IBM Model 1 and the EM Algorithm

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Lexical Translation

- How to translate a word → look up in dictionary

  Haus — house, building, home, household, shell.

- Multiple translations
  - some more frequent than others
  - for instance: house, and building most common
  - special cases: Haus of a snail is its shell

- Note: In all lectures, we translate from a foreign language into English

Collect Statistics

Look at a parallel corpus (German text along with English translation)

Translation of *Haus*	**Count**
house	8,000
building	1,600
home	200
household	150
shell	50

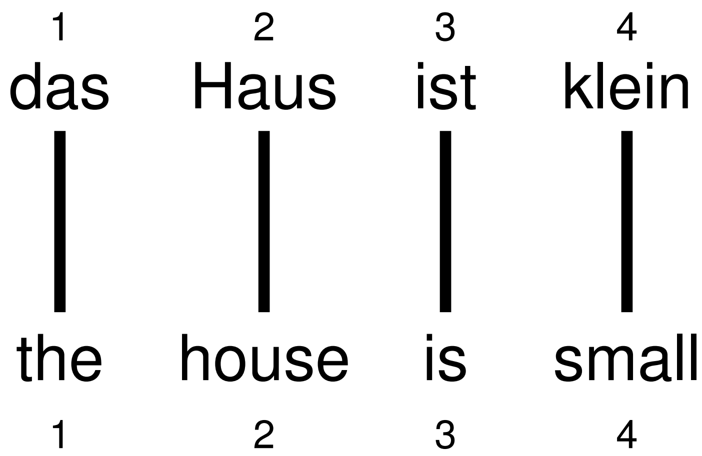
Estimate Translation Probabilities

Maximum likelihood estimation

$$p_f(e) = \begin{cases} 0.8 & \text{if } e = \text{house}, \\ 0.16 & \text{if } e = \text{building}, \\ 0.02 & \text{if } e = \text{home}, \\ 0.015 & \text{if } e = \text{household}, \\ 0.005 & \text{if } e = \text{shell}. \end{cases}$$

Alignment

- In a parallel text (or when we translate), we align words in one language with the words in the other



- Word positions are numbered 1–4

Alignment Function

- Formalizing alignment with an alignment function

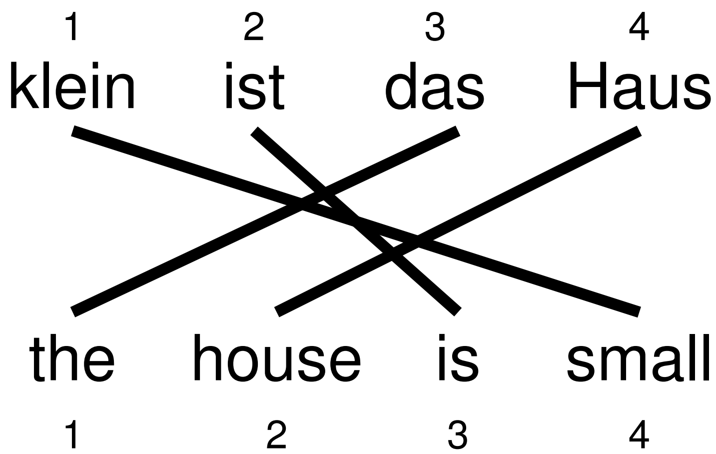
- Mapping an English target word at position i to a German source word at position j with a function $a : i \to j$

- Example

$$a : \{1 \to 1, 2 \to 2, 3 \to 3, 4 \to 4\}$$

Reordering

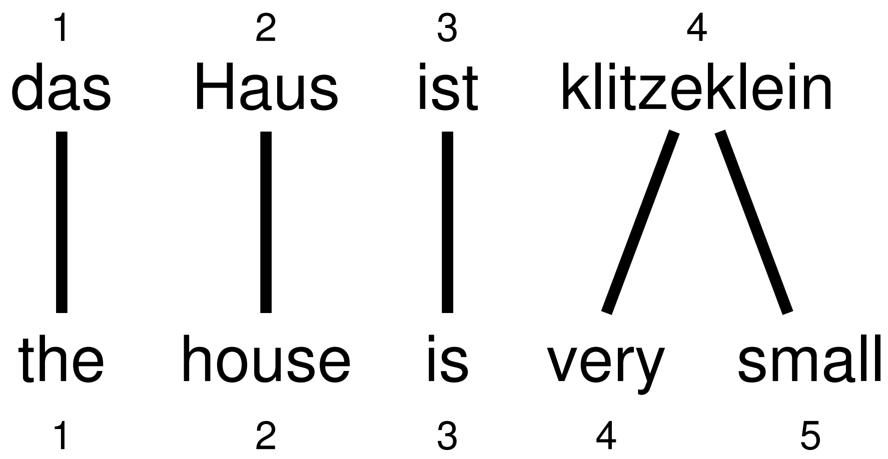
Words may be reordered during translation



$$a : \{1 \rightarrow 3, 2 \rightarrow 4, 3 \rightarrow 2, 4 \rightarrow 1\}$$

One-to-Many Translation

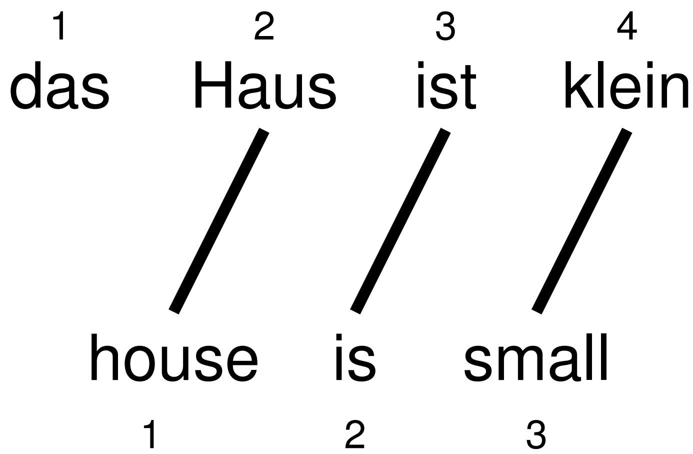
A source word may translate into multiple target words



$$a : \{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 4\}$$

Dropping Words

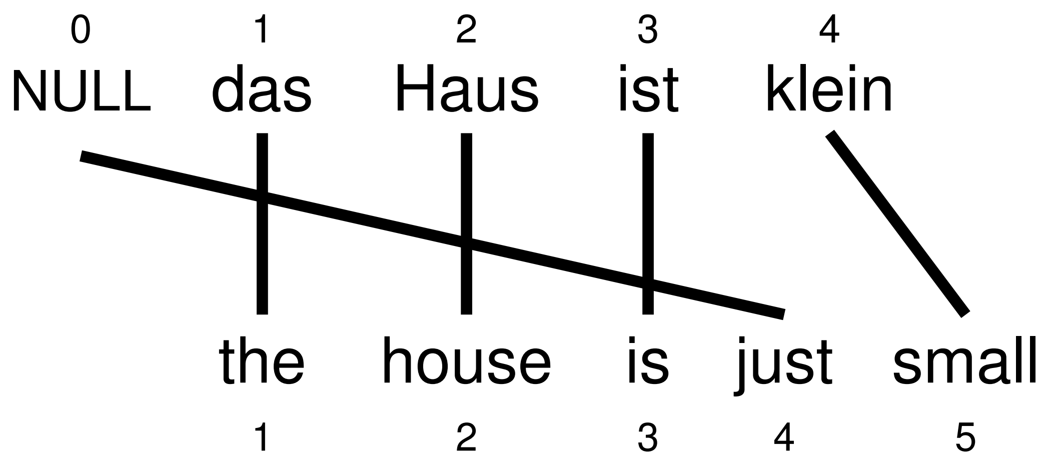
Words may be dropped when translated
(German article das is dropped)



$$a : \{1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4\}$$

Inserting Words

- Words may be added during translation
  - The English just does not have an equivalent in German
  - We still need to map it to something: special NULL token



$$a : \{1 \to 1, 2 \to 2, 3 \to 3, 4 \to 0, 5 \to 4\}$$

IBM Model 1

- Generative model: break up translation process into smaller steps
  - IBM Model 1 only uses lexical translation

- Translation probability
  - for a foreign sentence $\mathbf{f} = (f_1, ..., f_{l_f})$ of length l_f
  - to an English sentence $\mathbf{e} = (e_1, ..., e_{l_e})$ of length l_e
  - with an alignment of each English word e_j to a foreign word f_i according to the alignment function $a : j \to i$

$$p(\mathbf{e}, a|\mathbf{f}) = \frac{\epsilon}{(l_f + 1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

  - parameter ϵ is a normalization constant

Example

das	
e	$t(e\|f)$
the	0.7
that	0.15
which	0.075
who	0.05
this	0.025

Haus	
e	$t(e\|f)$
house	0.8
building	0.16
home	0.02
household	0.015
shell	0.005

ist	
e	$t(e\|f)$
is	0.8
's	0.16
exists	0.02
has	0.015
are	0.005

klein	
e	$t(e\|f)$
small	0.4
little	0.4
short	0.1
minor	0.06
petty	0.04

$$
\begin{aligned}
p(e,a|f) &= \frac{\epsilon}{5^4} \times t(\text{the}|\text{das}) \times t(\text{house}|\text{Haus}) \times t(\text{is}|\text{ist}) \times t(\text{small}|\text{klein}) \\
&= \frac{\epsilon}{5^4} \times 0.7 \times 0.8 \times 0.8 \times 0.4 \\
&= 0.00029\epsilon
\end{aligned}
$$

finding translations

Centauri-Arcturan Parallel Text

1a. ok-voon ororok sprok .
1b. at-voon bichat dat .

2a. ok-drubel ok-voon anok plok sprok .
2b. at-drubel at-voon pippat rrat dat .

3a. erok sprok izok hihok ghirok .
3b. totat dat arrat vat hilat .

4a. ok-voon anok drok brok jok .
4b. at-voon krat pippat sat lat .

5a. wiwok farok izok stok .
5b. totat jjat quat cat .

6a. lalok sprok izok jok stok .
6b. wat dat krat quat cat .

7a. lalok farok ororok lalok sprok izok enemok .
7b. wat jjat bichat wat dat vat eneat .

8a. lalok brok anok plok nok .
8b. iat lat pippat rrat nnat .

9a. wiwok nok izok kantok ok-yurp .
9b. totat nnat quat oloat at-yurp .

10a. lalok mok nok yorok ghirok clok .
10b. wat nnat gat mat bat hilat .

11a. lalok nok crrrok hihok yorok zanzanok .
11b. wat nnat arrat mat zanzanat .

12a. lalok rarok nok izok hihok mok .
12b. wat nnat forat arrat vat gat .

Translation challenge: **farok crrrok hihok yorok clok kantok ok-yurp**

(from Knight (1997): Automating Knowledge Acquisition for Machine Translation)

em algorithm

Learning Lexical Translation Models

- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus

- ... but we do not have the alignments

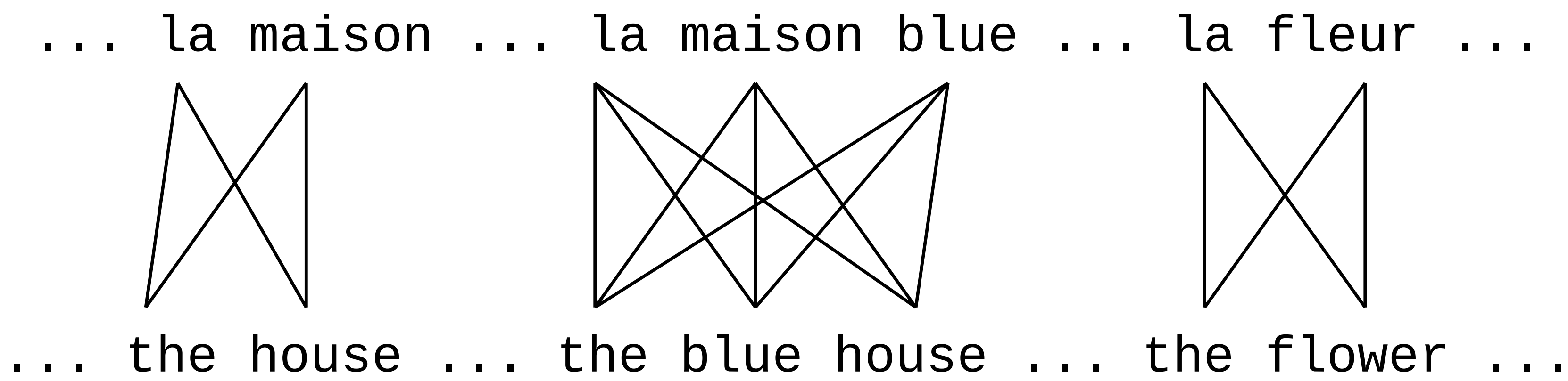
- Chicken and egg problem
  - if we had the *alignments*,
    $\rightarrow$ we could estimate the *parameters* of our generative model
  - if we had the *parameters*,
    $\rightarrow$ we could estimate the *alignments*

EM Algorithm

- Incomplete data
  - if we had *complete data,* would could estimate *model*
  - if we had *model,* we could fill in the *gaps in the data*

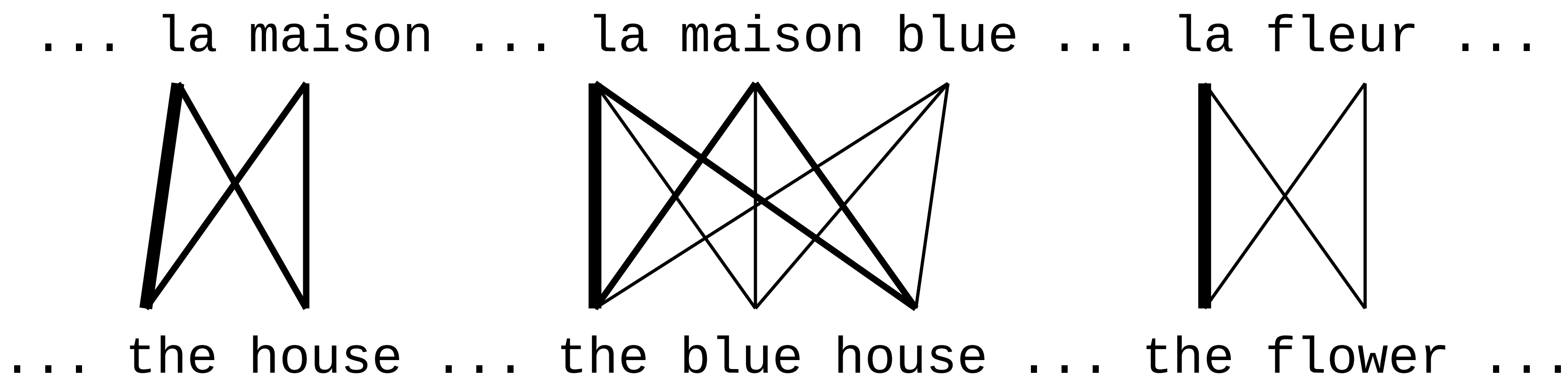
- Expectation Maximization (EM) in a nutshell
  1. initialize model parameters (e.g. uniform)
  2. assign probabilities to the missing data
  3. estimate model parameters from completed data
  4. iterate steps 2–3 until convergence

EM Algorithm



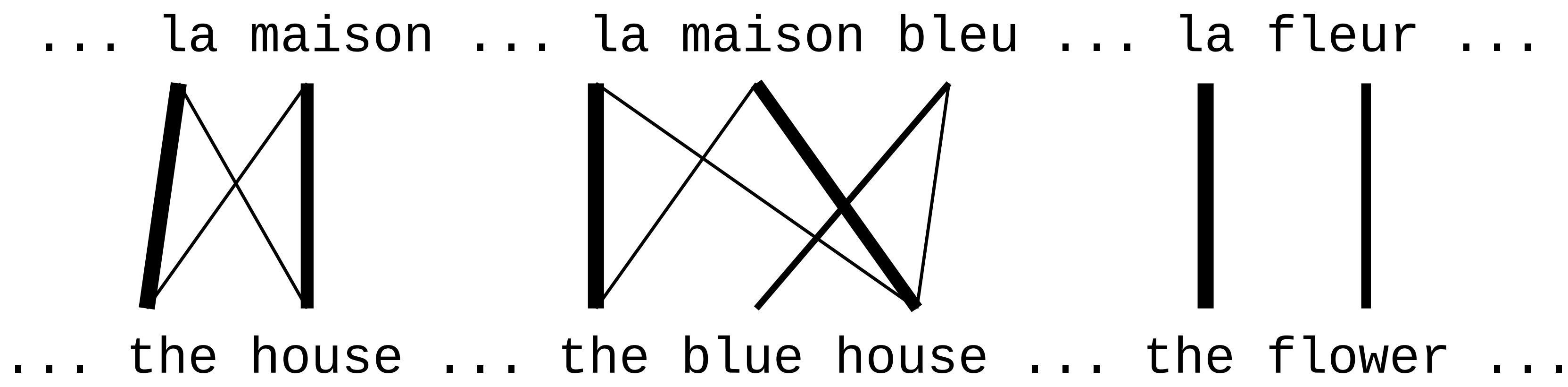
- Initial step: all alignments equally likely
- Model learns that, e.g., la is often aligned with the

EM Algorithm



- After one iteration
- Alignments, e.g., between la and the are more likely

EM Algorithm



- After another iteration

- It becomes apparent that alignments, e.g., between fleur and flower are more likely (pigeon hole principle)

EM Algorithm

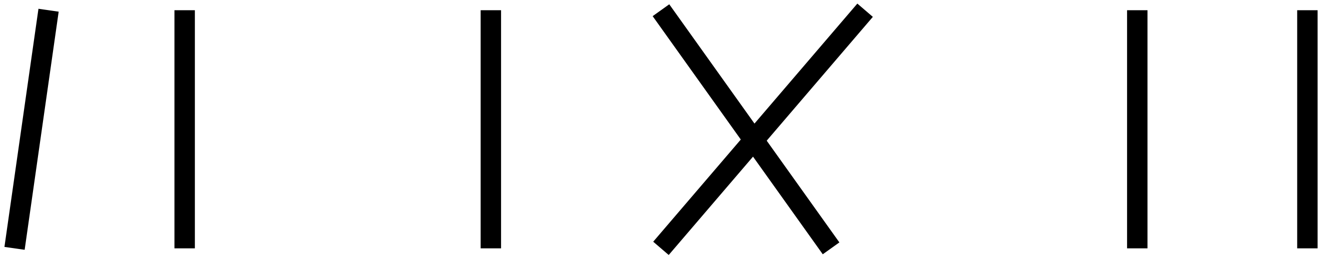
```
... la maison ... la maison bleu ... la fleur ...
```

```
... the house ... the blue house ... the flower ...
```

- Convergence
- Inherent hidden structure revealed by EM

EM Algorithm

```
... la maison ... la maison bleu ... la fleur ...
```



```
... the house ... the blue house ... the flower ...
```

```
p(la|the) = 0.453
p(le|the) = 0.334
p(maison|house) = 0.876
p(bleu|blue) = 0.563
...
```

- Parameter estimation from the aligned corpus

IBM Model 1 and EM

- EM Algorithm consists of two steps
- Expectation-Step: Apply model to the data
  - parts of the model are hidden (here: alignments)
  - using the model, assign probabilities to possible values
- Maximization-Step: Estimate model from data
  - take assign values as fact
  - collect counts (weighted by probabilities)
  - estimate model from counts
- Iterate these steps until convergence

IBM Model 1 and EM

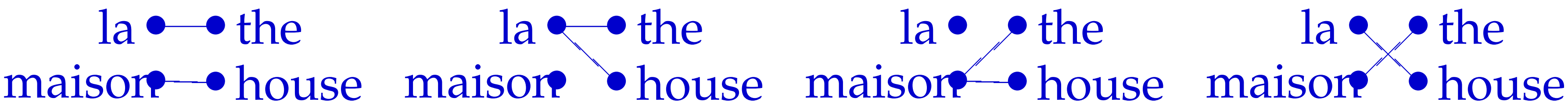
- We need to be able to compute:
  - Expectation-Step: probability of alignments
  - Maximization-Step: count collection

IBM Model 1 and EM

- **Probabilities**

$$p(\text{the}|\text{la}) = 0.7 \qquad p(\text{house}|\text{la}) = 0.05$$
$$p(\text{the}|\text{maison}) = 0.1 \qquad p(\text{house}|\text{maison}) = 0.8$$

- **Alignments**



$$p(\mathbf{e}, a|\mathbf{f}) = 0.56 \qquad p(\mathbf{e}, a|\mathbf{f}) = 0.035 \qquad p(\mathbf{e}, a|\mathbf{f}) = 0.08 \qquad p(\mathbf{e}, a|\mathbf{f}) = 0.005$$

$$p(a|\mathbf{e}, \mathbf{f}) = 0.824 \qquad p(a|\mathbf{e}, \mathbf{f}) = 0.052 \qquad p(a|\mathbf{e}, \mathbf{f}) = 0.118 \qquad p(a|\mathbf{e}, \mathbf{f}) = 0.007$$

- **Counts**

$$c(\text{the}|\text{la}) = 0.824 + 0.052 \qquad c(\text{house}|\text{la}) = 0.052 + 0.007$$
$$c(\text{the}|\text{maison}) = 0.118 + 0.007 \qquad c(\text{house}|\text{maison}) = 0.824 + 0.118$$

IBM Model 1 and EM: Expectation Step

- We need to compute $p(a|\mathbf{e},\mathbf{f})$
- Applying the chain rule:

$$p(a|\mathbf{e},\mathbf{f}) = \frac{p(\mathbf{e},a|\mathbf{f})}{p(\mathbf{e}|\mathbf{f})}$$

- We already have the formula for $p(\mathbf{e},\mathbf{a}|\mathbf{f})$ (definition of Model 1)

IBM Model 1 and EM: Expectation Step

- We need to compute $p(\mathbf{e}|\mathbf{f})$

$$
\begin{aligned}
p(\mathbf{e}|\mathbf{f}) &= \sum_a p(\mathbf{e}, a|\mathbf{f}) \\
&= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} p(\mathbf{e}, a|\mathbf{f}) \\
&= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})
\end{aligned}
$$

IBM Model 1 and EM: Expectation Step

$$
\begin{aligned}
p(\mathbf{e}|\mathbf{f}) &= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)}) \\
&= \frac{\epsilon}{(l_f+1)^{l_e}} \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \prod_{j=1}^{l_e} t(e_j|f_{a(j)}) \\
&= \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} \sum_{i=0}^{l_f} t(e_j|f_i)
\end{aligned}
$$

- Note the trick in the last line
  - removes the need for an exponential number of products

$\rightarrow$ this makes IBM Model 1 estimation tractable

The Trick

(case $l_e = l_f = 2$)

$$\sum_{a(1)=0}^{2} \sum_{a(2)=0}^{2} = \frac{\epsilon}{3^2} \prod_{j=1}^{2} t(e_j|f_{a(j)}) =$$

$$= t(e_1|f_0)\ t(e_2|f_0) + t(e_1|f_0)\ t(e_2|f_1) + t(e_1|f_0)\ t(e_2|f_2) +$$

$$+ t(e_1|f_1)\ t(e_2|f_0) + t(e_1|f_1)\ t(e_2|f_1) + t(e_1|f_1)\ t(e_2|f_2) +$$

$$+ t(e_1|f_2)\ t(e_2|f_0) + t(e_1|f_2)\ t(e_2|f_1) + t(e_1|f_2)\ t(e_2|f_2) =$$

$$= t(e_1|f_0)\ (t(e_2|f_0) + t(e_2|f_1) + t(e_2|f_2)) +$$

$$+ t(e_1|f_1)\ (t(e_2|f_0) + t(e_2|f_1) + t(e_2|f_2)) +$$

$$+ t(e_1|f_2)\ (t(e_2|f_0) + t(e_2|f_1) + t(e_2|f_2)) =$$

$$= \ (t(e_1|f_0) + t(e_1|f_1) + t(e_1|f_2))\ (t(e_2|f_0) + t(e_2|f_1) + t(e_2|f_2))$$

IBM Model 1 and EM: Expectation Step

- Combine what we have:

$$p(\mathbf{a}|\mathbf{e},\mathbf{f}) = p(\mathbf{e},\mathbf{a}|\mathbf{f})/p(\mathbf{e}|\mathbf{f})$$

$$= \frac{\frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})}{\frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} \sum_{i=0}^{l_f} t(e_j|f_i)}$$

$$= \prod_{j=1}^{l_e} \frac{t(e_j|f_{a(j)})}{\sum_{i=0}^{l_f} t(e_j|f_i)}$$

IBM Model 1 and EM: Maximization Step

- Now we have to collect counts

- Evidence from a sentence pair **e,f** that word e is a translation of word f:

$$c(e|f;\mathbf{e},\mathbf{f}) = \sum_a p(a|\mathbf{e},\mathbf{f}) \sum_{j=1}^{l_e} \delta(e,e_j)\delta(f,f_{a(j)})$$

- With the same simplication as before:

$$c(e|f;\mathbf{e},\mathbf{f}) = \frac{t(e|f)}{\sum_{i=0}^{l_f} t(e|f_i)} \sum_{j=1}^{l_e} \delta(e,e_j) \sum_{i=0}^{l_f} \delta(f,f_i)$$

IBM Model 1 and EM: Maximization Step

After collecting these counts over a corpus, we can estimate the model:

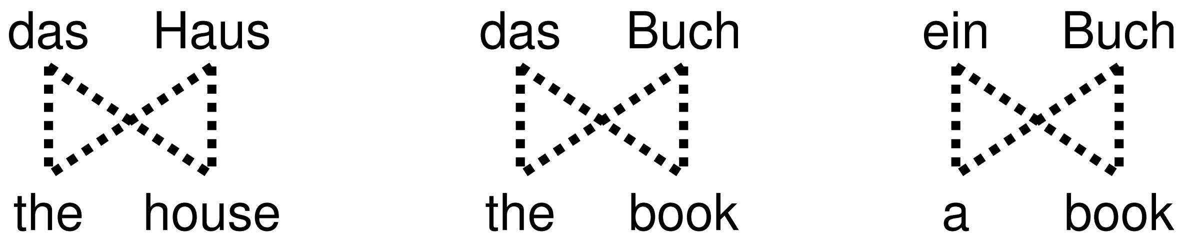
$$t(e|f;\mathbf{e},\mathbf{f}) = \frac{\sum_{(\mathbf{e},\mathbf{f})} c(e|f;\mathbf{e},\mathbf{f}))}{\sum_e \sum_{(\mathbf{e},\mathbf{f})} c(e|f;\mathbf{e},\mathbf{f}))}$$

IBM Model 1 and EM: Pseudocode

Input: set of sentence pairs $(\mathbf{e}, \mathbf{f})$
Output: translation prob. $t(e|f)$

initialize $t(e|f)$ uniformly
while not converged **do**
    // initialize
    count$(e|f)$ = 0 **for all** e, f
    total(f) = 0 **for all** f
    for all sentence pairs $(\mathbf{e},\mathbf{f})$ **do**
        // compute normalization
        for all words e in $\mathbf{e}$ **do**
            s-total(e) = 0
            for all words f in $\mathbf{f}$ **do**
                s-total(e) += $t(e|f)$
            end for
        end for
        // collect counts
        for all words e in $\mathbf{e}$ **do**
            for all words f in $\mathbf{f}$ **do**
                count$(e|f)$ += $\frac{t(e|f)}{\text{s-total}(e)}$
                total(f) += $\frac{t(e|f)}{\text{s-total}(e)}$
            end for
        end for
    end for
    // estimate probabilities
    for all foreign words f **do**
        for all English words e **do**
            $t(e|f) = \frac{\text{count}(e|f)}{\text{total}(f)}$
        end for
    end for
end while

Convergence



e	f	initial	1st it.	2nd it.	3rd it.	...	final
the	das	0.25	0.5	0.6364	0.7479	...	1
book	das	0.25	0.25	0.1818	0.1208	...	0
house	das	0.25	0.25	0.1818	0.1313	...	0
the	buch	0.25	0.25	0.1818	0.1208	...	0
book	buch	0.25	0.5	0.6364	0.7479	...	1
a	buch	0.25	0.25	0.1818	0.1313	...	0
book	ein	0.25	0.5	0.4286	0.3466	...	0
a	ein	0.25	0.5	0.5714	0.6534	...	1
the	haus	0.25	0.5	0.4286	0.3466	...	0
house	haus	0.25	0.5	0.5714	0.6534	...	1

Perplexity

- How well does the model fit the data?
- Perplexity: derived from probability of the training data according to the model

$$\log_2 PP = -\sum_s \log_2 p(\mathbf{e}_s|\mathbf{f}_s)$$

- Example (ϵ=1)

	initial	1st it.	2nd it.	3rd it.	...	final
p(the haus|das haus)	0.0625	0.1875	0.1905	0.1913	...	0.1875
p(the book|das buch)	0.0625	0.1406	0.1790	0.2075	...	0.25
p(a book|ein buch)	0.0625	0.1875	0.1907	0.1913	...	0.1875
perplexity	4095	202.3	153.6	131.6	...	113.8

Higher IBM Models

IBM Model 1	lexical translation
IBM Model 2	adds absolute reordering model
IBM Model 3	adds fertility model
IBM Model 4	relative reordering model
IBM Model 5	fixes deficiency

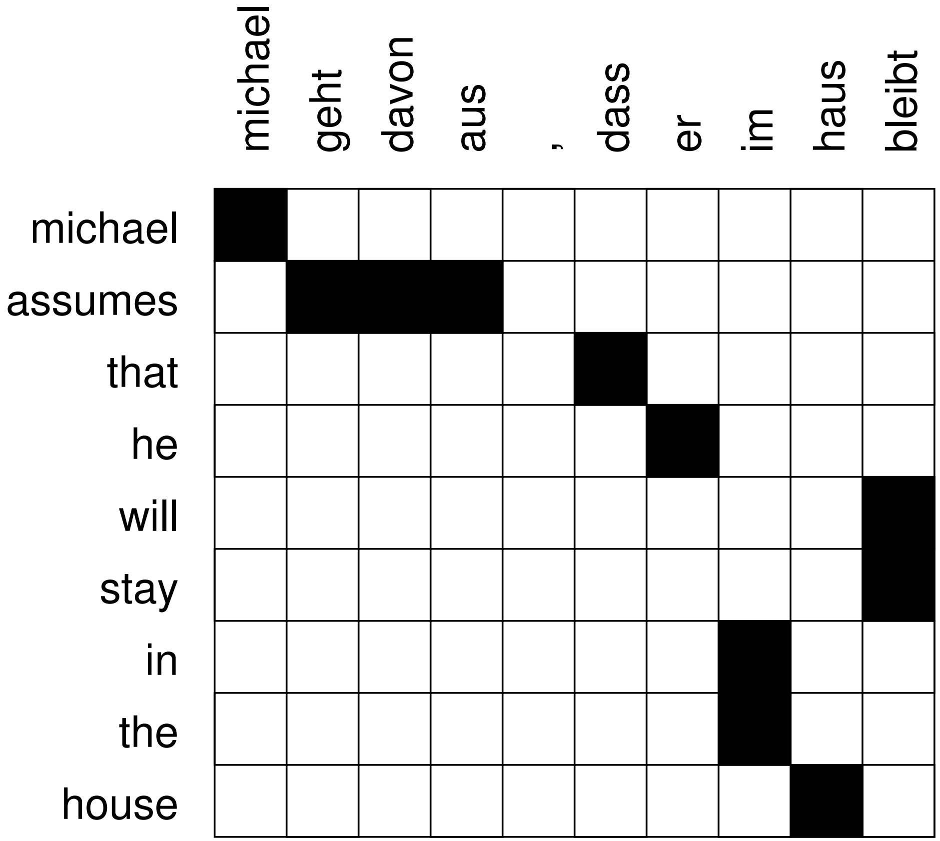
- Only IBM Model 1 has global maximum
  - training of a higher IBM model builds on previous model

- Compuationally biggest change in Model 3
  - trick to simplify estimation does not work anymore
  - $\rightarrow$ exhaustive count collection becomes computationally too expensive
  - sampling over high probability alignments is used instead

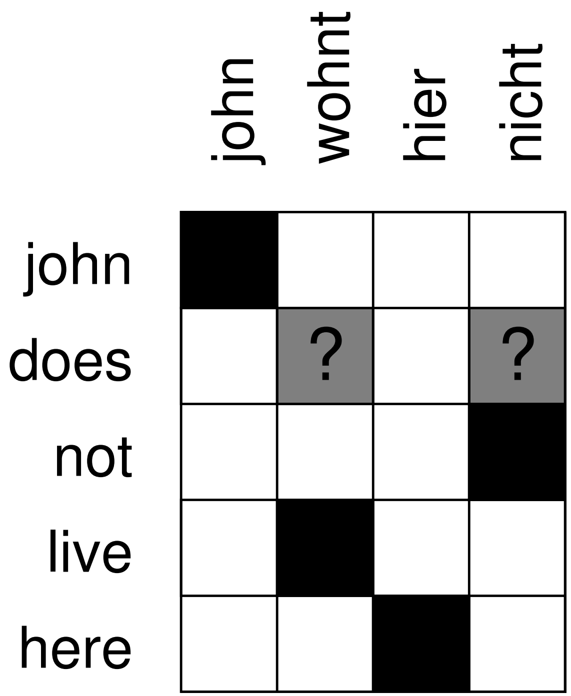
word alignment

Word Alignment

Given a sentence pair, which words correspond to each other?

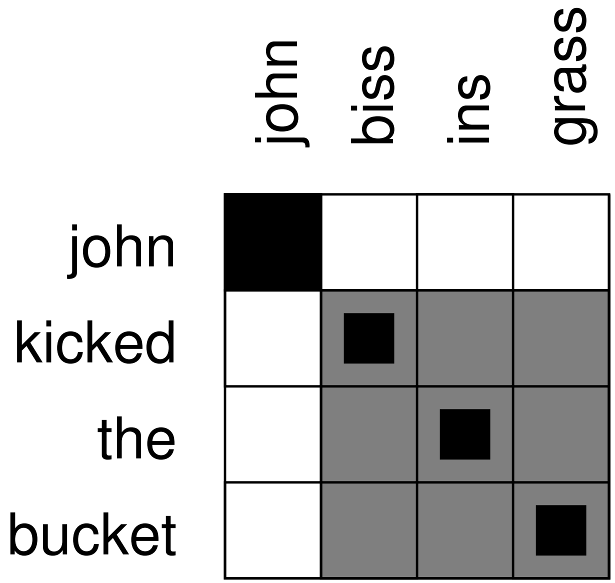


Word Alignment?



Is the English word does aligned to
the German wohnt (verb) or nicht (negation) or neither?

Word Alignment?



How do the idioms kicked the bucket and biss ins grass match up?
Outside this exceptional context, bucket is never a good translation for grass

Measuring Word Alignment Quality

- Manually align corpus with *sure* (S) and *possible* (P) alignment points ($S \subseteq P$)

- Common metric for evaluation word alignments: Alignment Error Rate (AER)

$$\text{AER}(S, P; A) = 1 - \frac{|A \cap S| + |A \cap P|}{|A| + |S|}$$

- AER = 0: alignment A matches all sure, any possible alignment points

- However: different applications require different precision/recall trade-offs

symmetrization

Word Alignment with IBM Models

- IBM Models create a **many-to-one** mapping
  - words are aligned using an alignment function
  - a function may return the same value for different input (one-to-many mapping)
  - a function can not return multiple values for one input (no many-to-one mapping)
- Real word alignments have **many-to-many** mappings

Symmetrization

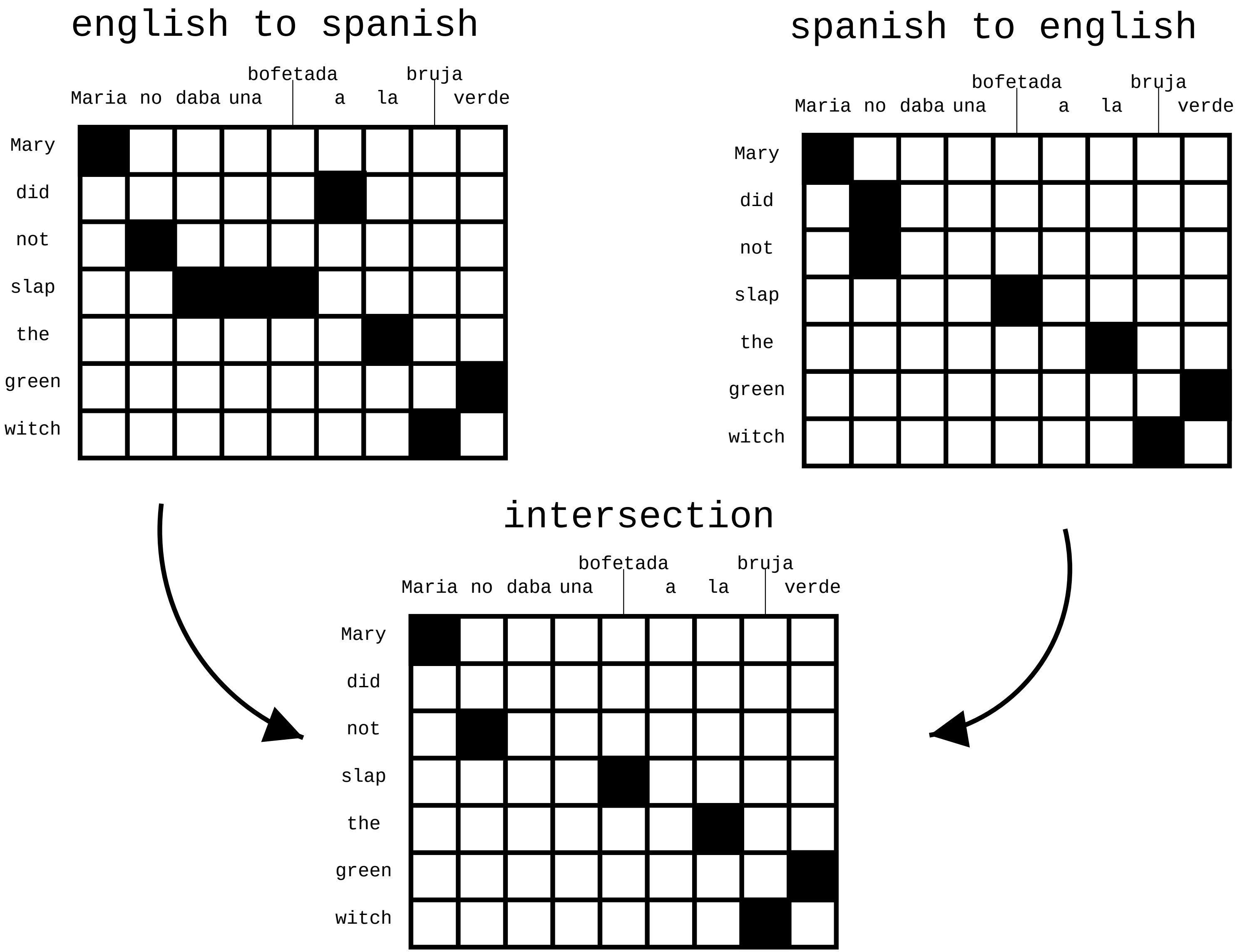
- Run IBM Model training in both directions

$\rightarrow$ two sets of word alignment points

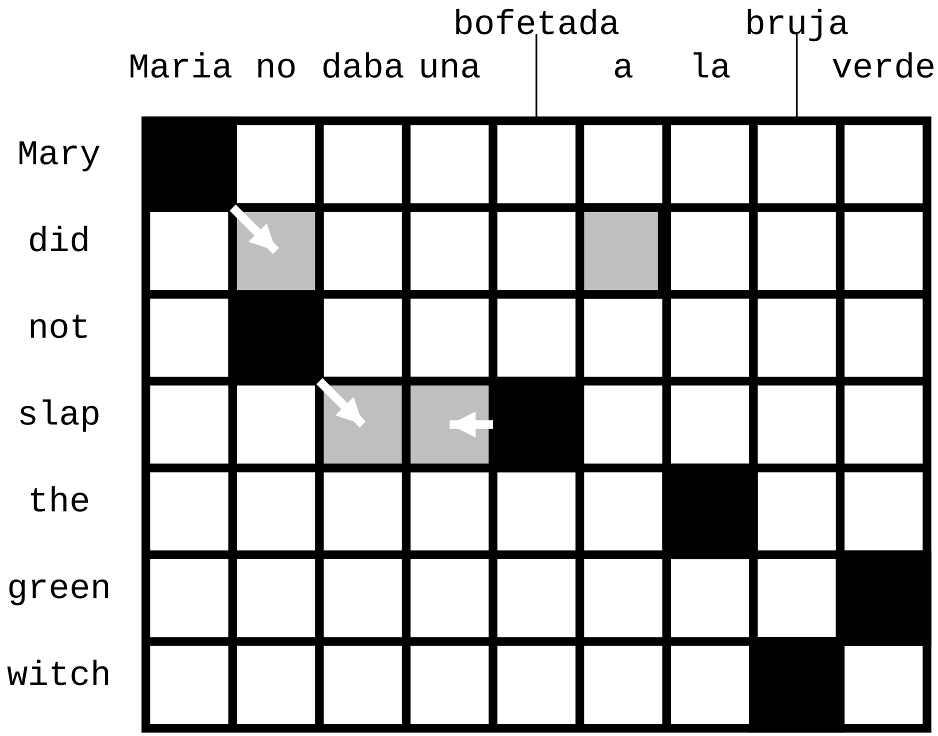
- Intersection: high precision alignment points
- Union: high recall alignment points

- Refinement methods explore the sets between intersection and union

Example


english to spanish
spanish to english
intersection
Maria no daba una bofetada a la bruja verde
Mary
did
not
slap
the
green
witch

Growing Heuristics



black: intersection **grey**: additional points in union

- Add alignment points from union based on heuristics:
  - directly/diagonally neighboring points
  - finally, add alignments that connect unaligned words in source and/or target
- Popular method: grow-diag-final-and