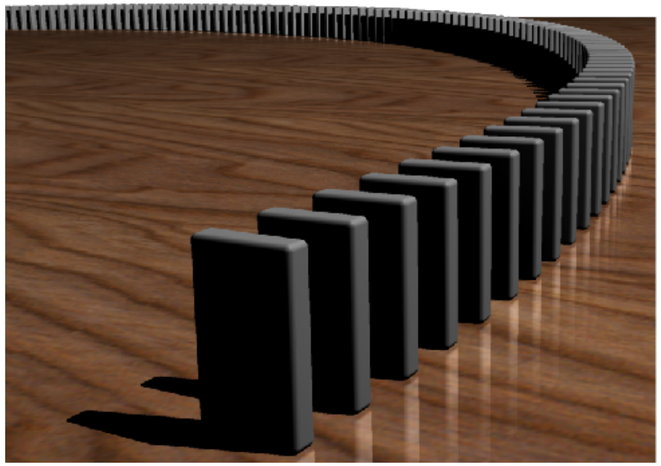
Lecture 18: Induction and command line tools

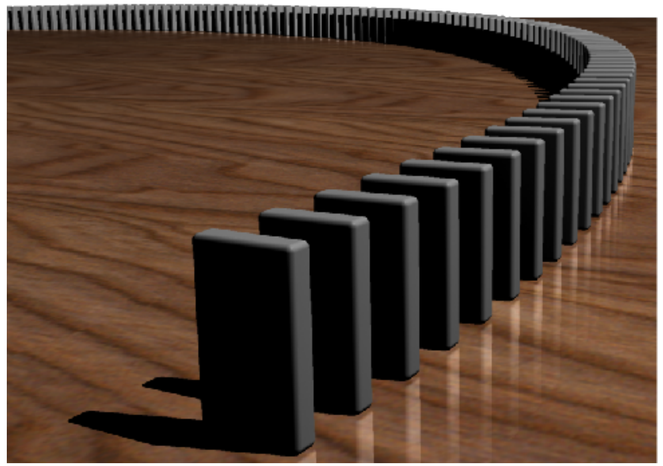
Instructor: Michael Szell
Nov 3, 2023

```
> >> awk cat cd cp
cut diff du grep
head less ls man mv
nl rm sed sort tr
uniq wc |
```



Today we learn induction proofs and command line exploration

Induction



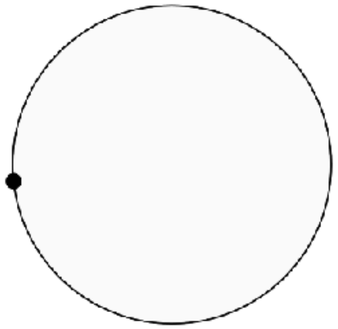
Exploratory data analysis

Command line tools

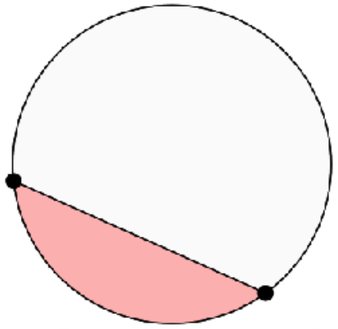
```
> >> awk cat cd cp
cut diff du grep
head less ls man mv
nl rm sed sort tr
uniq wc |
```

Induction

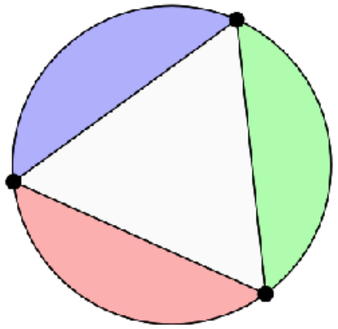
Let's explore this mathematical problem



$n=1, c=0, r_G=1$



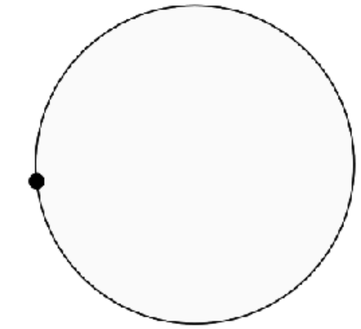
$n=2, c=1, r_G=2$



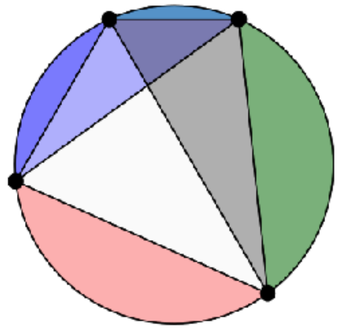
$n=3, c=3, r_G=4$

What is the maximum number of areas r we can divide a circle into by connecting chords between n points?

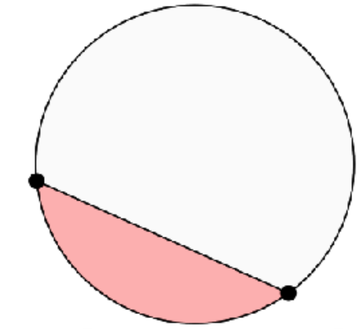
Let's explore this mathematical problem



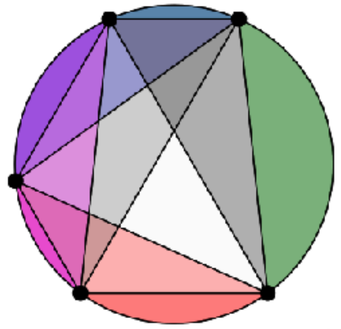
$n=1, c=0, r_G=1$



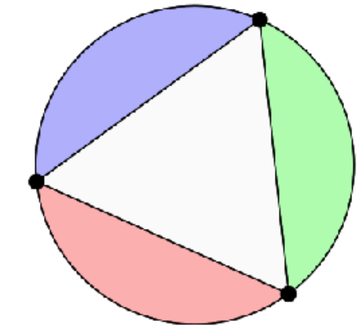
$n=4, c=6, r_G=8$



$n=2, c=1, r_G=2$



$n=5, c=10, r_G=16$



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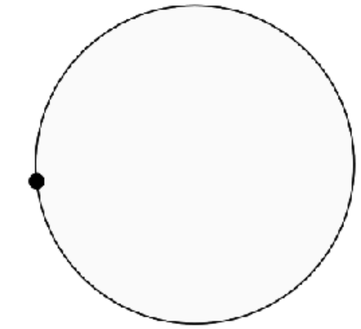
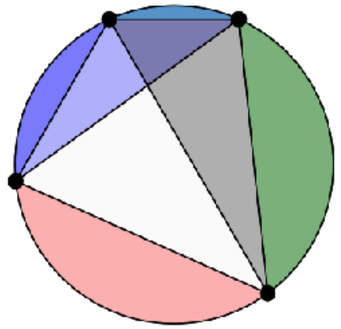
What is the maximum number of areas r we can divide a circle into by connecting chords between n points?

Ideas for a general relation?

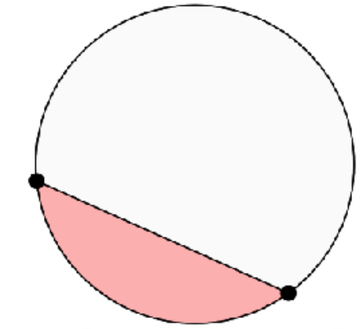
Let's explore this mathematical problem



$n=1, c=0, r_G=1$



$n=4, c=6, r_G=8$



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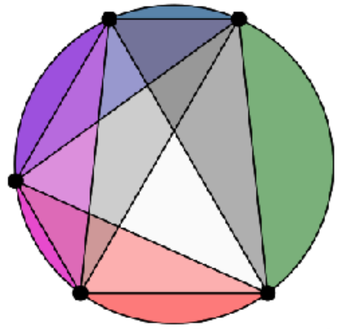

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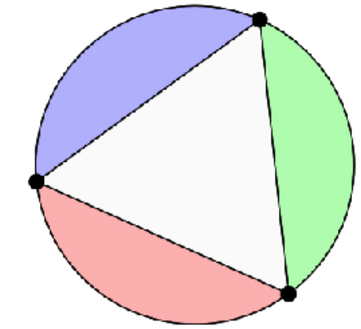
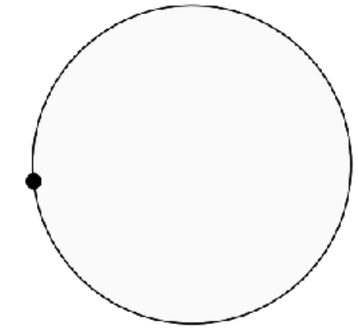


$n=3, c=3, r_G=4$

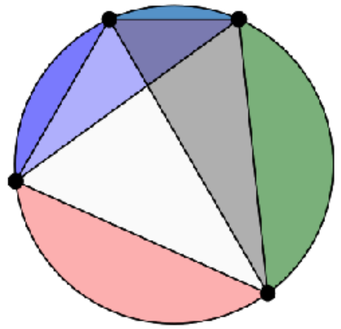
What is the maximum number of areas r we can divide a circle into by connecting chords between n points?

Looks like: $r = 2^{n-1}$

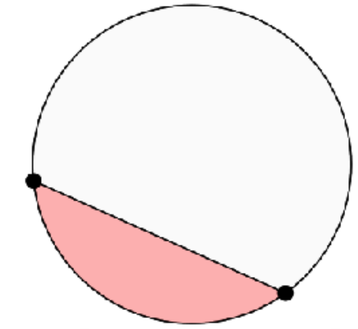
Let's explore Moser's circle problem



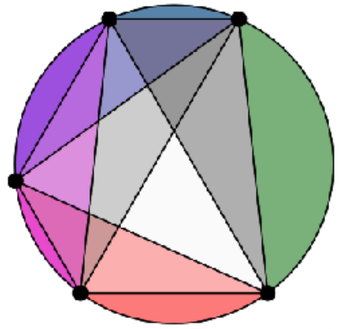
$n=1, c=0, r_G=1$



$n=4, c=6, r_G=8$



$n=2, c=1, r_G=2$



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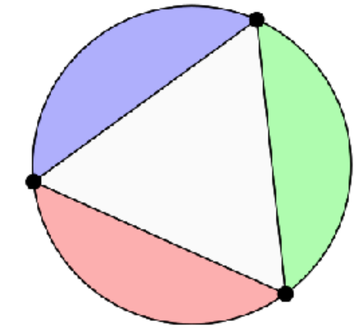
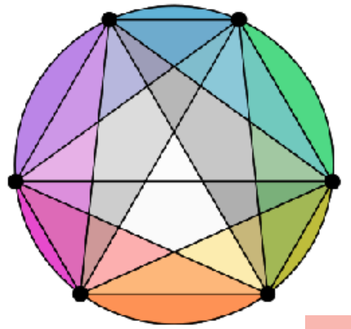


$n=3, c=3, r_G=4$



$n=6, c=15, r_G=31$

What is the maximum number of areas r we can divide a circle into by connecting chords between n points?

~~Looks like: $r = 2^{n-1}$~~

Borwein integral

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

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$$\frac{467807924713440738696537864469\pi}{935615849440640907310521750000} \approx 0.9999999999852937186\,\frac{\pi}{2}$$

https://math.stackexchange.com/questions/111440/examples-of-patterns-that-eventually-fail

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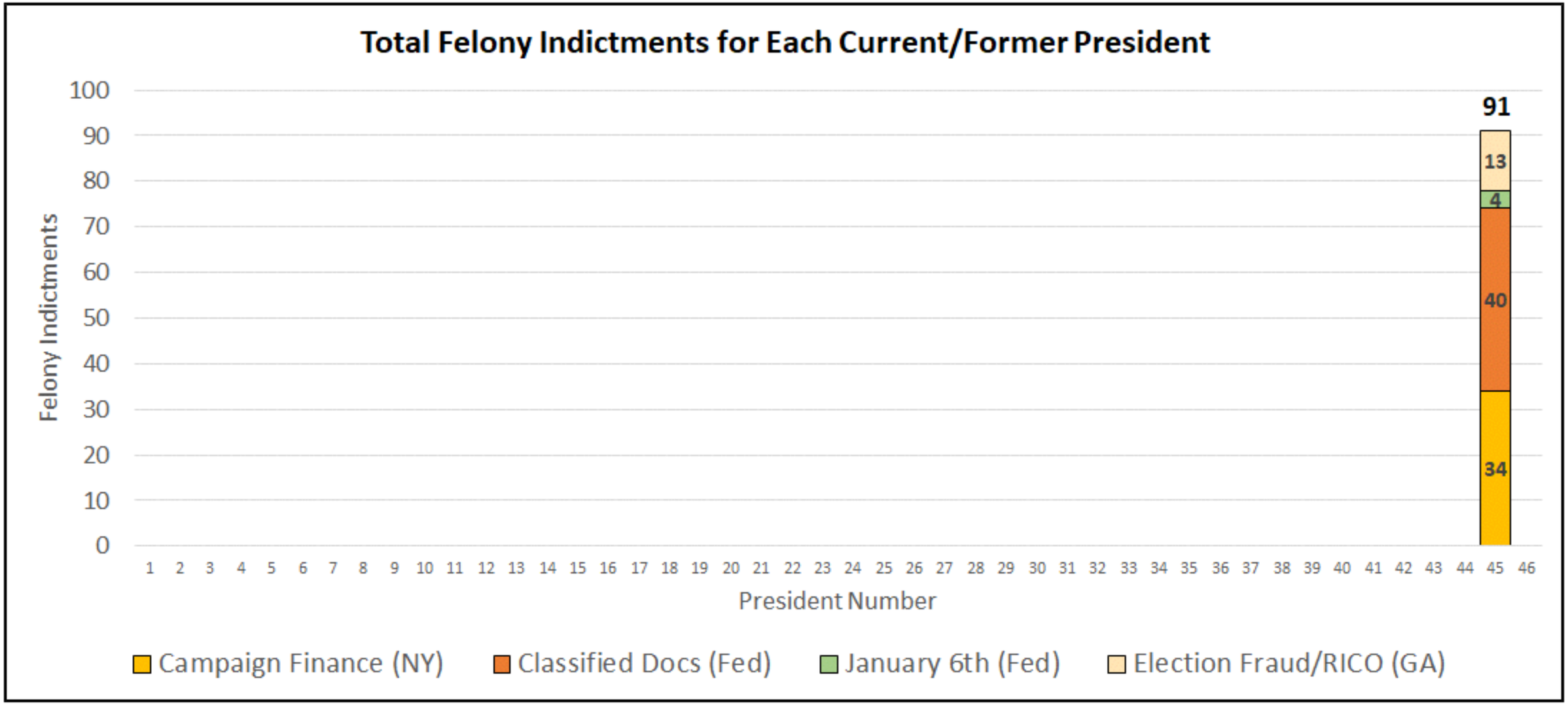
$$\frac{467807924713440738696537864469\pi}{935615849440640907310521750000} \approx 0.9999999999852937186\,\frac{\pi}{2}$$

Out of **infinite** cases, if you find just **one**, you disproved the claimed rule

Counterexample

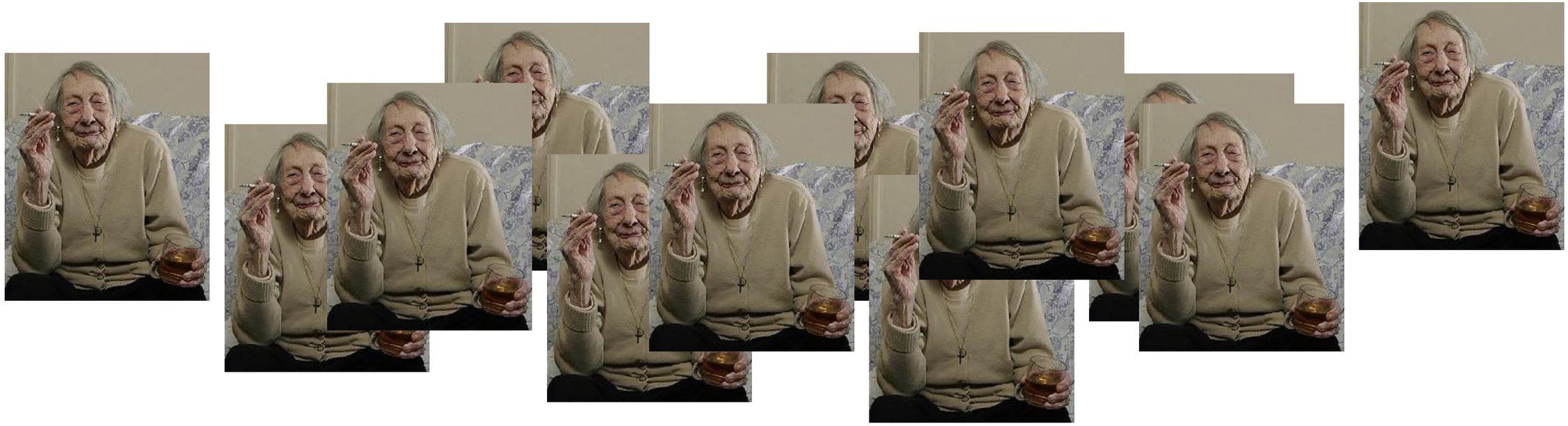
https://math.stackexchange.com/questions/111440/examples-of-patterns-that-eventually-fail

"Not happened" is not proof for "never happening"





How much data do we need to collect to *prove* a relation?



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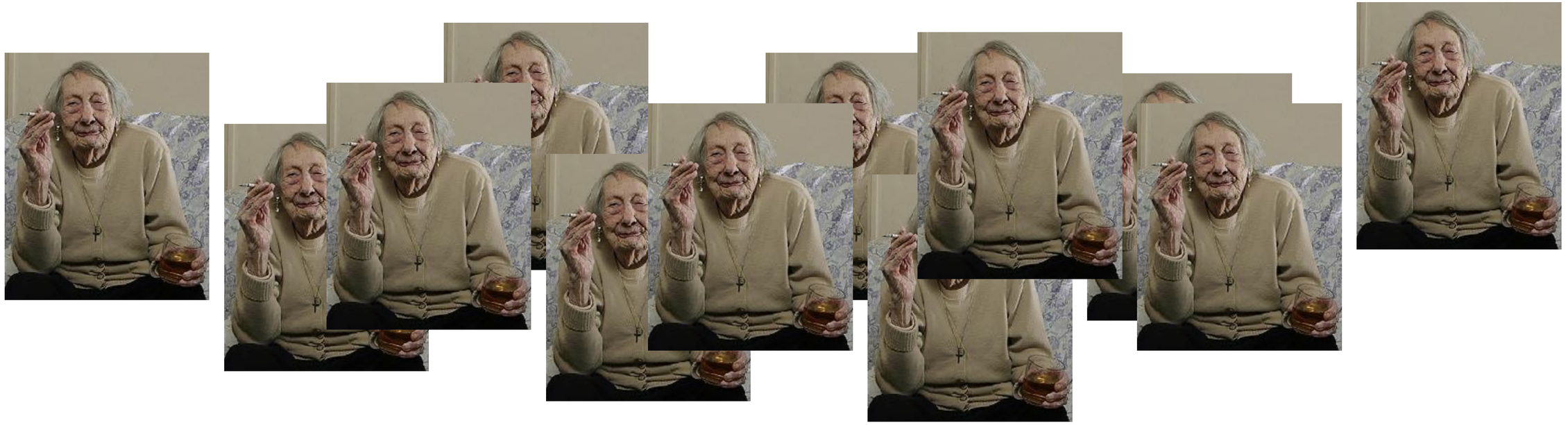
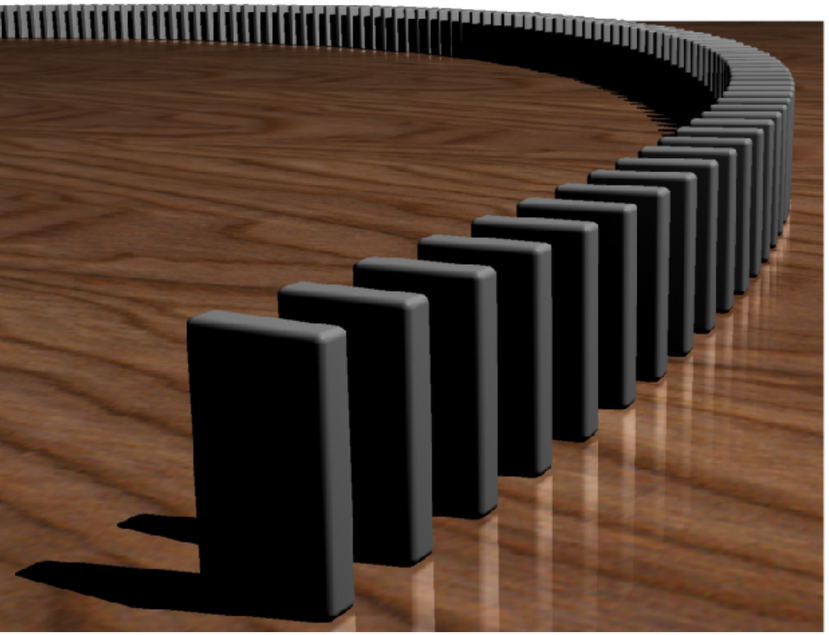


How much data do we need to collect to *prove* a relation?

In statistics, there are rules of thumb but they are arbitrary.

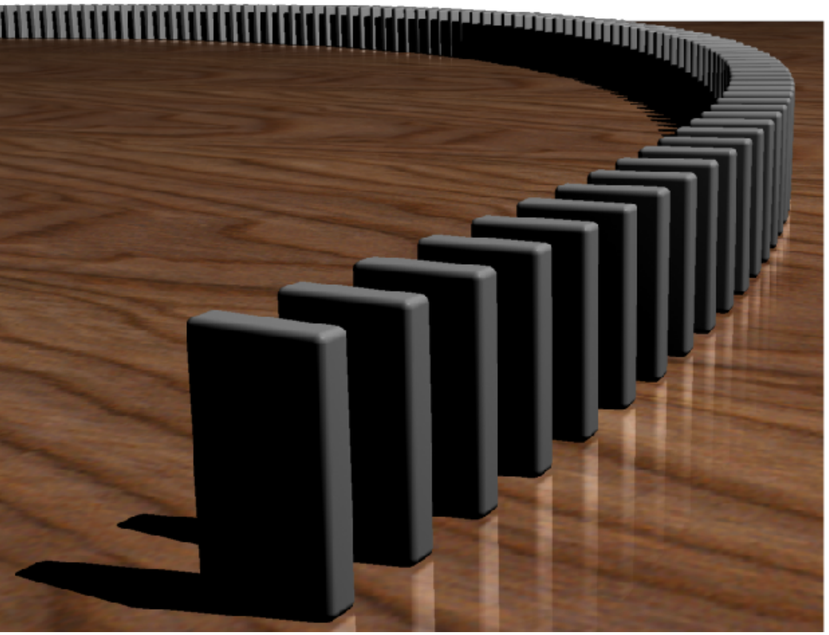
In maths, we can prove things *without doubt,* with induction



1) Base case

Show that a statement holds for $n = 0$ (or a fixed number)

In maths, we can prove things *without doubt,* with induction



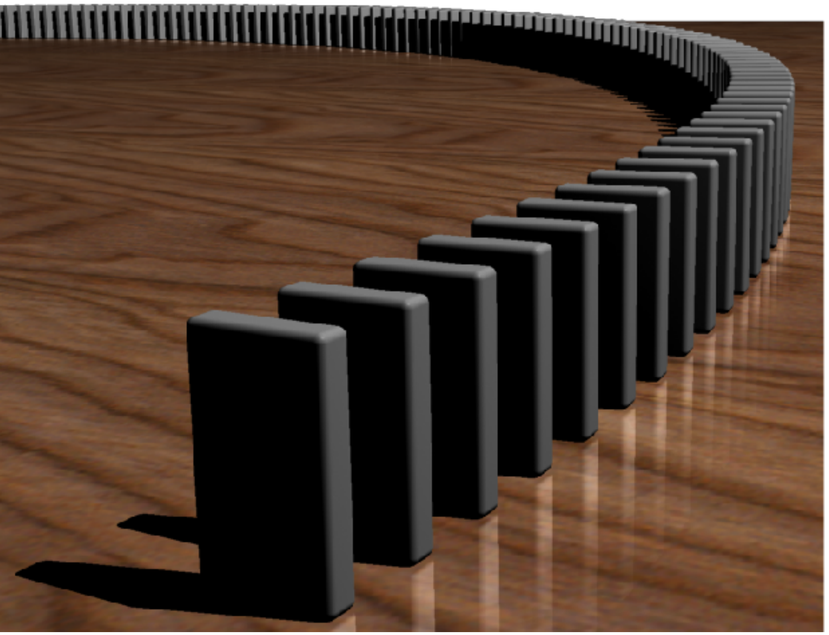
1) Base case

Show that a statement holds for $n = 0$ (or a fixed number)

2) Induction step

Show that IF a statement holds for any given $n = k$
THEN it holds for $n = k + 1$

In maths, we can prove things *without doubt,* with induction



1) Base case

Show that a statement holds for $n = 0$ (or a fixed number)

2) Induction step

Show that IF a statement holds for any given $n = k$
THEN it holds for $n = k + 1$

If you do that, you *Prove* the statement for all. infinite. numbers. forever.

Works because numbers are well-ordered: 0,1,2,3,...

Example: Gauss' sum formula

$$1 + 2 + 3 + \cdots + 100 = ?$$

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$$1 + 2 + 3 + \cdots + 100 = 5050$$

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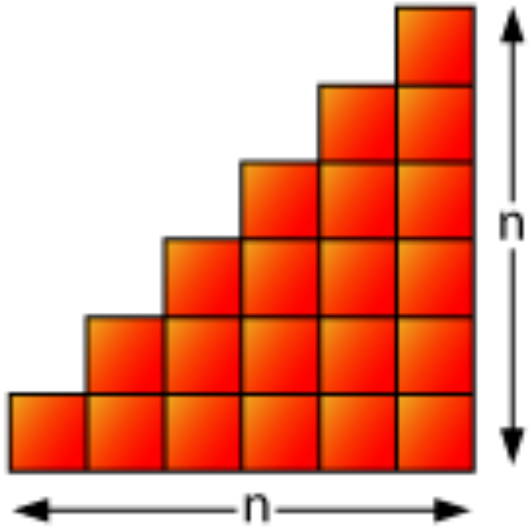
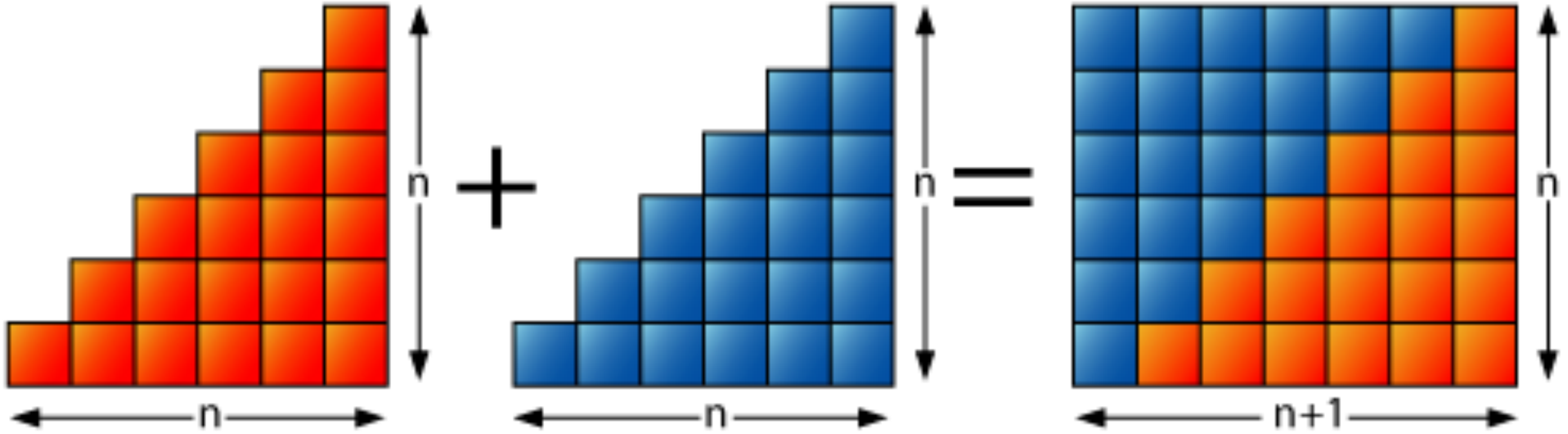


Example: Gauss' sum formula

$$1 + 2 + 3 + \cdots + 100 = 5050$$

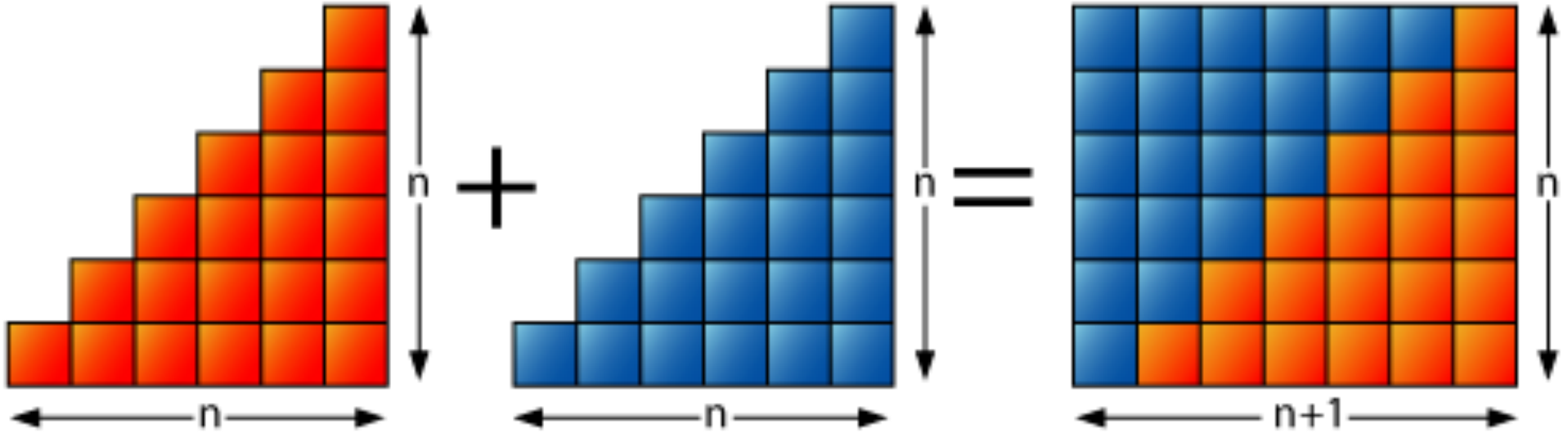
$$1 + 2 + 3 + \cdots + n = ?$$



Example: Gauss' sum formula

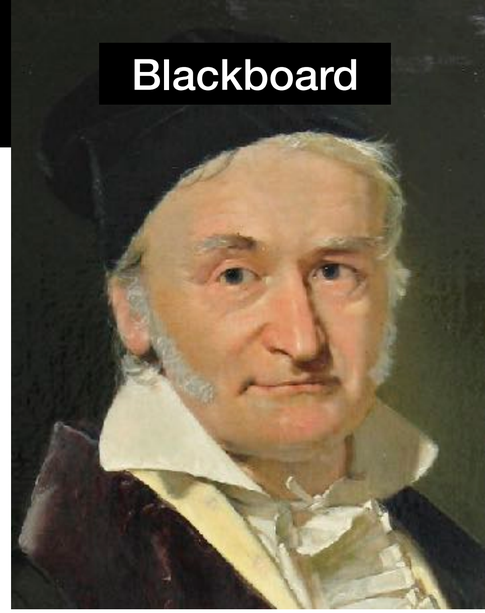
$$1 + 2 + 3 + \cdots + 100 = 5050$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$



Visual proof

Example: Gauss' sum formula



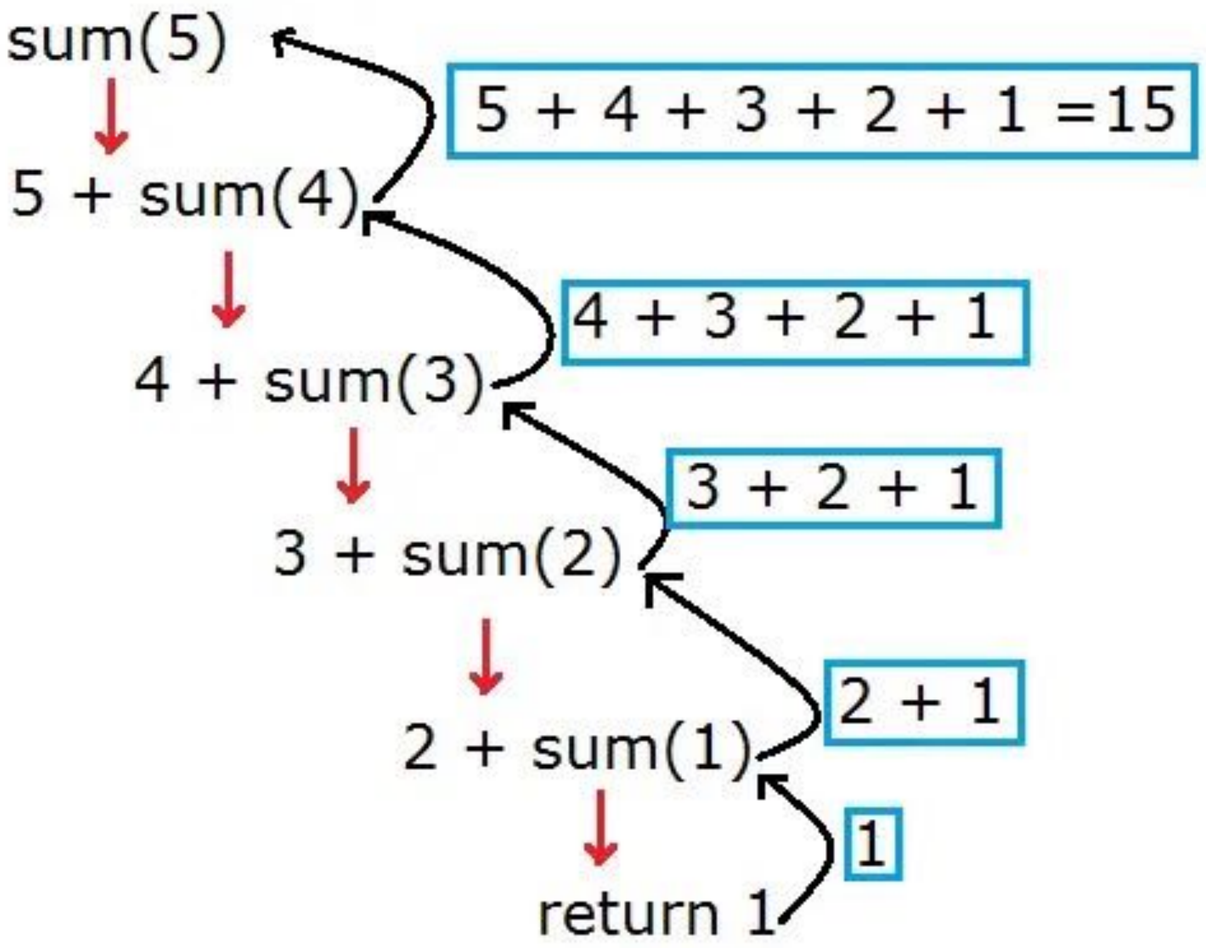
$$1 + 2 + 3 + \cdots + 100 = 5050$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Induction proof..

Induction in maths is what recursion is in computer science

Blackboard



https://pencilprogrammer.com/cpp-tutorials/recursion/

Example 2

Blackboard

Question 4: Induction (55 points)

Prove the following by induction. Explicitly state the base case and the induction step.

Prove for all $n \geq 4$, that:

$$2n < n!$$

Data Science at the command line

Command line tools are really fast

Agile

Augmenting

Scalable

Extensible

Ubiquitous

https://www.datascienceatthecommandline.com/1e/chapter-1-introduction.html#why-data-science-at-the-command-line

Command line tools are really fast

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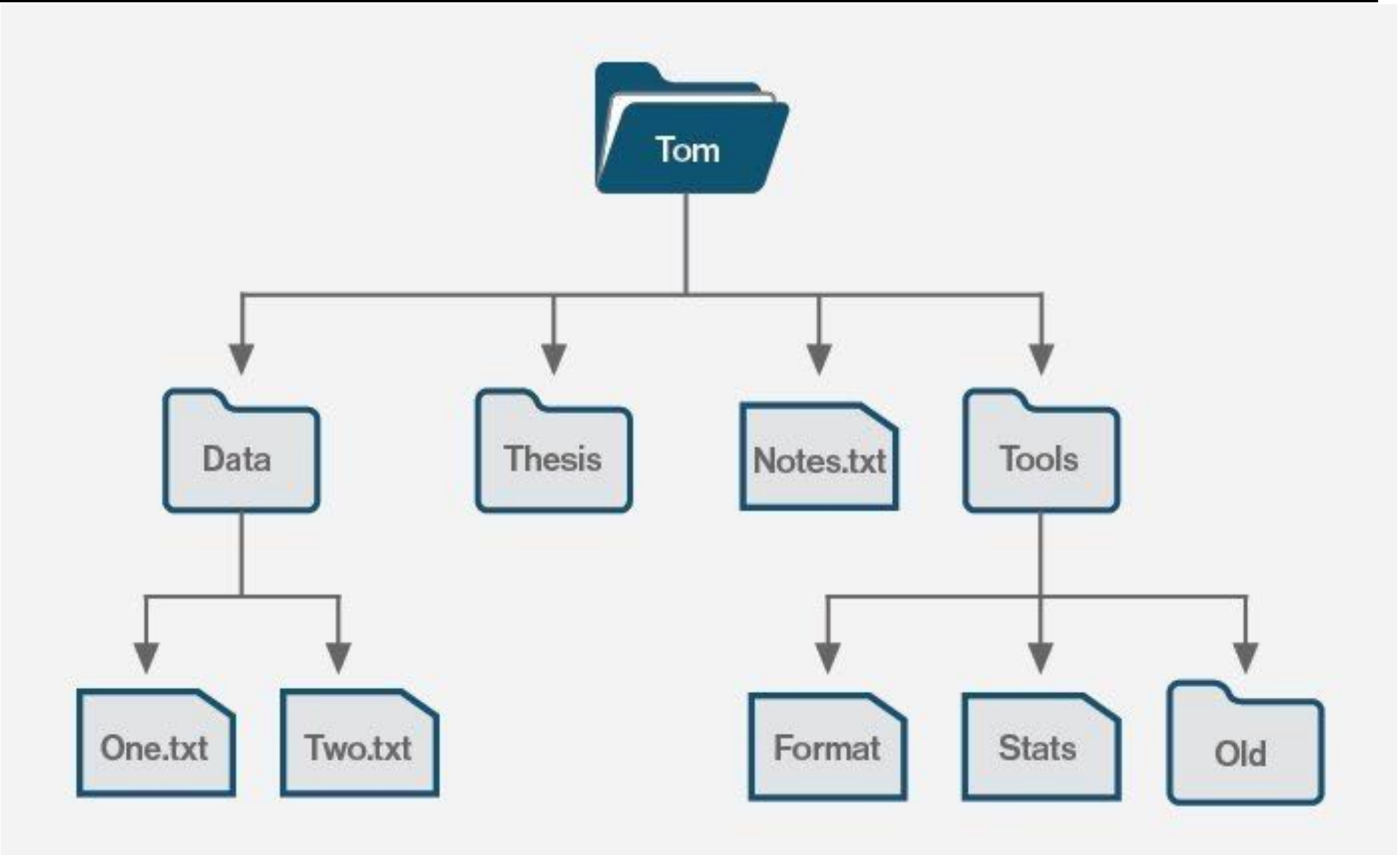
Ubiquitous

Choice between command line, Python, or even Excel depends on size of your data and what you want to do.

https://www.datascienceatthecommandline.com/1e/chapter-1-introduction.html#why-data-science-at-the-command-line

The file system is the way how files are stored logically



File system operations you should become familiar with

Terminal

`cd`	change directory
`mv`	move
`rm`	remove
`cp`	copy
`ls`	list directory
`du`	disk usage
`cat`	concatenate, or just output content

Getting help for a command `cmd`

Terminal

```
man cmd

cmd --help

cmd -h
```

Many commands have useful options

home

ls

ls -a

Many commands have useful options

Terminal

home

`ls`

`ls -a`

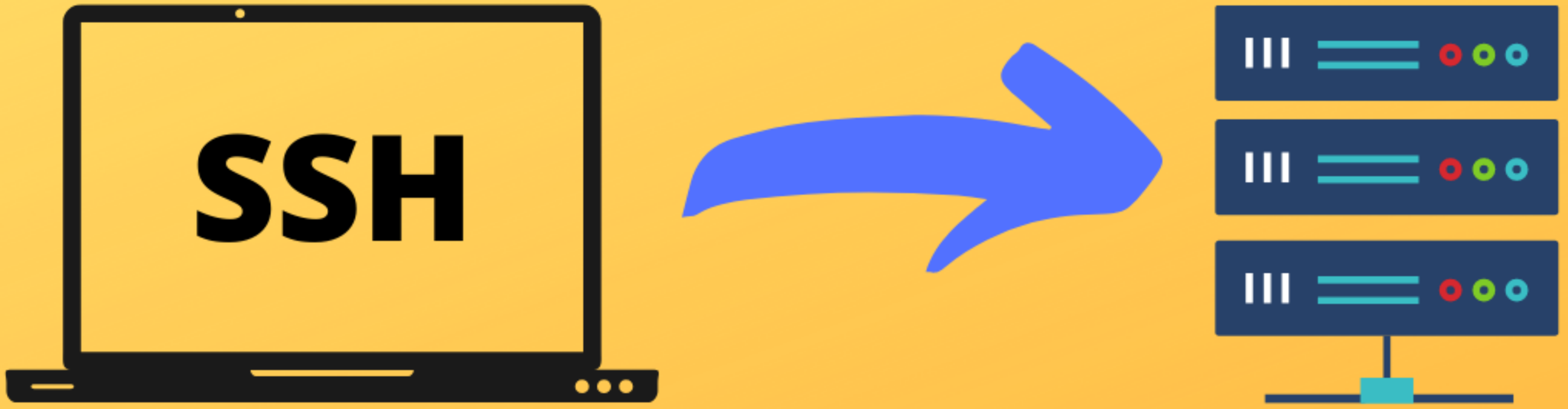
Show all files
(including hidden ones)

long format

`ls -ahl`

human-readable

You access most servers with `ssh`



Use high-performance computing (HPC) for big computations

Terminal

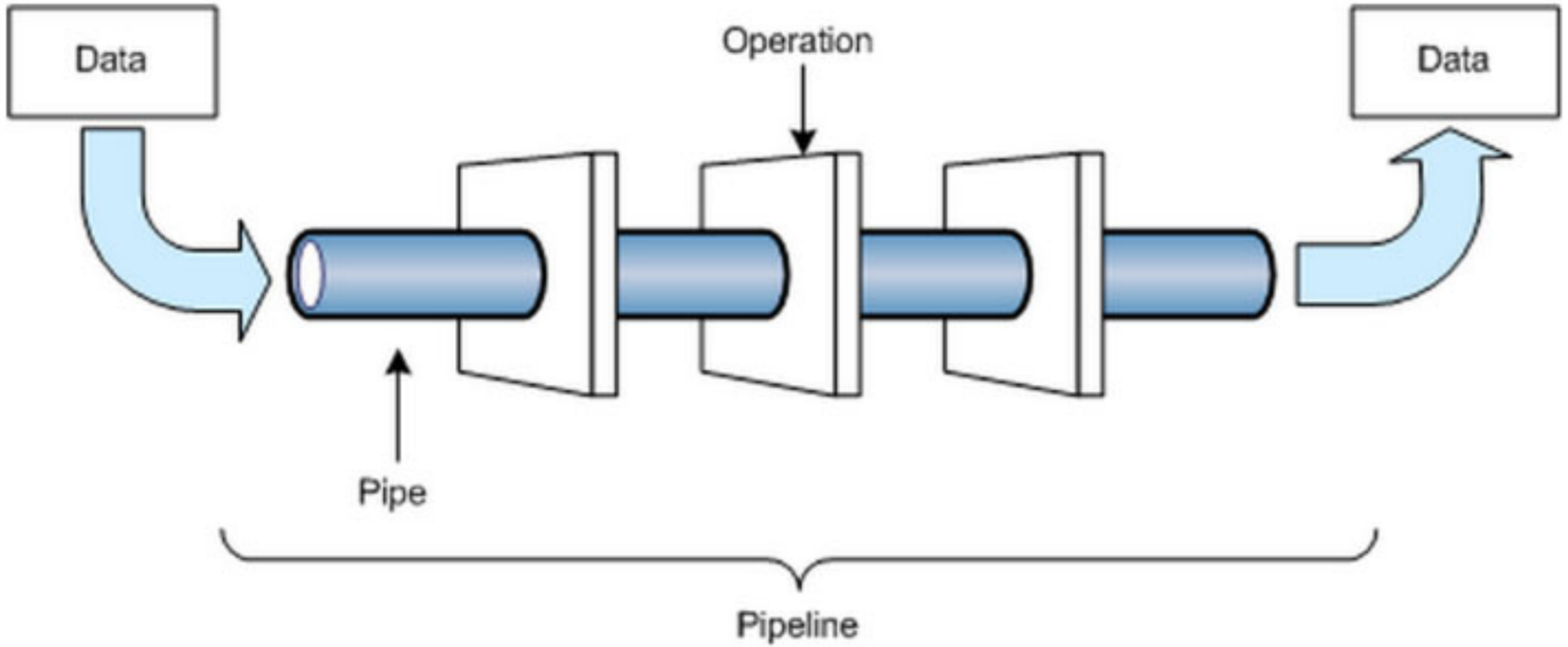
The HPC cluster offers computational resources for researchers for research projects and students in courses with computational needs.

Most important tools for data exploration

`head`	returns top lines
`less`	navigate file content
`wc, wc -l`	word count, line count
`tr`	string replacement
`sort`	sort
`uniq`	find unique (adjacent!) lines
`cut`	cut
`diff`	file difference
`nl`	line number

The pipe | combines commands into a pipeline

```
cmd1 | cmd2 | cmd3 | ...
```



https://en.wikipedia.org/wiki/Pipeline_(Unix)

The > redirects your output, >> appends

```
cmd infile.txt > outfile.txt

cmd infile.txt >> outfile.txt
```

Tools you should have heard about

`grep` globally search for a regular expression and print matching lines

`sed` stream editor

additionally to grep, good for replacing strings

awk is a scripting languauge, powerful data extraction tool

Example: print the second column in a CSV file

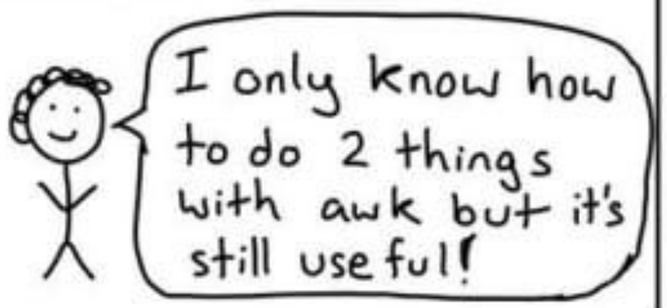
```
awk -F',' '{print $2}' mycsv.csv
```

https://en.wikipedia.org/wiki/AWK

awk

Julia Evans
@b0rk

awk is a tiny programming language for manipulating columns of data



basic awk program structure

```
BEGIN { ... }
CONDITION {action}
CONDITION {action}
END {...}
```

do action on lines matching CONDITION

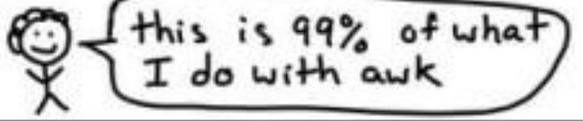
extract a column of text with awk

```
awk -F, '{print $5}'
```

column separator

single quotes!

print the 5th column



SO MANY unix commands print columns of text (ps! ls!)

so being able to get the column you want with awk is GREAT

A few more awk programs →

sum the numbers in the 3rd column

```
{s += $3}
END {print s}
```

action

at the end, print the sum!

print every line over 80 characters

```
length($0) > 80
```

condition

(there's an implicit {print} as the action)

Your Data Science command line alphabet

```
cd mv rm cp ls du cat grep sed man awk head
less wc tr sort uniq cut diff nl | > >>
```

Your Data Science command line alphabet

Terminal

```
cd mv rm cp ls du cat grep sed man awk head
less wc tr sort uniq cut diff nl | > >>
```

Exercise (together): Make a pipeline that takes dsalphabet.txt and creates dsalphabet_ordered.txt:

```
> >> awk cat cd cp cut diff du grep head less
ls man mv nl rm sed sort tr uniq wc |
```

Your Data Science command line alphabet

Terminal

```
cd mv rm cp ls du cat grep sed man awk head
less wc tr sort uniq cut diff nl | > >>
```

Exercise (together): Make a pipeline that takes dsalphabet.txt and creates dsalphabet_ordered.txt:

```
> >> awk cat cd cp cut diff du grep head less
ls man mv nl rm sed sort tr uniq wc |
```

Solution: `cat dsalphabet.txt | tr " " "\n" | sort | tr "\n" " " > dsalphabet_ordered.txt`

Data set: Road collisions in the UK in 2019

Road Safety Data

Published by:	Department for Transport
Last updated:	08 January 2021
Topic:	Transport
Licence:	Open Government Licence

Summary

Road Safety Statistics releases

https://data.gov.uk/dataset/road-accidents-safety-data

Some exercises with the accidents data

Terminal

Exploring the data

Total number of casualties?

Which are the LSOAs with most accidents?

Some exercises with the accidents data

Terminal

Exploring the data

```
head files/accidents.csv
less files/accidents.csv
```

Total number of casualties?

```
awk -F, '{s+=$9} END {print s}' files/accidents.csv
```

Which are the LSOAs with most accidents?

```
awk -F, 'NR>1 {print $NF}' files/accidents.csv  | sort | uniq -c | sort -k 1 -r | head
```

Get familiar with terminal shortcuts to navigate fast

Terminal

LINUX TERMINAL SHORTCUTS CHEATSHEET

⇤ ⇥	Automatically complete the file, directory, or command you're typing.
Ctrl C	Kill the current foreground process running in terminal.
Ctrl Z	Suspend the current foreground process running in terminal.
Ctrl D	Delete the character at the cursor location.
Ctrl U	Erases the complete line.
Ctrl K	Erase the part of the line after the cursor.
Ctrl W	Erase the word before the cursor.
Ctrl Y	Paste the last thing you cut from the clipboard.
Ctrl L	Clears the screen just like clear command.
Ctrl A	Move the cursor to the beginning of the line.
Ctrl E	Move the cursor to the end of the line.
Ctrl P	Go to the previous command in the command history.
Ctrl N	Go to the next command in the command history.
Ctrl R	Search for a command in command history.

https://linuxhandbook.com/linux-shortcuts/

LINUX HANDBOOK

https://linuxhandbook.com/linux-shortcuts/

https://hackertyper.net/

Jupyter

In the next year, explore all "mandatory" Unix commands

List of Unix commands

From Wikipedia, the free encyclopedia

"Unix command" redirects here. For other uses, see Command (computing).

This is a list of Unix commands as specified by IEEE Std 1003.1-2008, which is part of the Single UNIX Specification (S
can be found on Unix operating systems and most Unix-like operating systems.

Contents [show]

List [edit]

IEEE Std 1003.1-2008 utilities

Name	Category	Status (Option code)	Description
alias	Misc	Mandatory	Define or display aliases
ar	Misc	Mandatory	Create and maintain library archives
at	Process management	Mandatory	Execute commands at a later time
awk	Text processing	Mandatory	Pattern scanning and processing language
basename	Filesystem	Mandatory	Return non-directory portion of a pathname; see also dirname
batch	Process management	Mandatory	Schedule commands to be executed in a batch queue
bc	Misc	Mandatory	Arbitrary-precision arithmetic language
cat	Filesystem	Mandatory	Concatenate and print files
cd	Filesystem	Mandatory	Change the working directory
chgrp	Filesystem	Mandatory	Change the file group ownership
chmod	Filesystem	Mandatory	Change the file modes/attributes/permissions
chown	Filesystem	Mandatory	Change the file ownership
cksum	Filesystem	Mandatory	Write file checksums and sizes

https://en.wikipedia.org/wiki/List_of_Unix_commands

Why we generally do not use Excel in Data Science

Lack of:

- Reproducibility
- Version control
- Testing
- Maintainability
- Accuracy

https://datascience.stackexchange.com/questions/5443/do-data-scientists-use-excel

Why we generally do not use Excel in Data Science

Comment | Open Access | Published: 23 August 2016

Gene name errors are widespread in the scientific literature

Mark Ziemann, Yotam Eren & Assam El-Osta

Genome Biology **17**, Article number: 177 (2016) | Cite this article

127k Accesses | **45** Citations | **2567** Altmetric | Metrics

Abstract

The spreadsheet software Microsoft Excel, when used with default settings, is known to convert gene names to dates and floating-point numbers. A programmatic scan of leading genomics journals reveals that approximately one-fifth of papers with supplementary Excel gene lists contain erroneous gene name conversions.

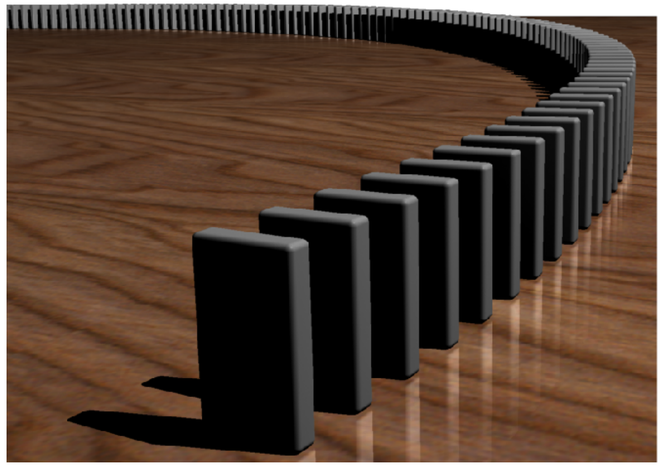
The problem of Excel software (Microsoft Corp., Redmond, WA, USA) inadvertently converting gene symbols to dates and floating-point numbers was originally described in 2004 [1]. For example, gene symbols such as *SEPT2* (Septin 2) and *MARCH1* [Membrane-Associated Ring Finger (C3HC4) 1, E3 Ubiquitin Protein Ligase] are converted by default to '2-Sep' and '1-Mar', respectively. Furthermore, RIKEN identifiers were described to be automatically converted to floating point numbers (i.e. from accession '2310009E13' to '2.31E+13'). Since that report, we have uncovered further instances where gene symbols were converted to dates in supplementary data of recently published papers (e.g. '*SEPT2*' converted to '2006/09/02'). This suggests that gene name errors continue to be a problem in supplementary files accompanying articles. Inadvertent gene symbol conversion is problematic because these supplementary files are an important resource in the genomics community that are frequently reused. Our aim here is to raise awareness of the problem.

https://genomebiology.biomedcentral.com/articles/10.1186/s13059-016-1044-7

Jupyter

Take home message for today

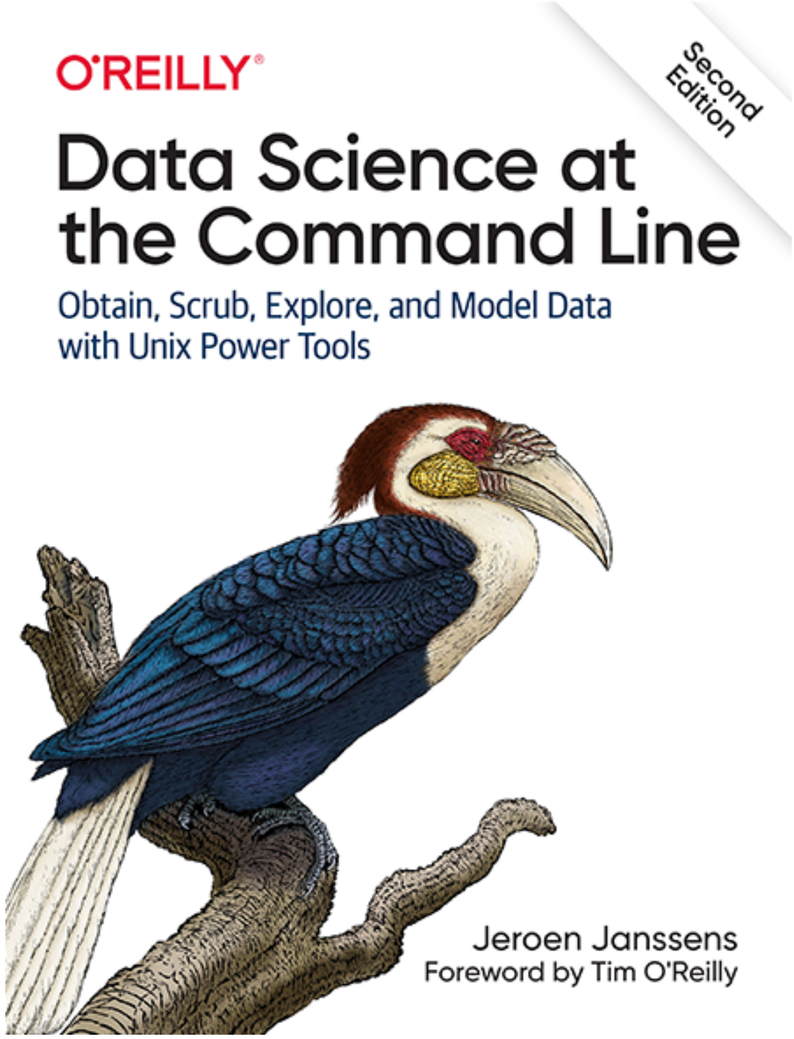
Induction truly proves mathematical relationships, not probabilistically



Command-line tools take learning but are the fastest way to explore big data (Beware of Excel)

Working with servers requires command line know-how

Sources and further materials for today's class



https://jeroenjanssens.com/dsatcl/

http://mywiki.wooledge.org/BashGuide/CommandsAndArguments

https://en.wikipedia.org/wiki/AWK

https://en.wikipedia.org/wiki/Pipeline_(Unix)

https://linuxhandbook.com/linux-shortcuts/

https://datascience.stackexchange.com/questions/5443/do-data-scientists-use-excel

https://genomebiology.biomedcentral.com/articles/10.1186/s13059-016-1044-7