Time Series Analytics

111-1 Homework #07
Due at 23h59, November 27, 2022; files uploaded to NTU-COOL

1. (10%) Given the model $y_t = a + bt + c_t + x_t$, where a, b are constants, c_t is deterministic and periodic with period s and x_t is a SARIMA$(p, 0, q) \times (P, 1, Q)_s$. What is the model for $w_t = y_t - y_{t-s}$?

2. (10%) Identify the following as certain multiplicative SARIMA models:
   (a) $y_t = 0.5y_{t-1} + y_{t-4} - 0.5y_{t-5} + a_t - 0.3a_{t-1}$
   (b) $y_t = y_{t-1} + y_{t-12} - y_{t-13} + a_t - 0.5a_{t-1} - 0.5a_{t-12} + 0.25a_{t-13}$

3. (10%) If the characteristic function of an AR time series model is
$$(1 - 1.6B + 0.7B^2)(1 - 0.8B^{12})$$
   (a) Is the model stationary?
   (b) Identify the model as a certain SARIMA model.

4. (15%) Suppose $y_t = y_{t-4} + a_t$ with $y_t = a_t$, for $t = 1, 2, 3, 4$.
   (a) Find the variance function for y_t.
   (b) Find the autocorrelation function for y_t.
   (c) Identify the model for y_t as a certain SARIMA model.

5. (15%) Consider the famous time series data "co2" (monthly carbon dioxide through 11 years in Alert, Canada).
   (a) Fit a deterministic regression model in terms of months and time. Are the regression coefficients significant? What is the adjusted R-squared? (Note that the month variable should be treated as categorical and transformed into 11 dummy variables.)
   (b) Identify, estimate the SARIMA model for the co2 level.
   (c) Compare the two models above, what do you observe?