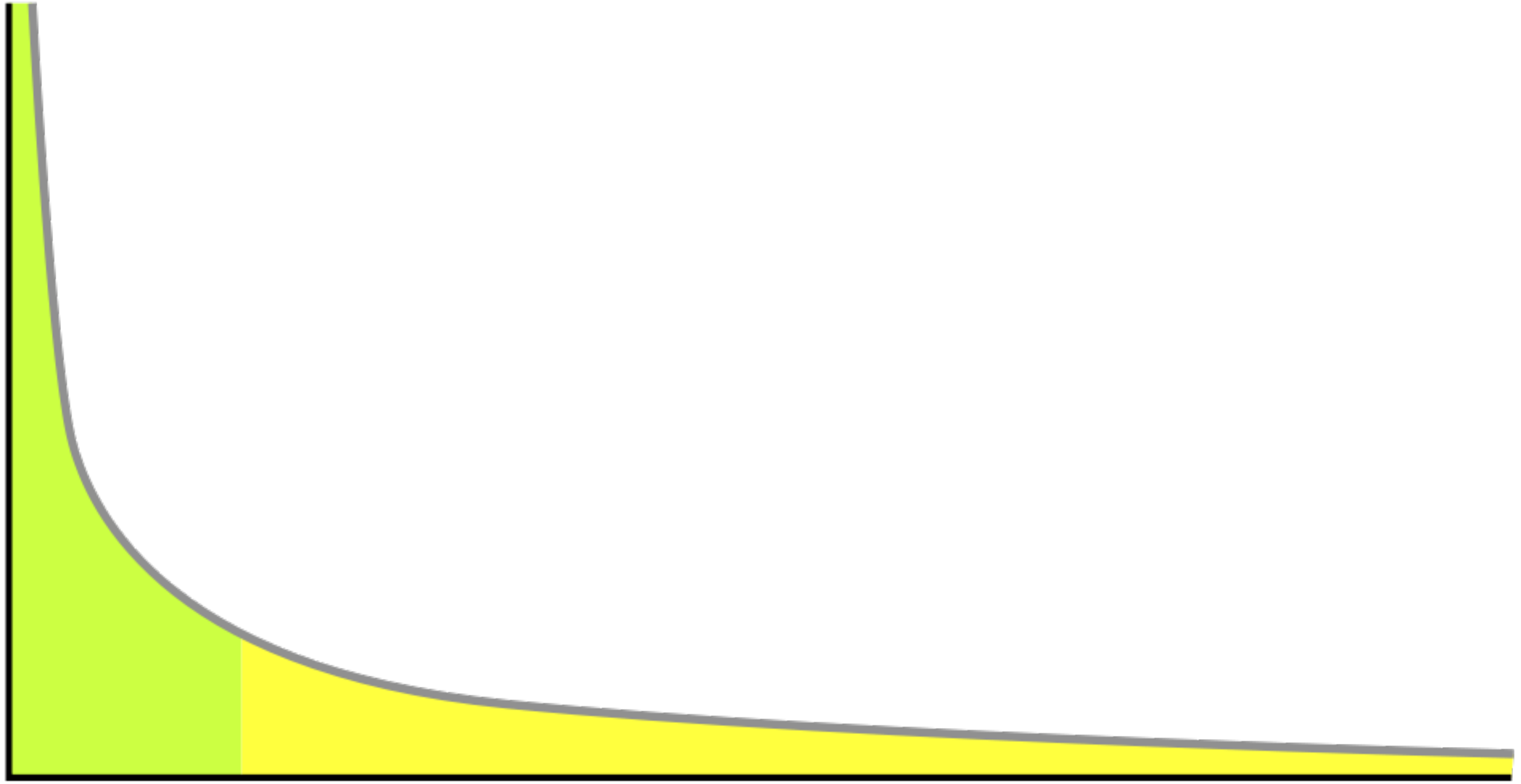
Data source: http://www.census.gov/hhes/www/cpstables/032011/hhinc/new06_000.htm

Skewed data has a long tail

Skewed data has a long tail

The abundance of long tails has a fundamental business impact

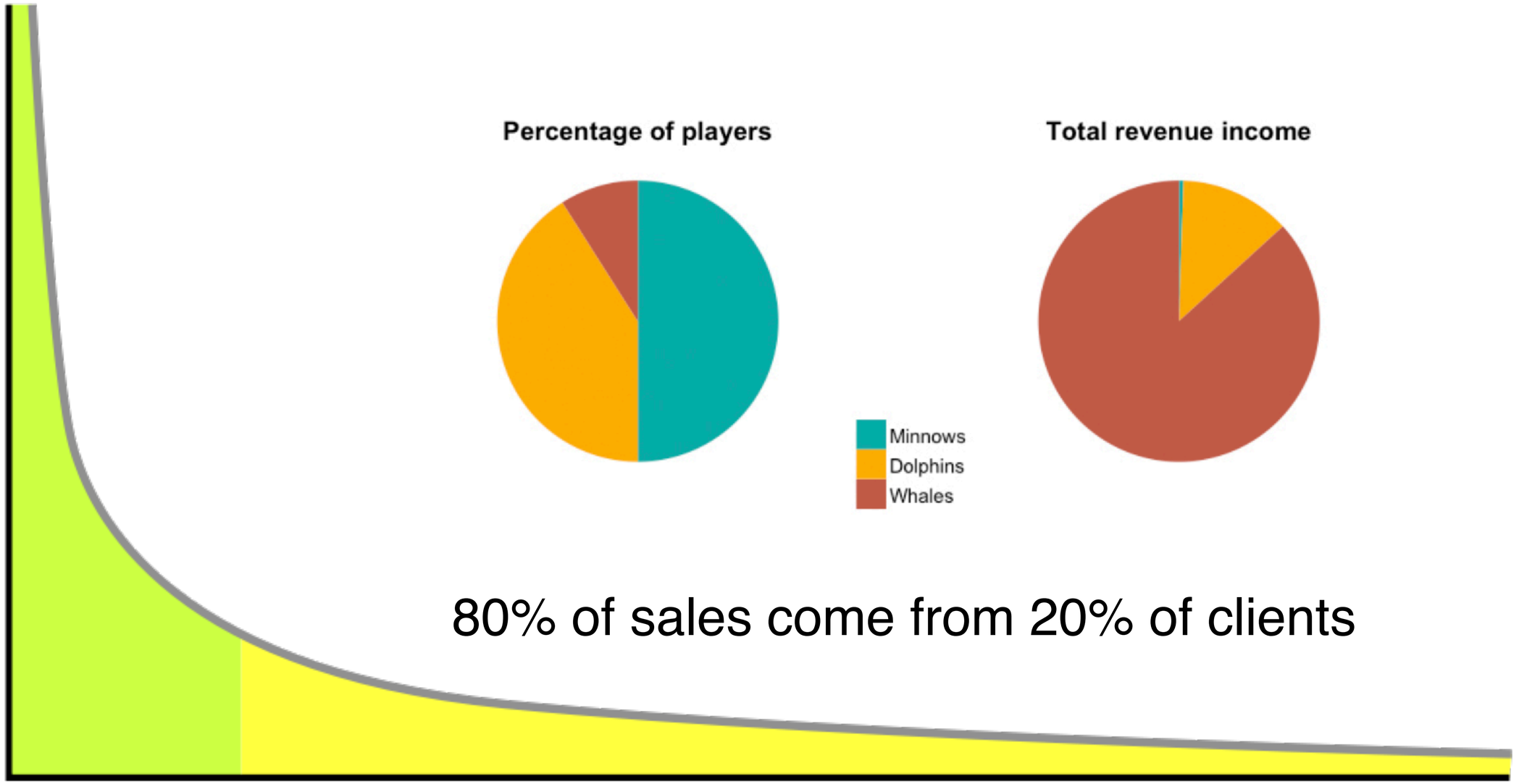
The pareto principle states:
80% of the effects come from 20% of the causes



The pareto principle states:
80% of the effects come from 20% of the causes

80% of sales come from 20% of clients

The pareto principle states:
80% of the effects come from 20% of the causes



https://gameanalytics.com/blog/how-to-identify-whales-in-your-game.html

The pareto principle states:
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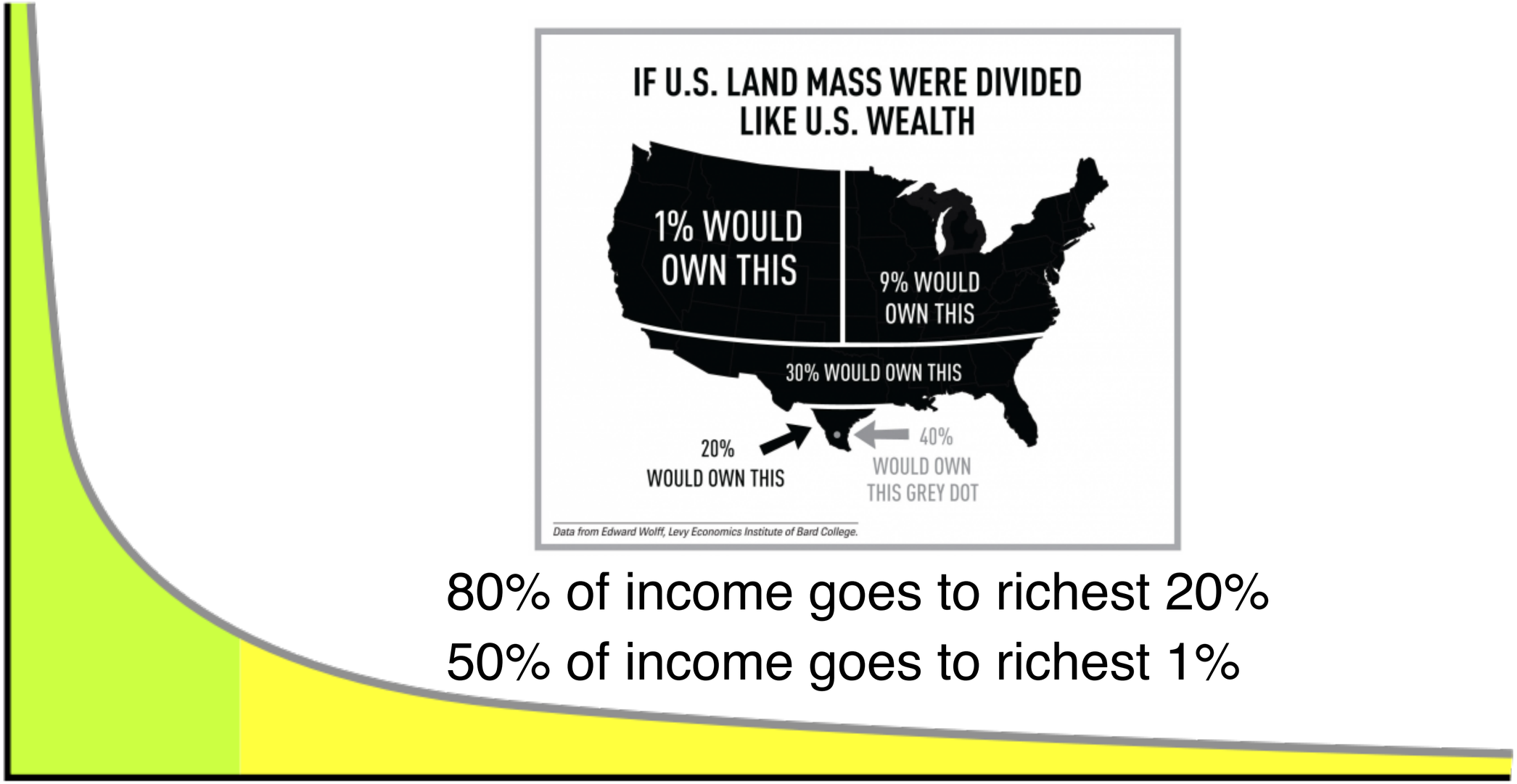
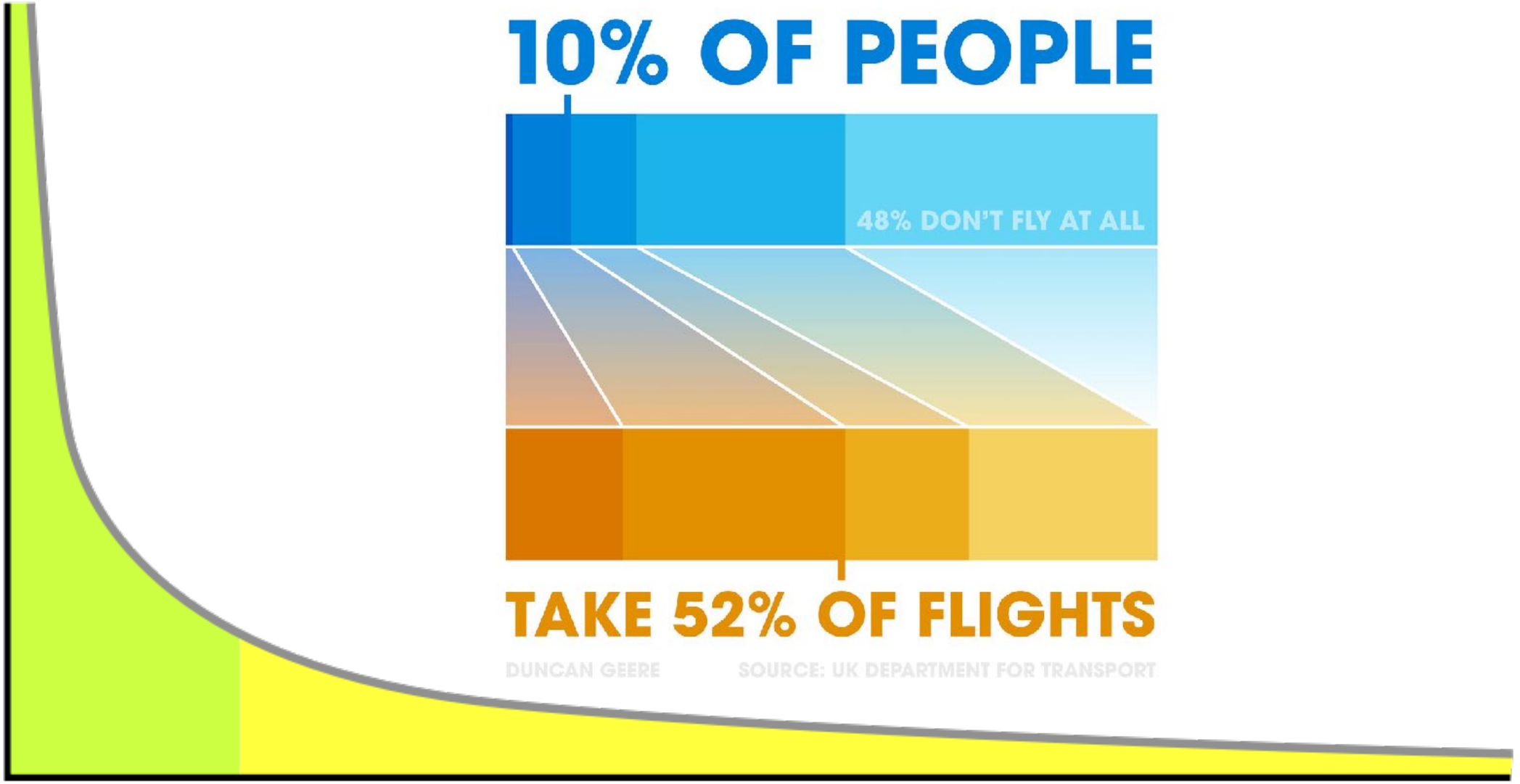


80% of income goes to richest 20%

50% of income goes to richest 1%

http://www.globalrichlist.com/

The pareto principle states:
80% of the effects come from 20% of the causes



The pareto principle states:
80% of the effects come from 20% of the causes

80% of errors come from 20% of bugs

In the past, selection of cars was limited

Hungary 1980s

In the past, media were limited by time or space

1971

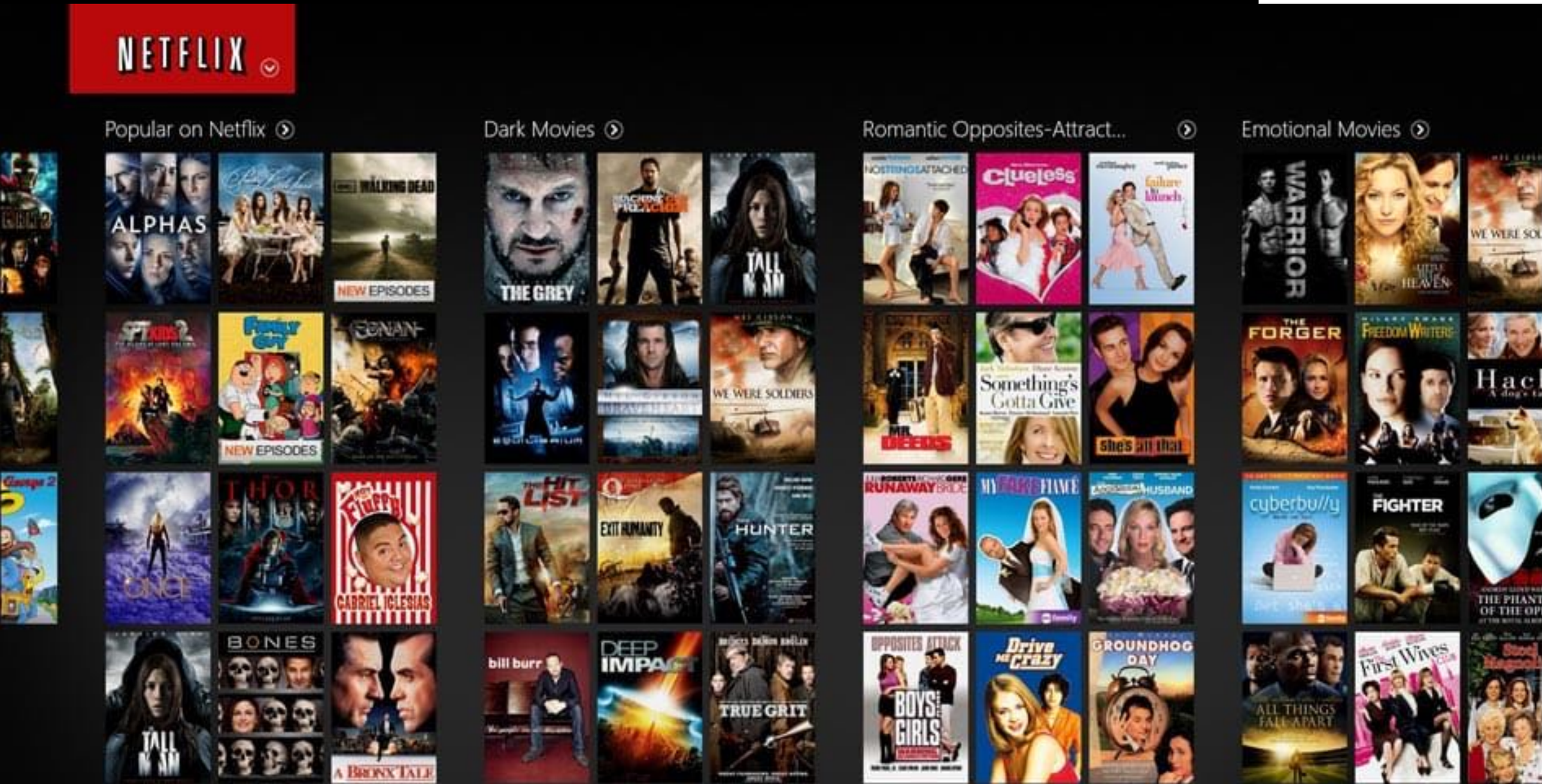
		7:30	8:00	8:30	9:00	9:30	10:00	10:30
SAT	ABC	Local	Getting Together	Movie of the Weekend			The Persuaders	
	CBS	Local	All in the Family	Funny Face	New Dick Van Dyke Show	Mary Tyler Moore Show	Mission: Impossible	
	NBC	Local	The Partners	The Good Life	NBC Saturday Night at the Movies			
SUN	ABC	Local	The FBI		The ABC Sunday Night Movie			
	CBS	The CBS Sunday Night Movies				Cade's County		Local
	NBC	The Wonderful World of Disney		Jimmy Stewart	Bonanza		The Bold Ones	
MON	ABC	Local	Nanny and the Professor	Local	NFL Monday Night Football (to be replaced by movies after Jan. 24)			
	CBS	Local	Gunsmoke		Here's Lucy	Doris Day Show	My Three Sons	Arnie
	NBC	Local	Rowan and Martin's Laugh-In		NBC Monday Night at the Movies			
TUE	ABC	The Mod Squad		Movie of the Week			Marcus Welby, M.D.	
	CBS	Glen Campbell Goodtime Hour		Hawaii Five-O		Cannon		Local
	NBC	Ironside		Sarge		The Funny Side		Local
WED	ABC	Local	Bewitched	Eddie's Father	The Smith Family	Shirley's World	The Man and the City	
	CBS	Local	Carol Burnett Show		Medical Center		Mannix	
	NBC	Local	Adam-12	NBC Mystery Movie			Night Gallery	
THU	ABC	Local	Alias Smith and Jones		Longstreet		Owen Marshall: Counselor at Law	
	CBS	Local	Bearcats!		The CBS Thursday Night Movies/CBS Reports (once a month)			
	NBC	Local	Flip Wilson Show		Nichols		Dean Martin Show	
FRI	ABC	Local	The Brady Bunch	The Partridge Family	Room 222	The Odd Couple	Love, American Style	
	CBS	Local	The Chicago Teddy Bears	O'Hara, United States Treasury		The New CBS Friday Night Movies		
	NBC	Local	The D.A.	NBC World Premiere Movie/Chronolog (once a month)				Local

In the past, retail of goods was limited by shelf space

Are people happy with a limited choice?

When diversity of supply increases, there is a market for almost everything

Digital media allow unlimited shelf space

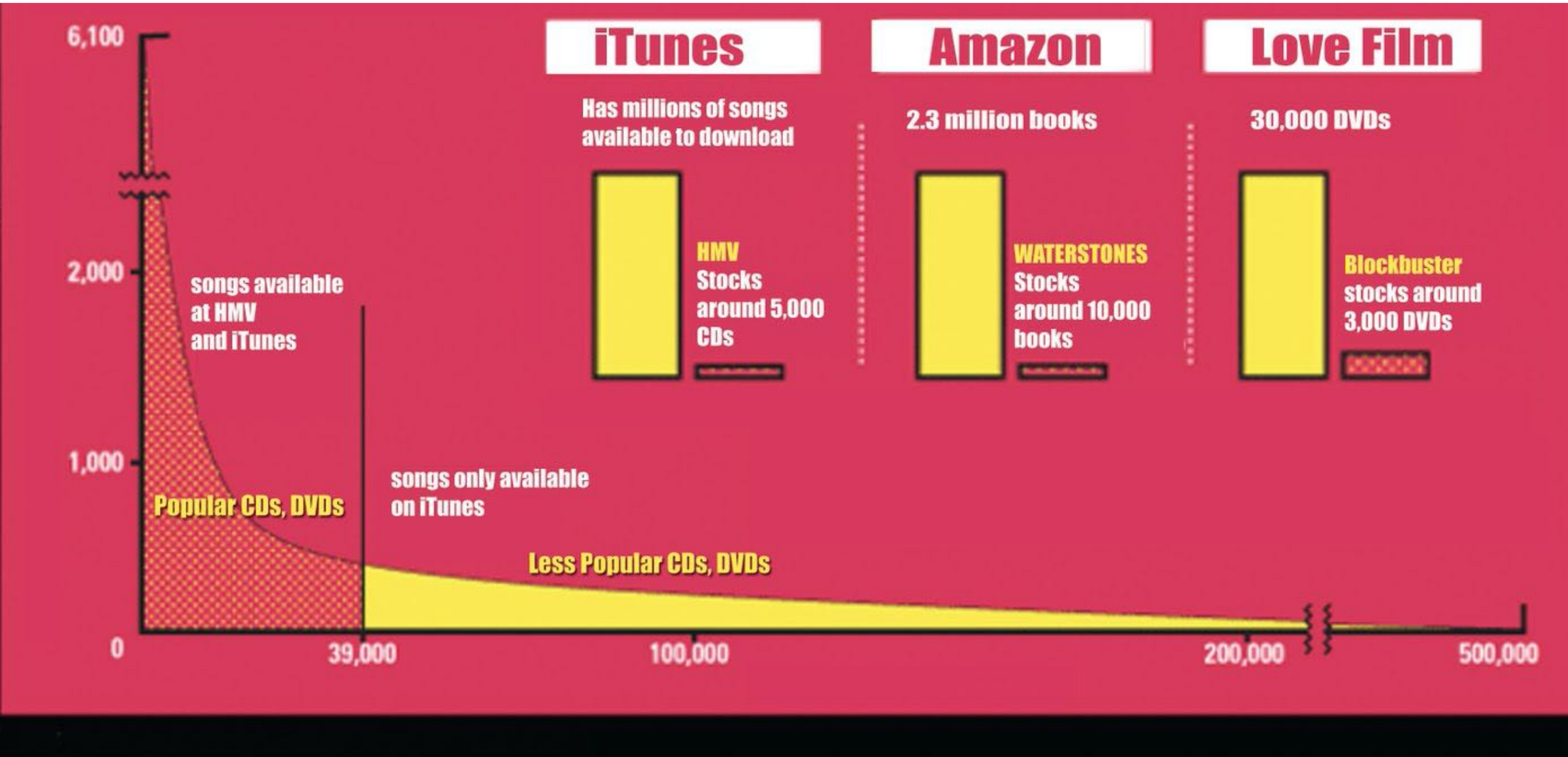


Digital media allow unlimited shelf space

This is why Amazon started and succeeded with books

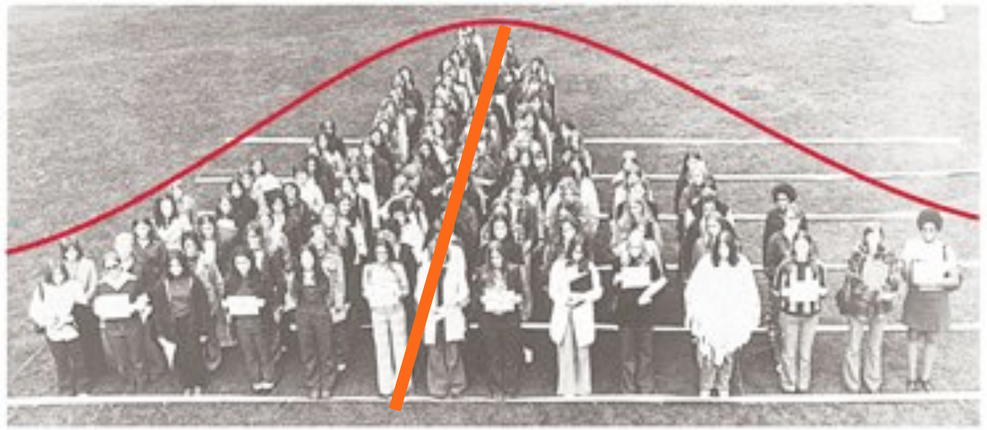
https://www.youtube.com/watch?v=rWRbTnE1PEM

Supply becomes unlimited. Demand becomes long-tailed.

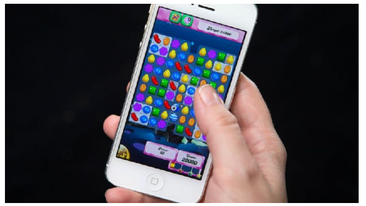


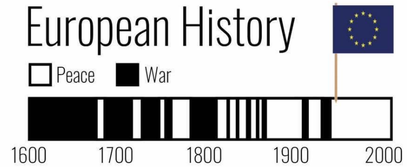
The long tail of demand poses a new challenge: How to best recommend products to people they will like?

Mediocristan



Extremistan





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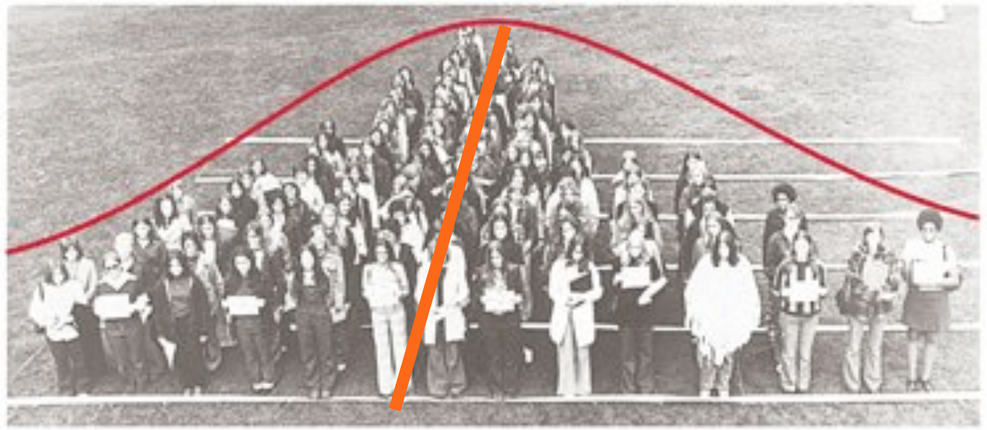

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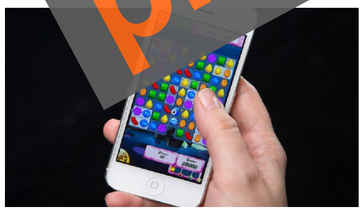
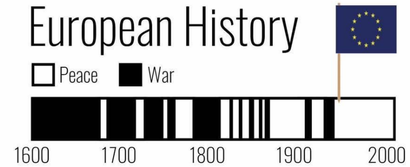


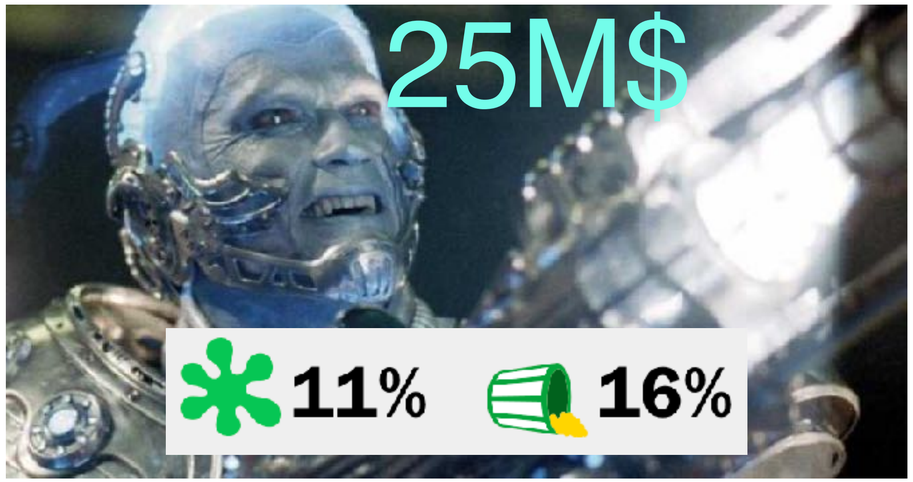
Social phenomena



Mediocristan

Low risk, low reward

Extremistan

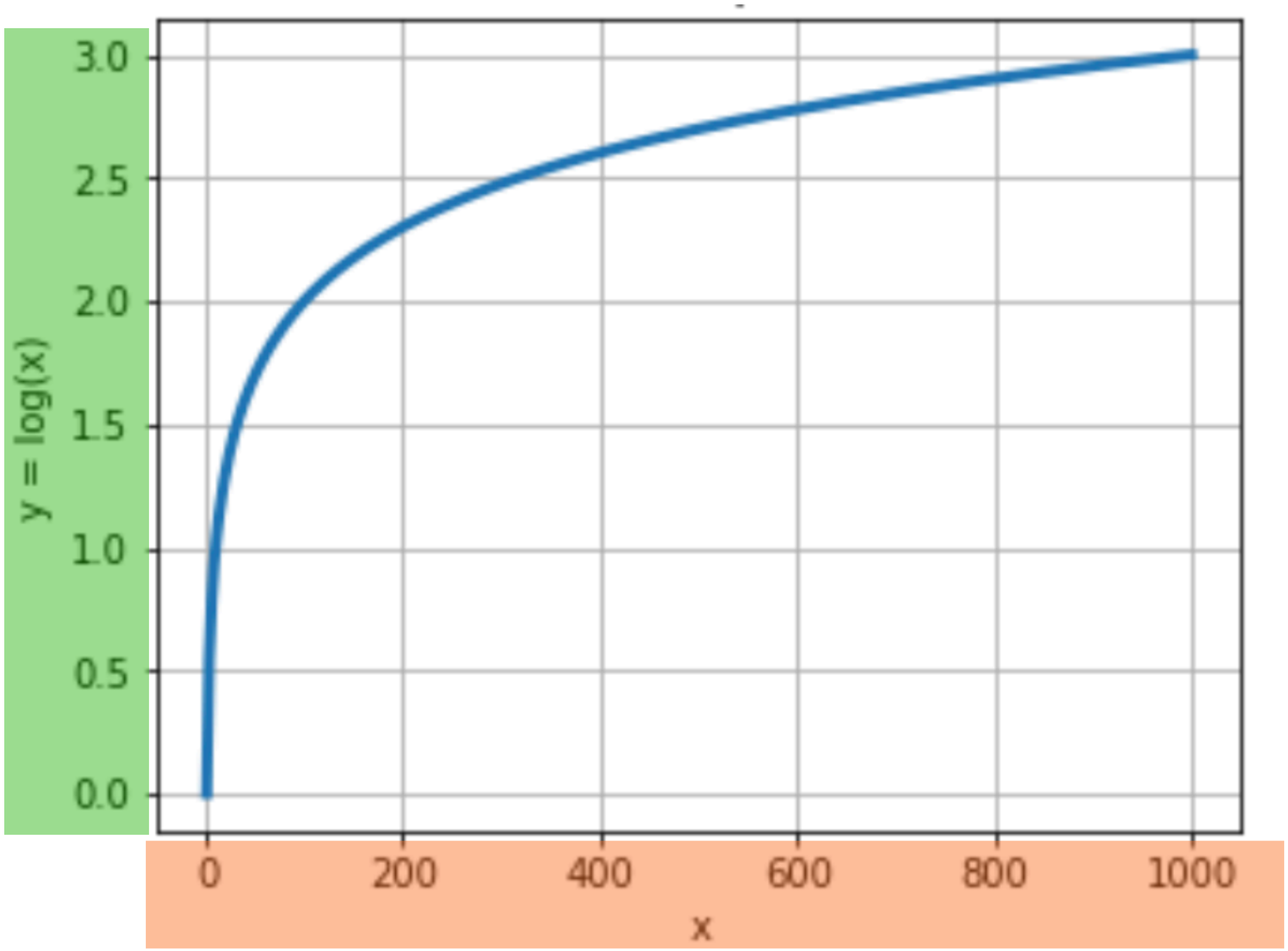


High risk, high reward

Tools to deal with skewed data

1) Logarithm transformation

The logarithm squashes a large range of values onto a small range

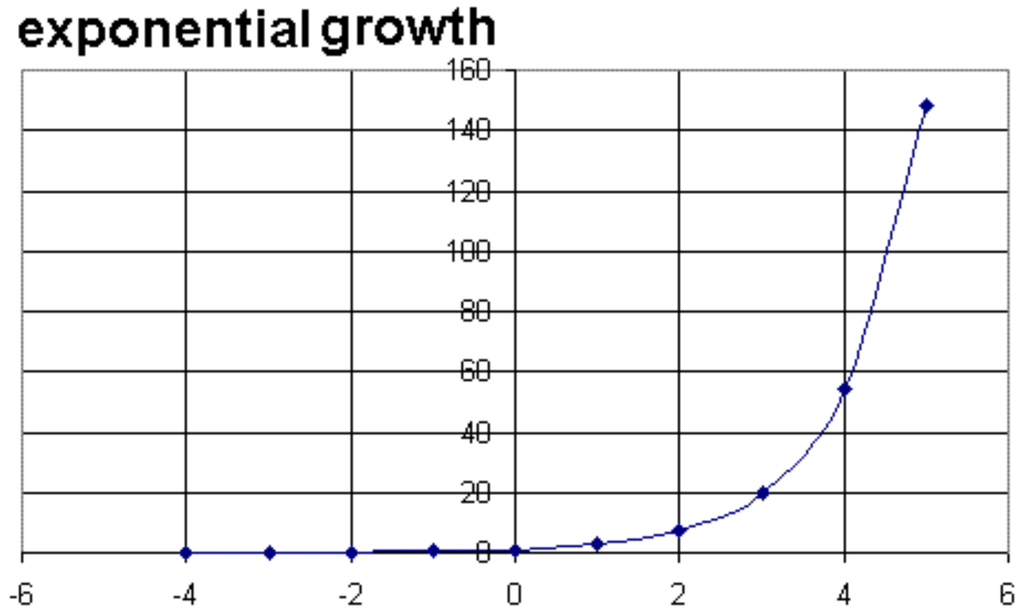


The log(arithm) transformation turns a linear into a log scale

$$y = ab^x$$

$y = e^x$

x	y
-4	0.02
-3	0.05
-2	0.14
-1	0.37
0	1
1	2.72
2	7.39
3	20.1
4	54.6
5	148



linear scales

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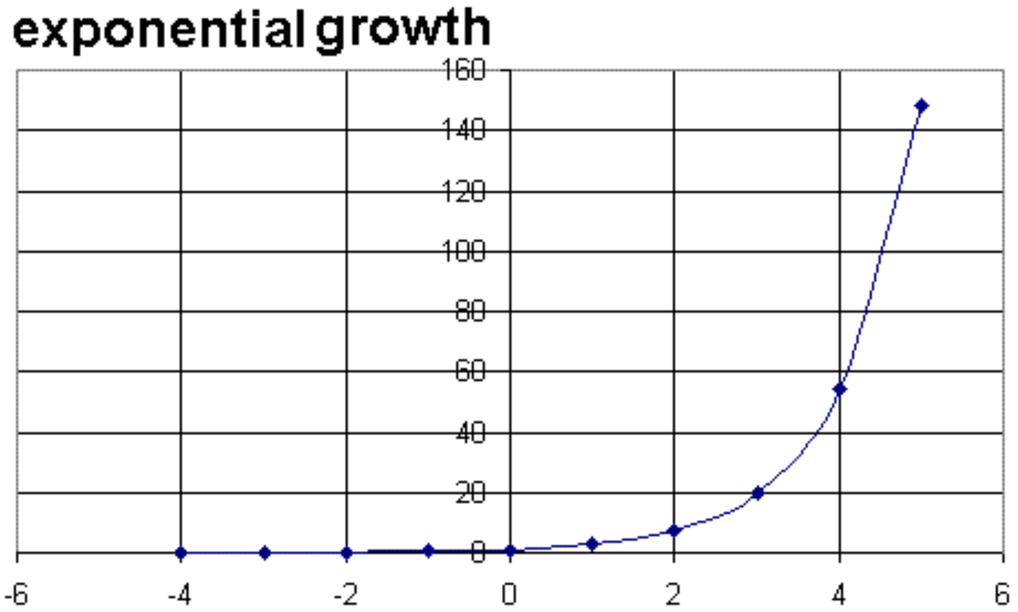
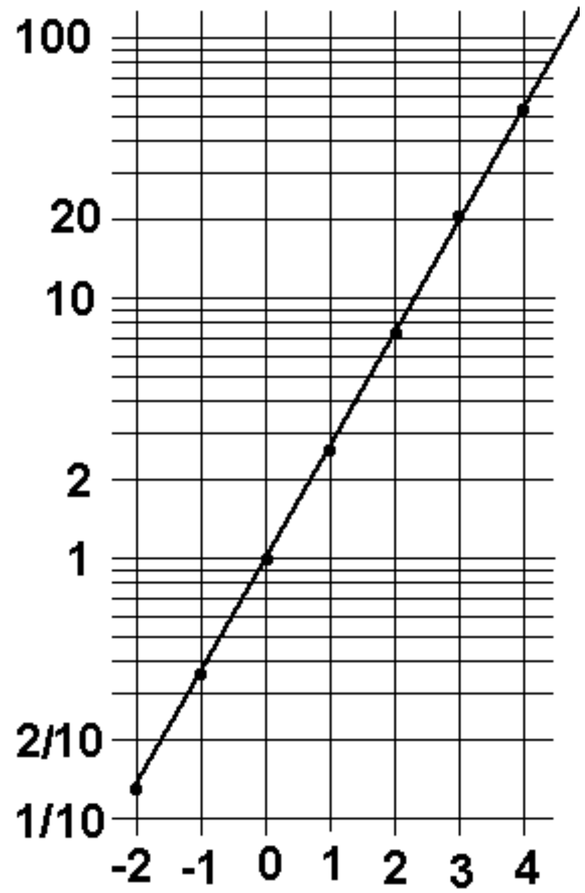




linear scales

semi-log scale

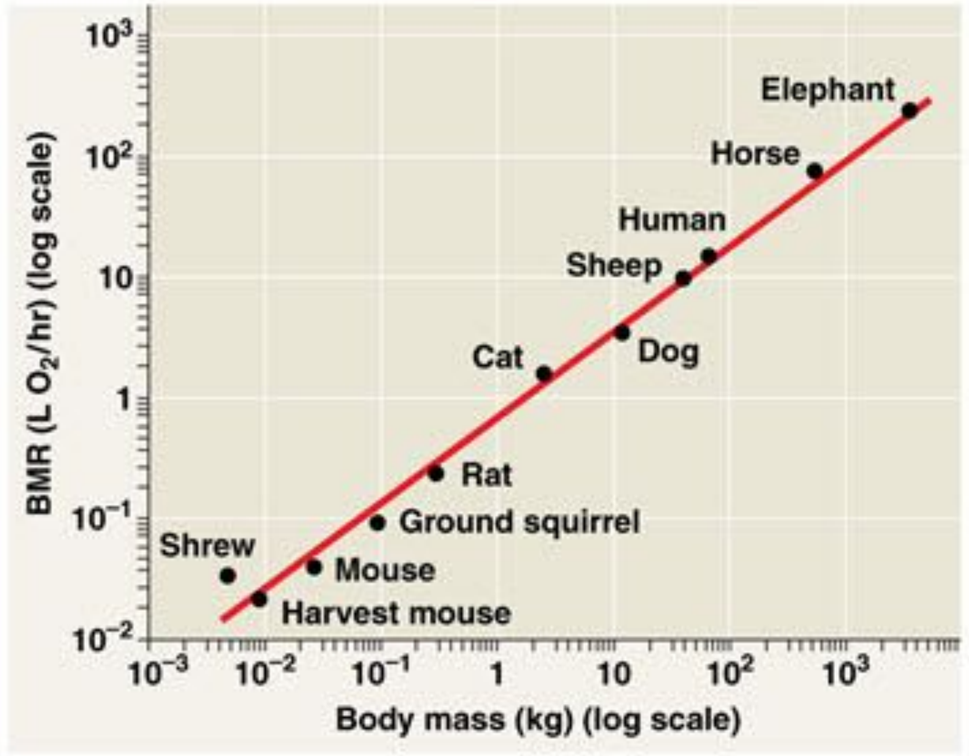
When we have a power law, we apply the log transform to both variables

$$y = ax^p$$

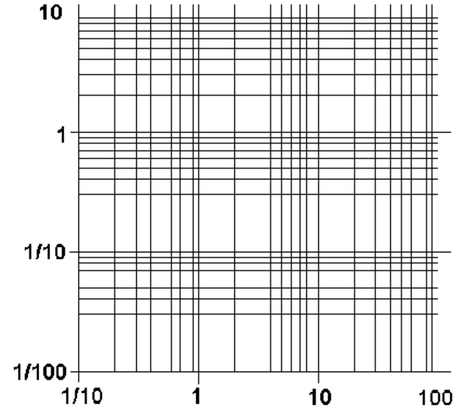
When we have a power law, we apply the log transform to both variables

$$y = ax^p$$

$$BMR = a \cdot BM^{3/4}$$



(a) Relationship of basal metabolic rate (BMR) to body size for various mammals



log-log scale

The log-transform turns bent curves into straight lines, making them easier to analyze

Exponential $y = ab^x$ → (log on right side) $y = a' + x$

Power law $y = ax^p$ → (log on both sides) $y' = a' + px'$

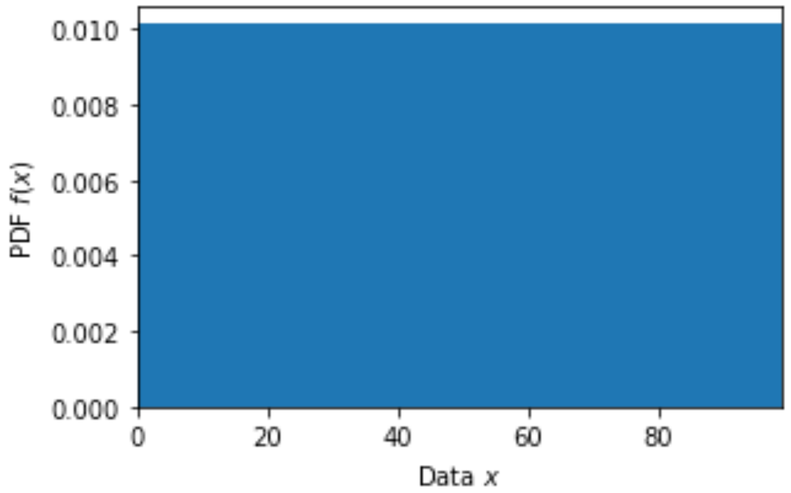
Jupyter

2) Complementary cumulative distribution function (CCDF)

PDF

Probability density function

$$f(x)$$



PDF

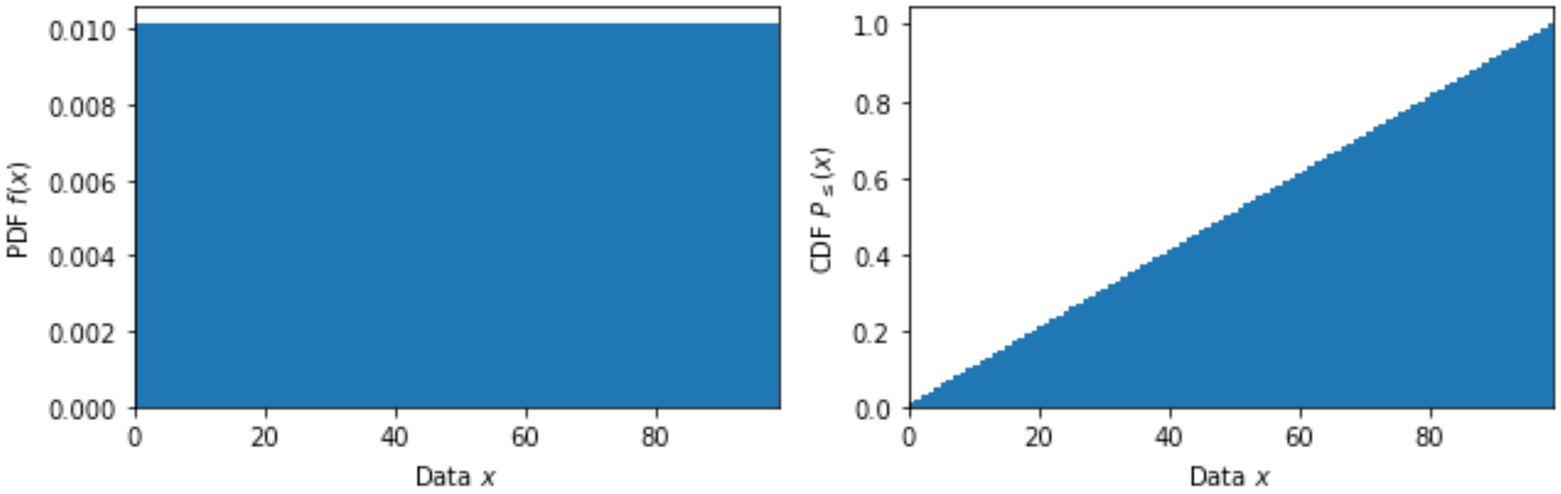
Probability
density function

CDF

Cumulative
distribution function

$f(x)$

$P_{\leq}(x)$



$P(X \leq x)$

Integrate

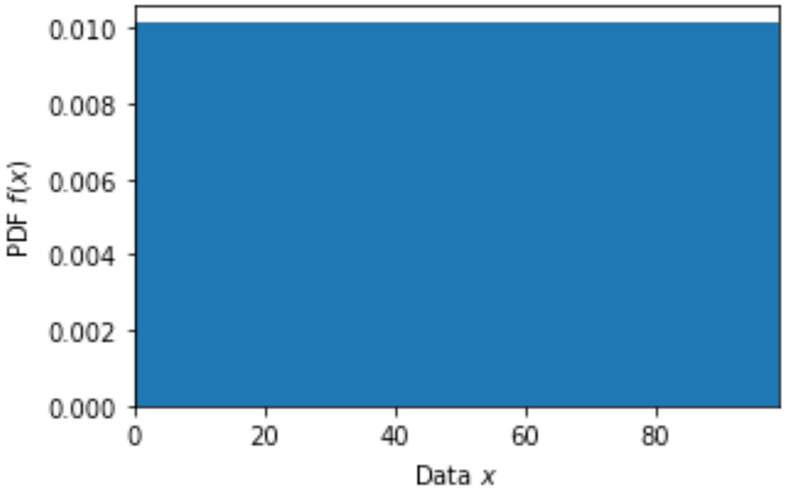
PDF

Probability density function

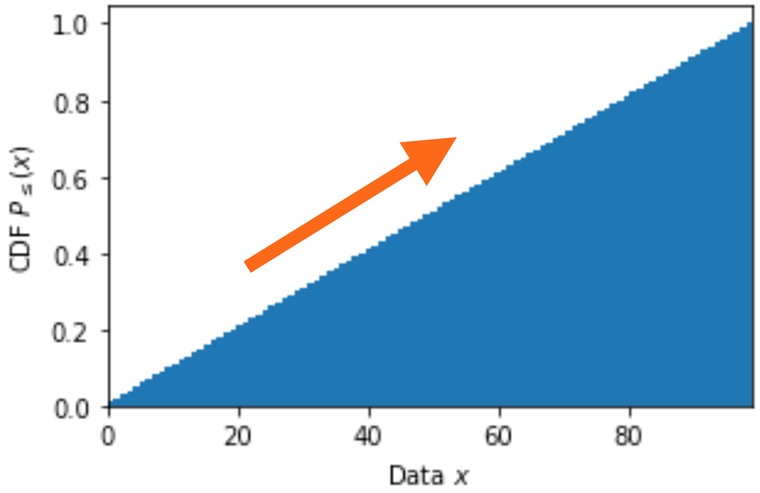
CDF

Cumulative distribution function

$f(x)$



$P_{\leq}(x)$



The CDF is monotonically non-decreasing

It starts at 0

Integrate

$P(X \leq x)$

PDF

Probability density function

$f(x)$

CDF

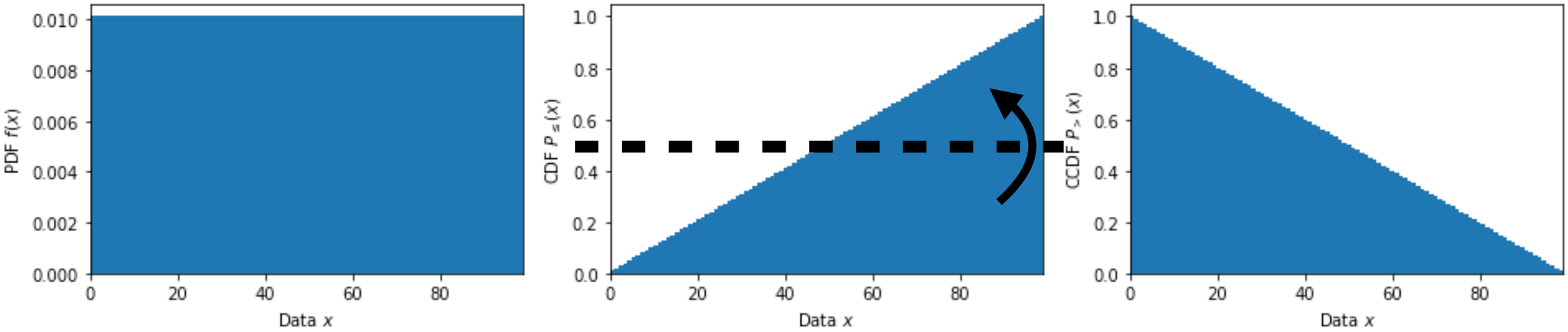
Cumulative distribution function

$P_{\leq}(x)$

CCDF

Complementary cumulative distribution function

$P_{>}(x)$



$P(X \leq x)$

$P(X > x)$

Integrate

Flip

PDF

Probability density function

$f(x)$

CDF

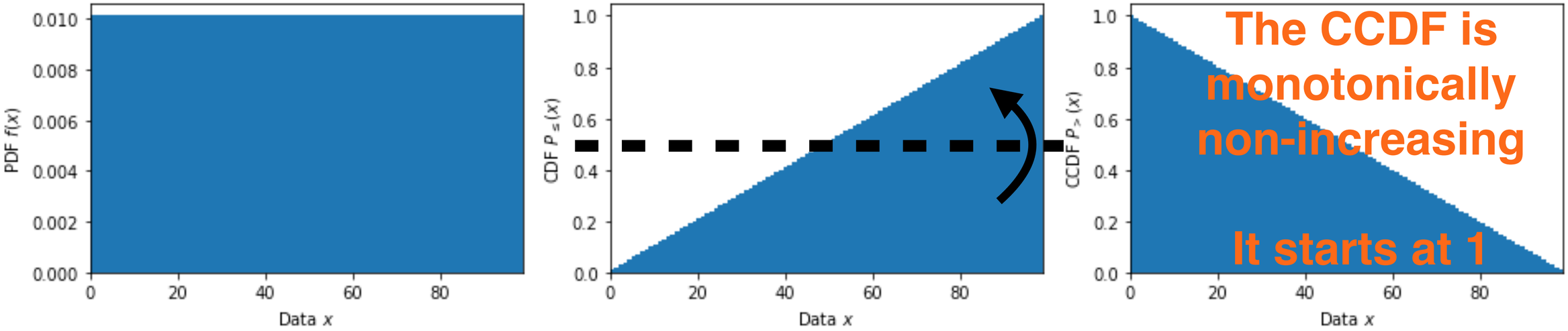
Cumulative distribution function

$P_{\leq}(x)$

CCDF

Complementary cumulative distribution function

$P_{>}(x)$



$P(X \leq x)$

$P(X > x)$

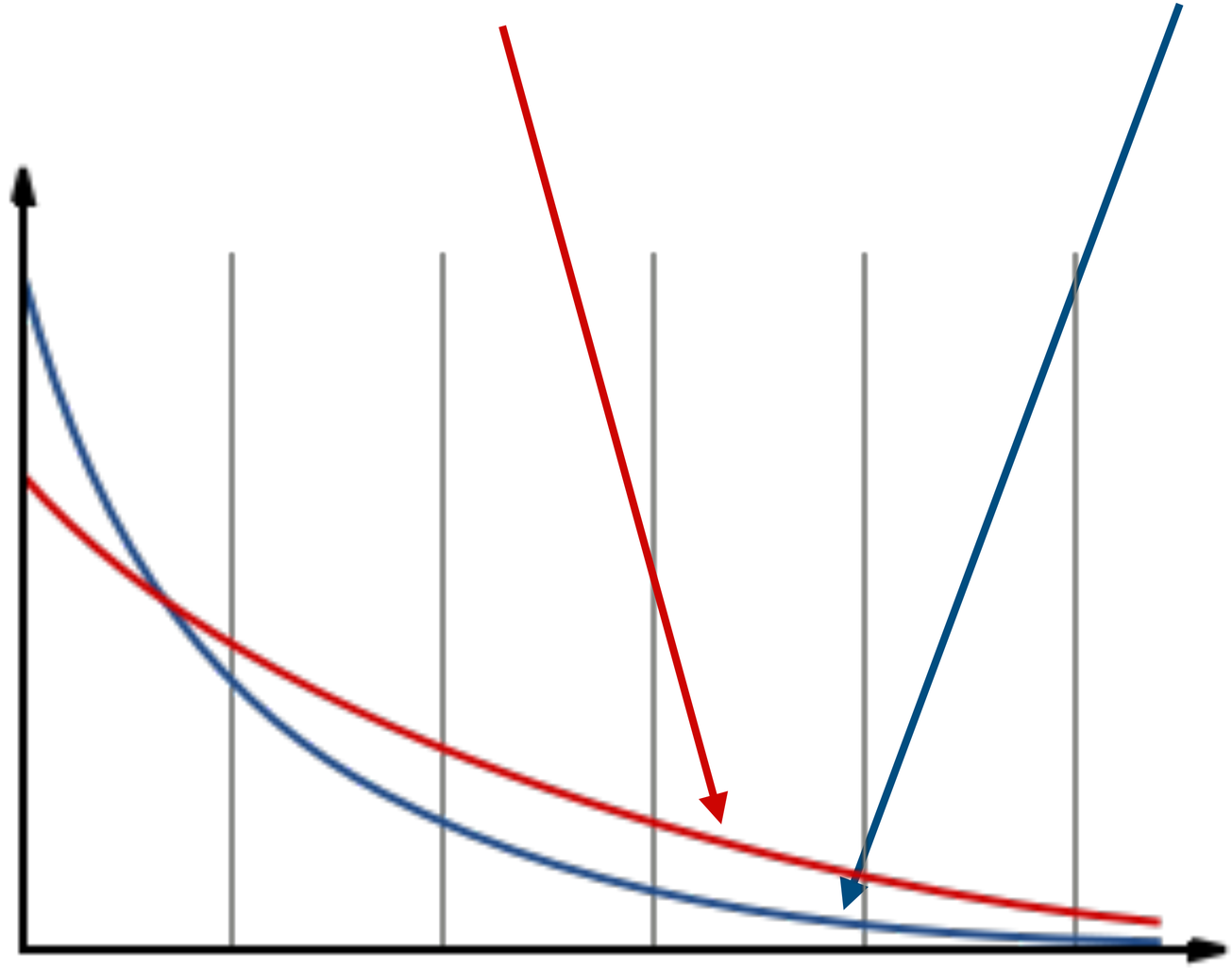
Integrate

Flip

Jupyter

Many real distributions are heavy-tailed

A distribution is heavy-tailed if its tail is not bounded by any exponential (from above)



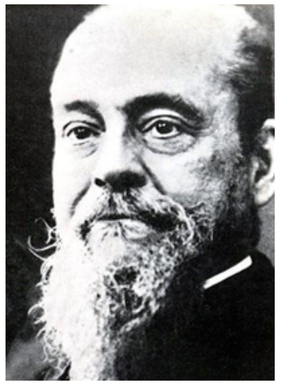
There are many names for certain heavy-tailed distributions

Fat tail

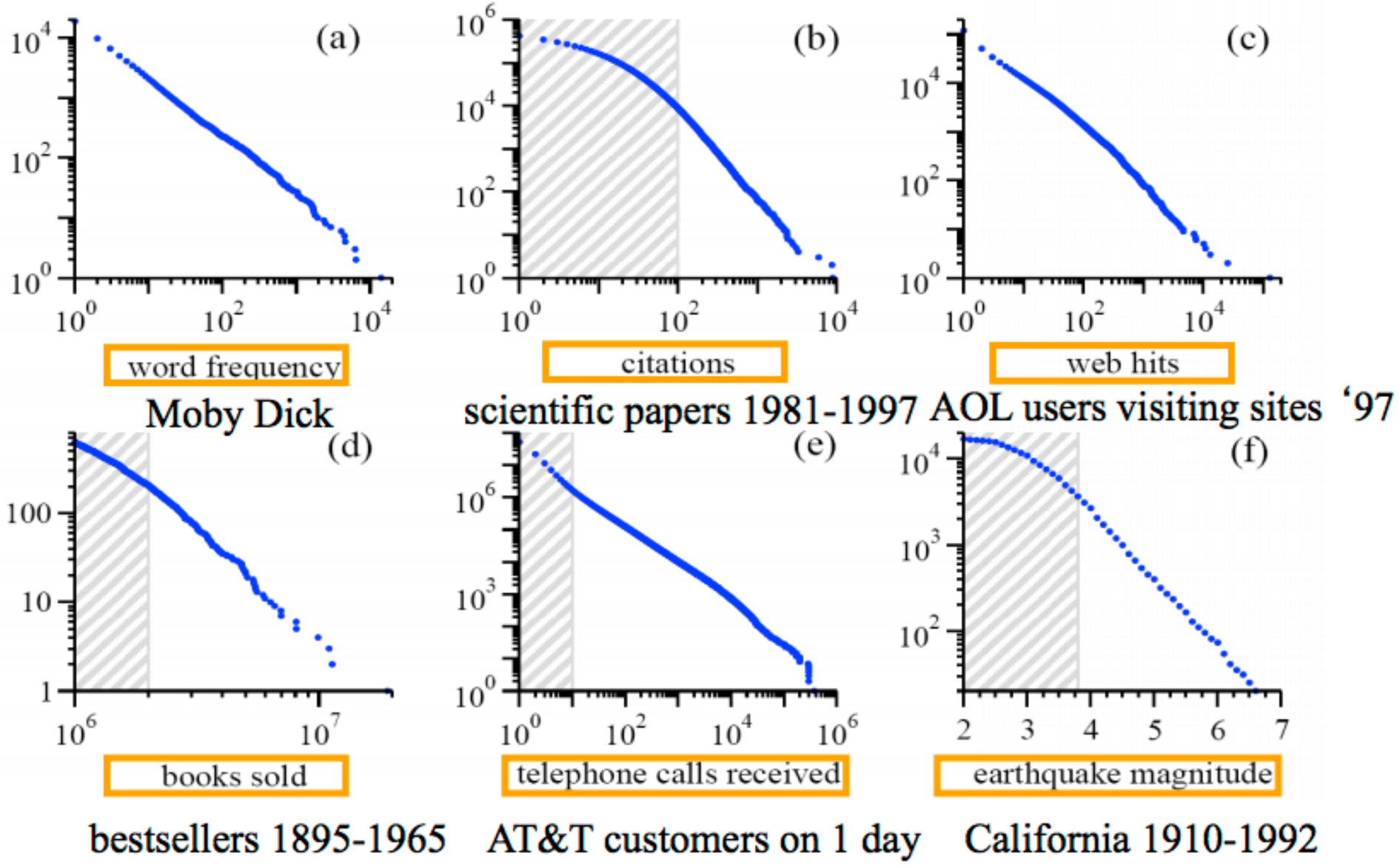
Power law

Long tail

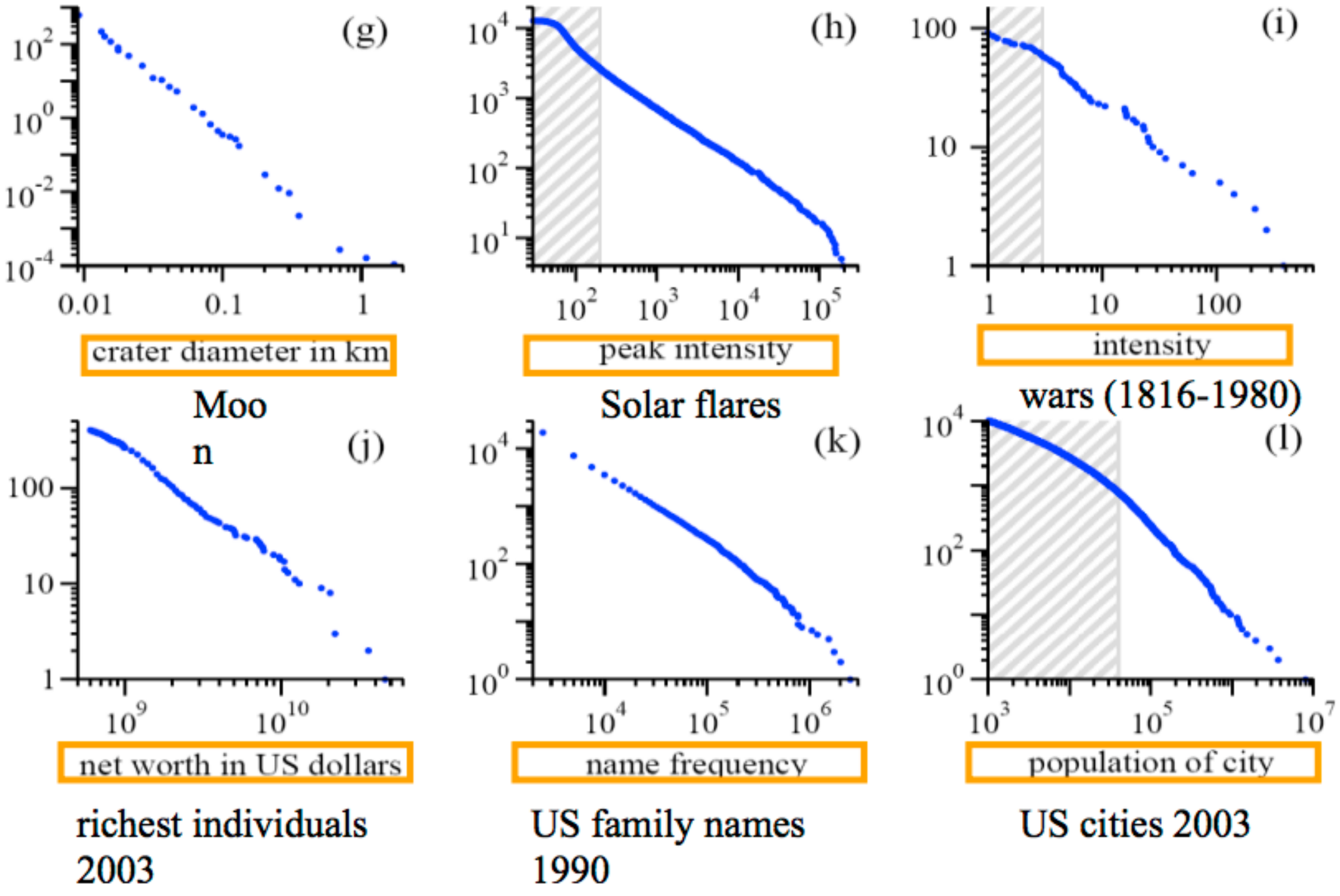
Pareto



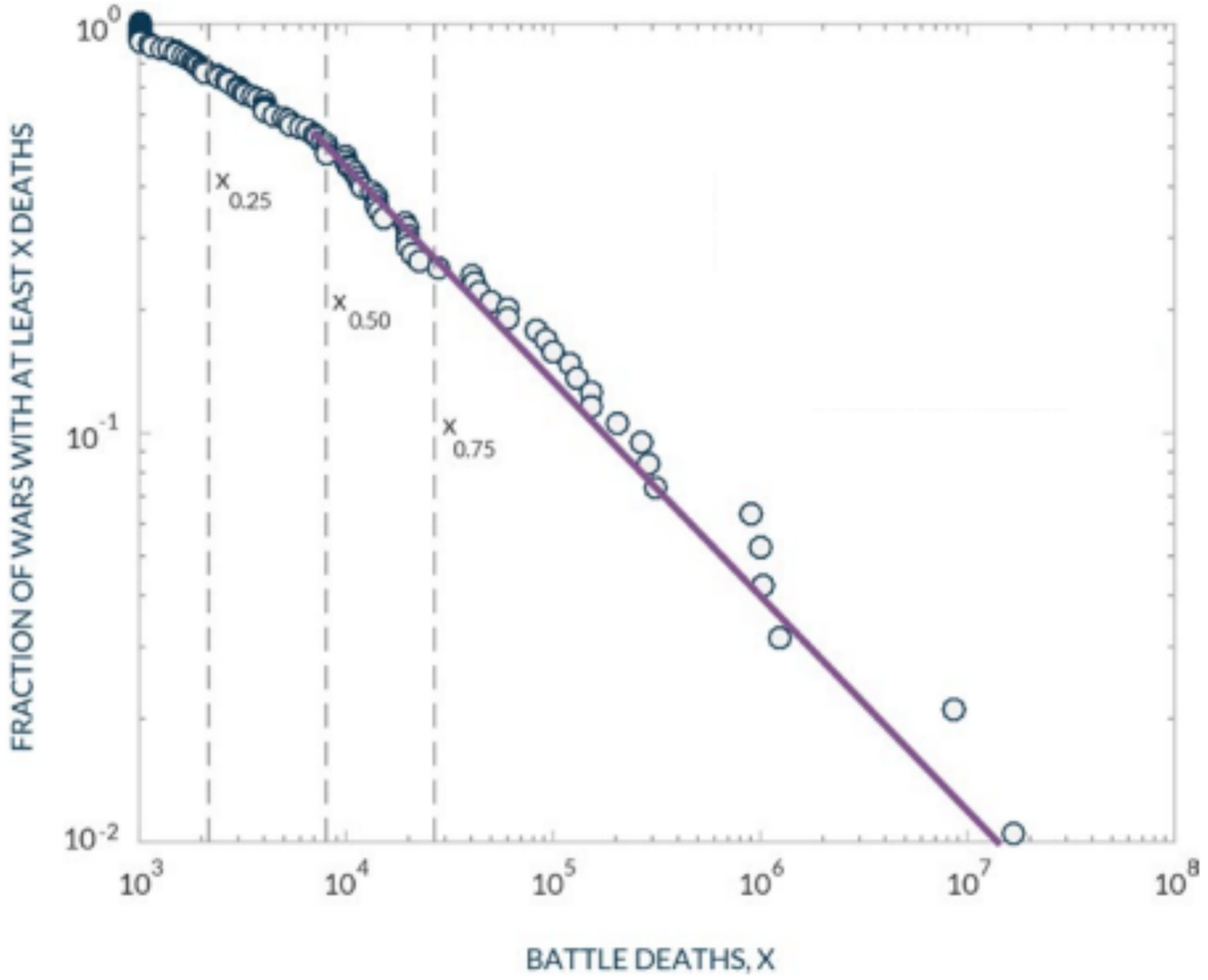
Heavy tails appear in all social systems, also in natural systems

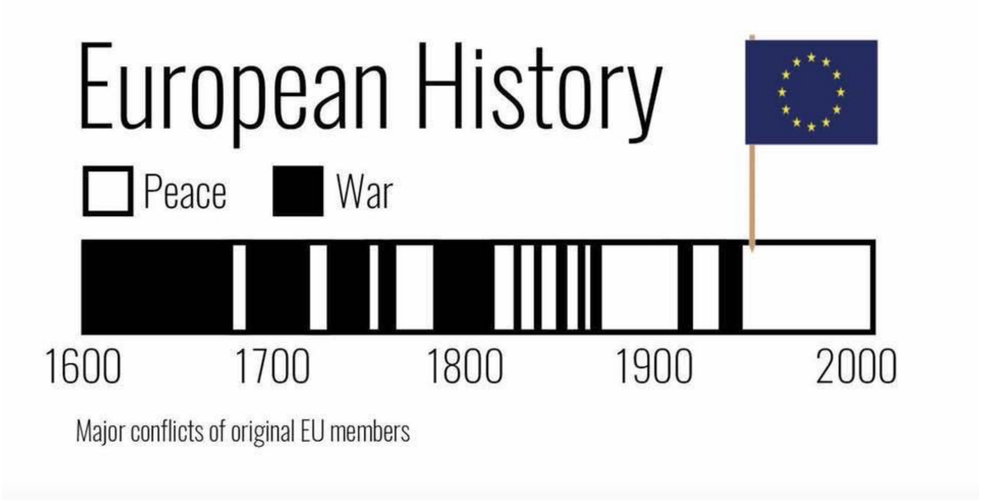


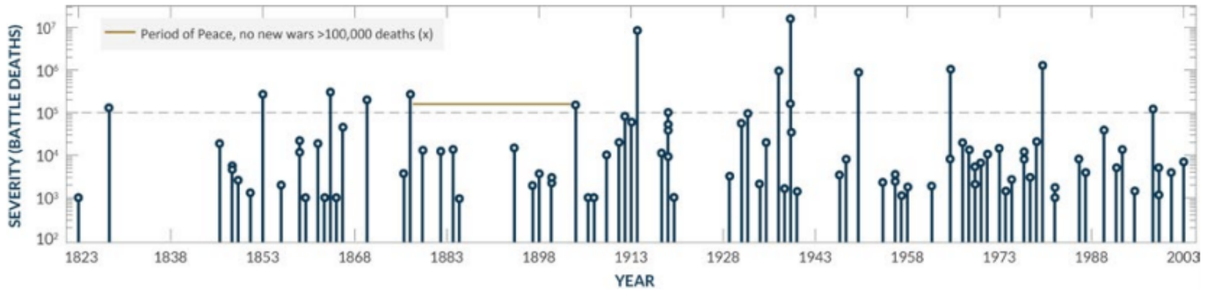
Heavy tails appear in all social systems, also in natural systems



Heavy tails appear in all social systems, also in natural systems

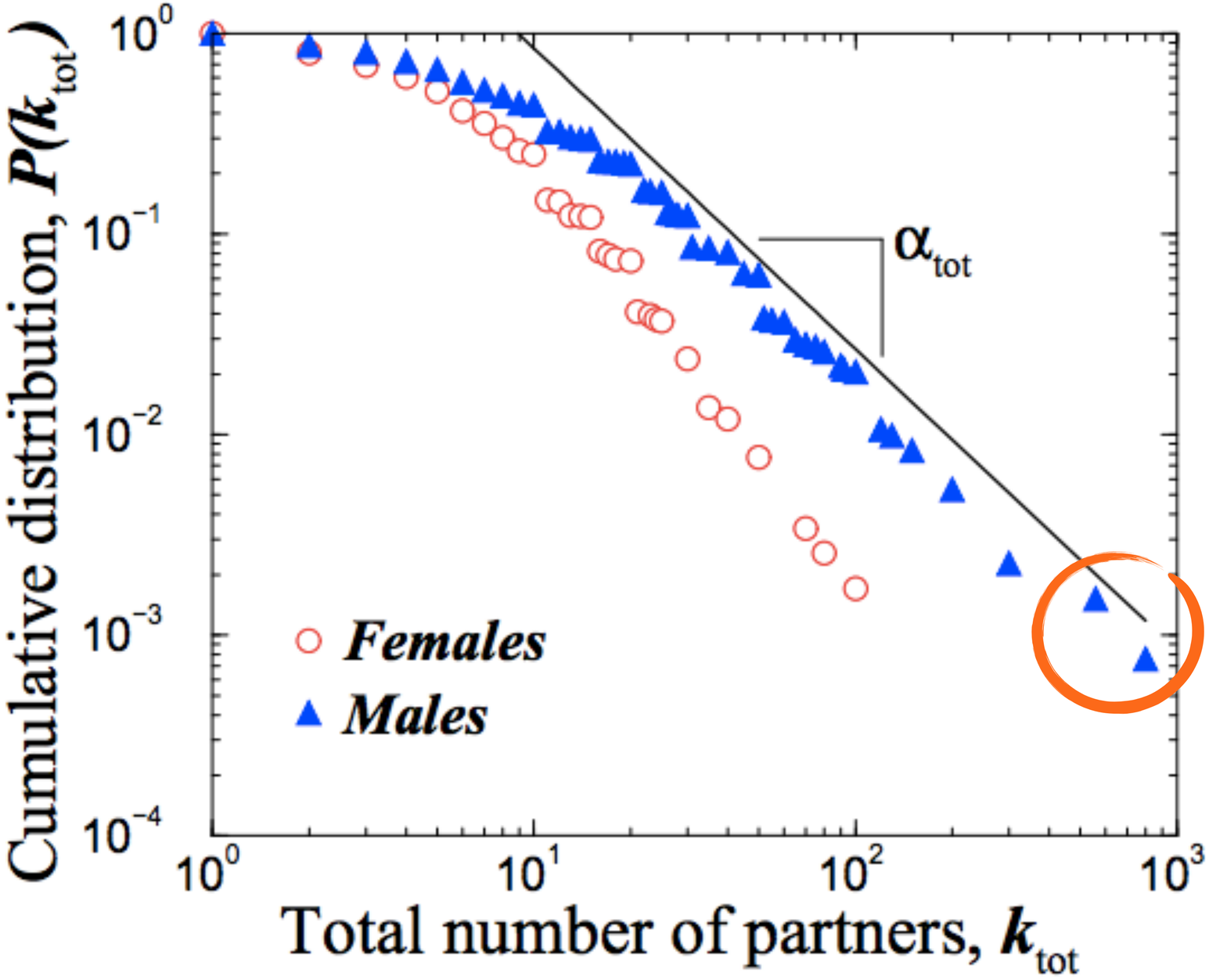






http://oefresearch.org/publications/enduring-threat-large-interstate-war

Extreme events are usually not outliers, but part of the system



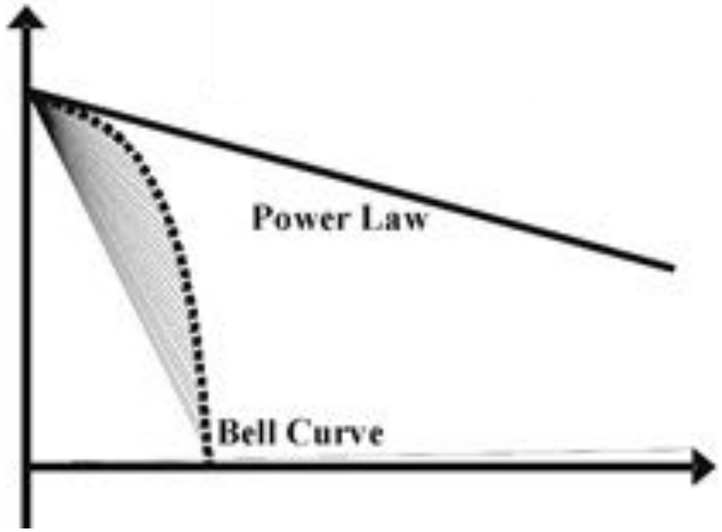
Extreme events are usually not outliers, but part of the system

20th century statistics

Strange outliers

Focus on the head/center

21th century statistics add:



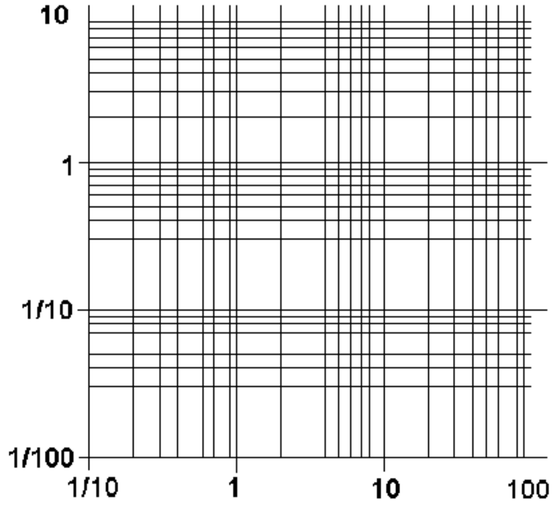
Not outliers but part of the system

Focus on the tail

Today we learned about the abundance of skewed data

Skewed data are inherent to social systems

The log transform helps to analyze skewed data



The CCDF allows to investigate heavy tails