A *forest* is a collection of disjoint trees. In other words, a forest is an undirected and acyclic graph, where all of its components are trees. Prove that a forest with $k$ edges has $n-k$ connected components, where $n$ denotes the number of vertices in the forest.

Figure 1: *A forest with 4 edges, 6 vertices, and 2 connected components.*

**Solution.** A tree containing $n$ vertices has $n-1$ edges. Let $C$ be the set of connected components. Let $v(c)$ and $e(c)$ denote the number of vertices and edges in the component $c \in C$ respectively.

Since the forest has $k$ edges, $\sum_{c \in C} e(c) = k$. Since the forest has $n$ vertices, $\sum_{c \in C} v(c) = n$.

$$\begin{aligned} k &= \sum_{c \in C} e(c) \\ &= \sum_{c \in C} (v(c) - 1) \\ &= \sum_{c \in C} v(c) - \sum_{c \in C} 1 \\ &= n - |C| \\ \therefore |C| &= n - k \end{aligned}$$

Therefore the number of connected components is $n-k$.