Notes on Norbert Wiener's *Cybernetics*

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1 Ergodic theory, the Fourier integral as an invariant under translations

A *character* of a transformation group, where transformations T transform coordinates x, are functions $f(x)$ such that

$$f(Tx) = \alpha(T)f(x)$$

where $|\alpha(T)| = |f(x)| = 1$ for all T and x. On p. 51, Wiener remarks, "clearly $f(x)g(x)$" is a character if $f(x)$ and $g(x)$ are characters. Here is a proof of this assertion.

Remark. *If $f(x)$ and $g(x)$ are characters, then $h(x) = f(x)g(x)$ is also a character of the group.*

Proof.

$$\begin{aligned} h(Tx) &= f(Tx)g(Tx) \\ &= \alpha_f(T)f(x) \cdot \alpha_g(T)g(x) \end{aligned}$$

Let $\alpha_h(T) = \alpha_f(T)\alpha_g(T)$. Then

$$\begin{aligned} |\alpha_h(T)| &= |\alpha_f(T)\alpha_g(T)| \\ &= |\alpha_f(T)| \cdot |\alpha_g(T)| \\ &= 1 \end{aligned}$$

Similarly, since the absolute value of $f(x)$ and $g(x)$ are both 1, $|h(x)| = 1$. We then have $h(Tx) = \alpha_h(T)h(x)$, where the $|\alpha_h(T)| = 1$ and $|h(x)| = 1$. Therefore $h(x)$ is also a character of the transformation group. $\square$

References

Wiener, N. (1961). *Cybernetics* (2nd ed.). Cambridge, MA: MIT Press.