Let f_1, f_2, g_1, g_2 be functions from $\mathbb{Z}^+$ to $\mathbb{Z}^+$, and suppose that $f_1 \in O(g_1)$ and $f_2 \in O(g_2)$.

(a) Show that $f_1 + f_2 \in O(g_1 + g_2)$.

(b) Show that $f_1 \cdot f_2 \in O(g_1 \cdot g_2)$.

Solution.

(a) Solution to part (a).

(b) Solution to part (b).