TSA_HW08

R10A21126
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Q2

Recall the dataset “robot” firstly introduced in TSA HW06.

In [ ]:
```
library(TSA)
library('lmtest')
# require(grDevices)
```

In [ ]:
```
options(repr.plot.width=15, repr.plot.height=5) # modify the plot size
```

(a) Use IMA(1, 1) to forecast five values ahead and calculate the 95% confidence intervals.

(b) Display the actual values, the five forecasts and the 95% confidence intervals of the five forecasts, all in one graph. What do you observe?

In [ ]:
```
data(robot)
```

In [ ]:
```
model = arima(robot,order=c(0,1,1))
model
```

```
Call:
arima(x = robot, order = c(0, 1, 1))

Coefficients:
          ma1
      -0.8713
s.e.   0.0389

sigma^2 estimated as 6.069e-06:  log likelihood = 1480.95,  aic = -2959.9
```

In [ ]:
```
plot(model,n.ahead=5,n1=c(300))$pred
plot(model,n.ahead=5,n1=300)$lpi;
```

```
plot(model,n.ahead=5,n1=300)$upi;
# n1 starting time point of the plot (default=earliest time point)
# pred the time series of predicted values
# lpi the corresponding lower 95% prediction limits
# upi the corresponding upper 95% prediction limits
#https://cran.r-project.org/web/packages/TSA/TSA.pdf P.45
```

A Time Series:

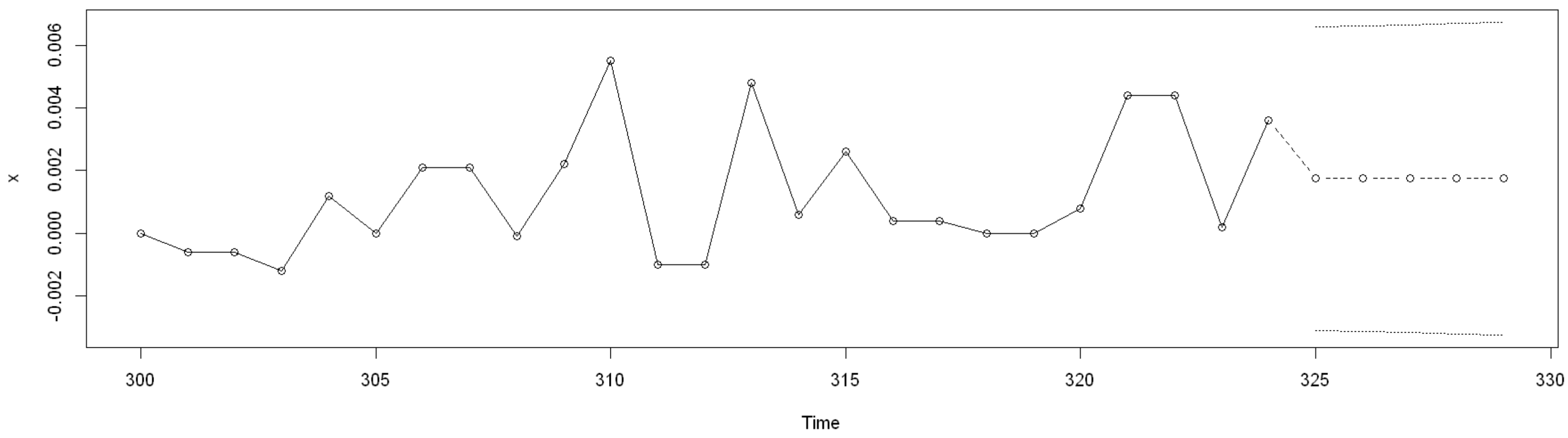
0.00174267189481582 · 0.00174267189481582 · 0.00174267189481582 · 0.00174267189481582 · 0.00174267189481582

A Time Series:

-0.00308600046365195 · -0.00312583931295607 · -0.00316535479775323 · -0.00320455466657593 · -0.00324344636335955

A Time Series:

0.00657134425328359 · 0.00661118310258771 · 0.00665069858738487 · 0.00668989845620757 · 0.00672879015299119



The forecast limits are quite wide in this fitted model and the forecasts are relatively constant.

(c) Use ARMA(1, 1) to forecast five values ahead and calculate the 95% confidence intervals. Compare the

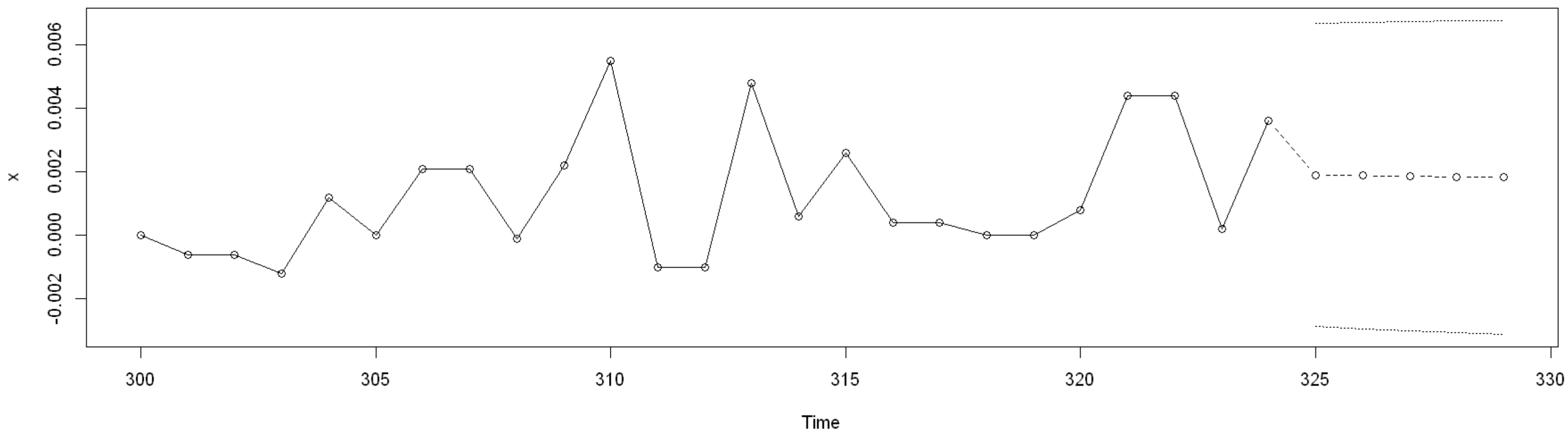
results with those in (a), what do you observe?

In [ ]:
```
model=arima(robot,order=c(1,0,1))
plot(model,n.ahead=5,n1=c(300))$pred
plot(model,n.ahead=5,n1=300)$lpi;
plot(model,n.ahead=5,n1=300)$upi;
```

A Time Series:
0.00190134843058797 · 0.00187944389457682 · 0.00185869511989562 · 0.00183904112446051 · 0.00182042414381957

A Time Series:
-0.00287877602479699 · -0.00294799357870244 · -0.00301080301733806 · -0.00306788937149357 · -0.00311985138858445

A Time Series:
0.00668147288597293 · 0.00670688136785608 · 0.0067281932571293 · 0.00674597162041459 · 0.00676069967622358



Two models give quite similar forecasts.

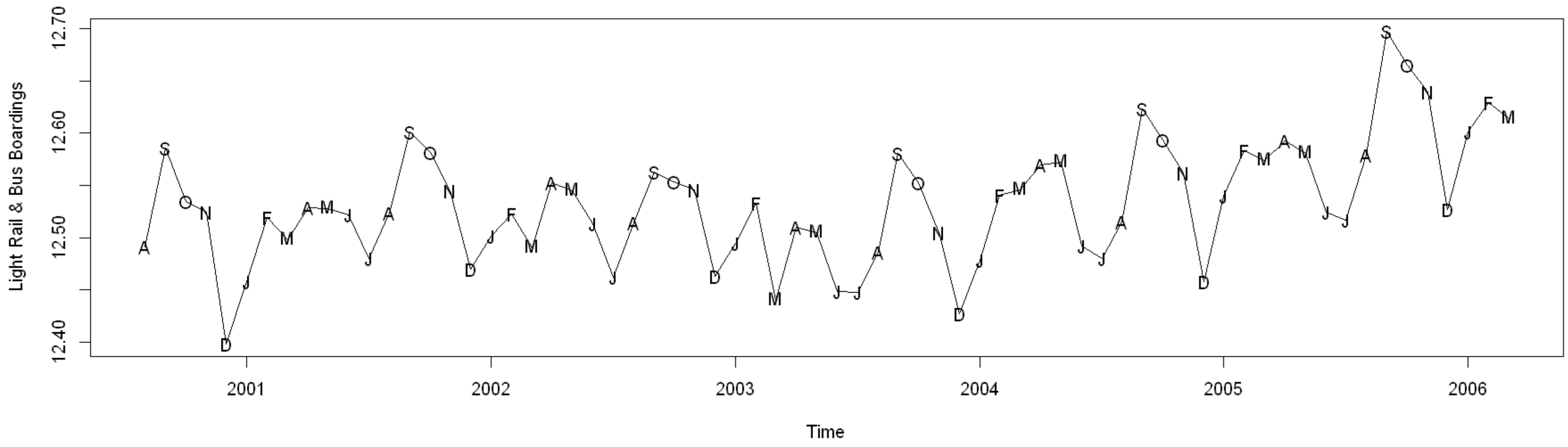
Q3

The dataset “boardings” contains the monthly number of passengers who boarded light rail trains and buses in Denver, Colorado, from August 2000 to March 2006.

(a) Plot the time series and tell your observation if there exists seasonality and if the series is stationary.

In [ ]:
```
data(boardings); series=boardings[,1]
```

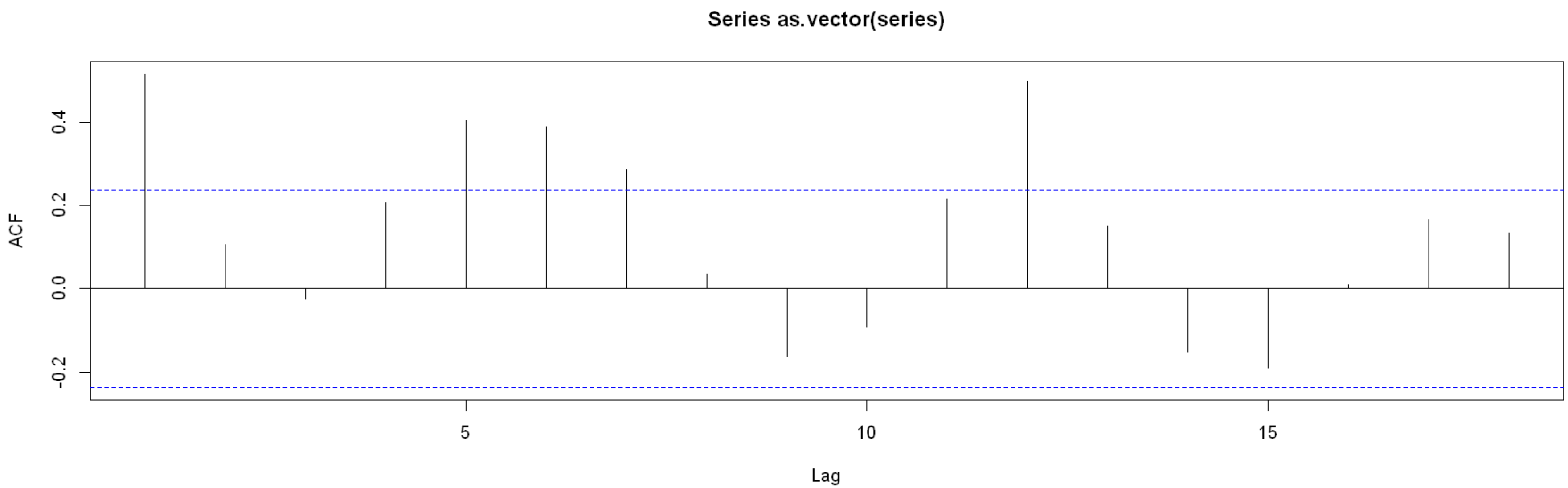
In [ ]:
```
plot(series,type='l',ylab='Light Rail & Bus Boardings')
points(series,x=time(series),pch=as.vector(season(series)))
```



There is substantial seasonality in this series. Decembers are generaly low due to the holidays and Septembers are usually quite high due to the start up of school. There may also be a gradual upward “trend” that may need to be modeled with some kind of nonstationarity.

(b) Plot the sample ACF and see what are the significant lags?

In [ ]:
```
acf(as.vector(series))
```



The significant autocorrelations indicate a seasonal ARMA model.

(c) Fit the data with ARMA $(0,3)\times(1,0)_{12}$, evaluate if the estimated coefficients $\left\{\hat{\theta}_1, \hat{\theta}_12, \hat{\theta}_3, \hat{\phi}_{12},\right\}$ are significant.

Hint: you need to check the associated standard errors “s.e.” to the estimated coefficients to know if the coefficients are significant, via hypothesis testing.

In [ ]:
```
model=arima(series,order=c(0,0,3),seasonal=list(order=c(1,0,0),period=12))
model
```

```
Call:
arima(x = series, order = c(0, 0, 3), seasonal = list(order = c(1, 0, 0), period = 12))

Coefficients:
         ma1     ma2     ma3    sar1  intercept
      0.7290  0.6116  0.2950  0.8776    12.5455
s.e.  0.1186  0.1172  0.1118  0.0507     0.0354

sigma^2 estimated as 0.0006542:  log likelihood = 143.54,  aic = -277.09
```

In [ ]:
```
coeftest(model);coefci(model);confint(model) #identical results
```

```
z test of coefficients:

          Estimate Std. Error  z value  Pr(>|z|)    
ma1        0.72897    0.11856   6.1487 7.812e-10 ***
ma2        0.61162    0.11718   5.2194 1.795e-07 ***
ma3        0.29503    0.11180   2.6390  0.008316 ** 
sar1       0.87761    0.05071  17.3064 < 2.2e-16 ***
intercept 12.54550    0.03543 354.0930 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

A matrix: 5 × 2 of type dbl

	2.5 %	97.5 %
ma1	0.49660242	0.9613358
ma2	0.38194697	0.8412979
ma3	0.07591171	0.5141480
sar1	0.77822267	0.9770035
intercept	12.47605729	12.6149402

A matrix: 5 × 2 of type dbl

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intercept	12.47605729	12.6149402

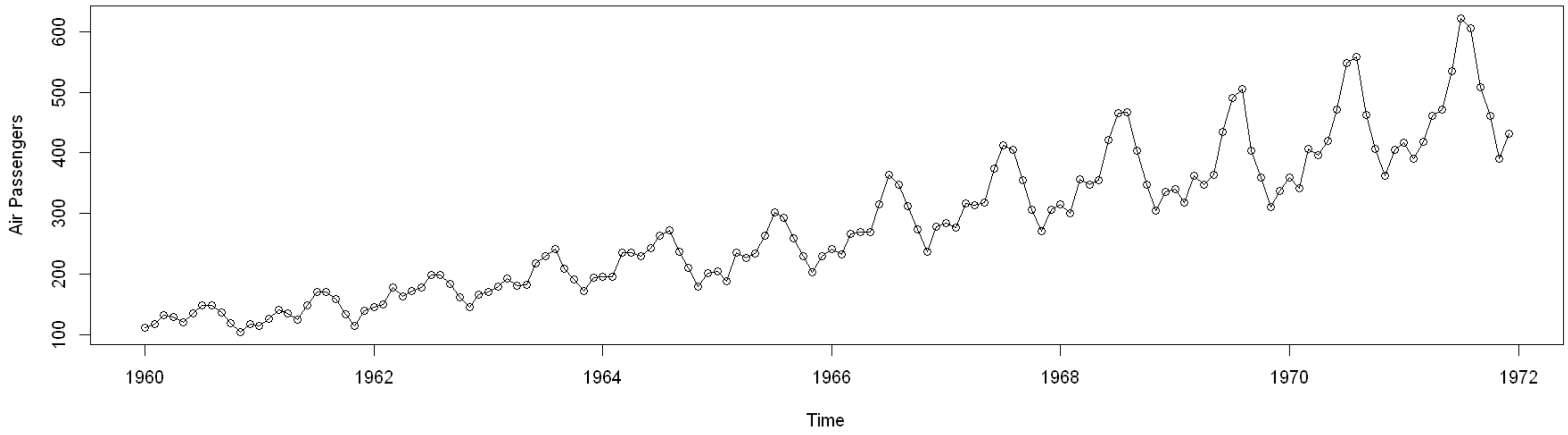
All of these coefficients are statistically significant at the usual significance levels.

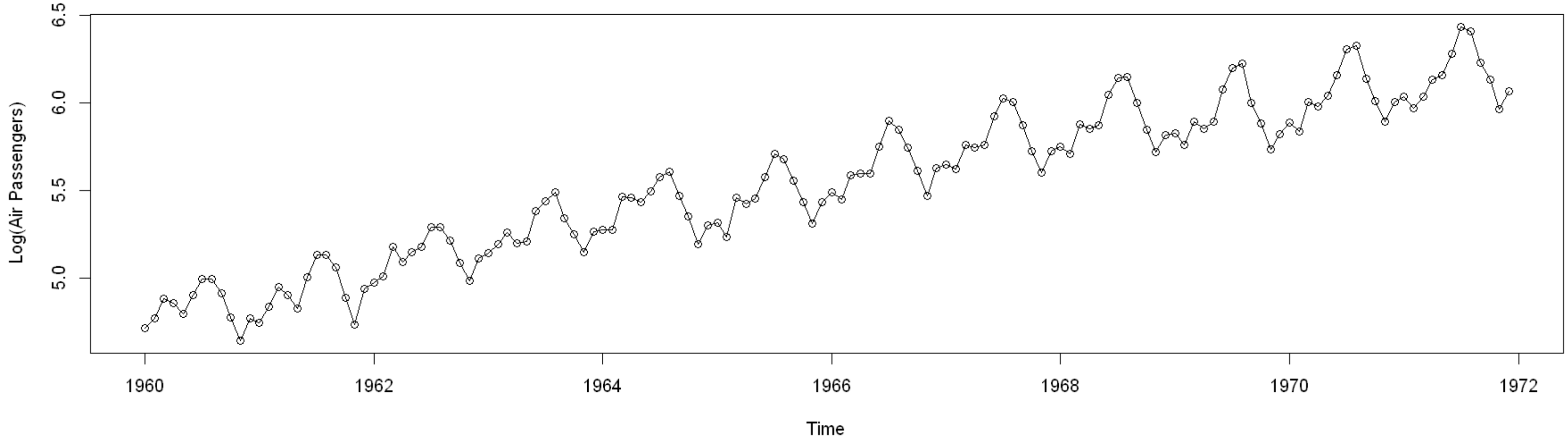
Q4

The monthly airline passengers, first investigated by Box and Jenkins in 1976, is considered as the classic time series dataset (see “TSA HW08.airpass.csv”).

(a) Plot the time series in its original scale and the log-transformed scale. Do you think making the log-transformation is appropriate?

In [ ]:
```
data(airpass)
plot(airpass, type='o',ylab='Air Passengers')
plot(log(airpass), type='o',ylab='Log(Air Passengers)')
```

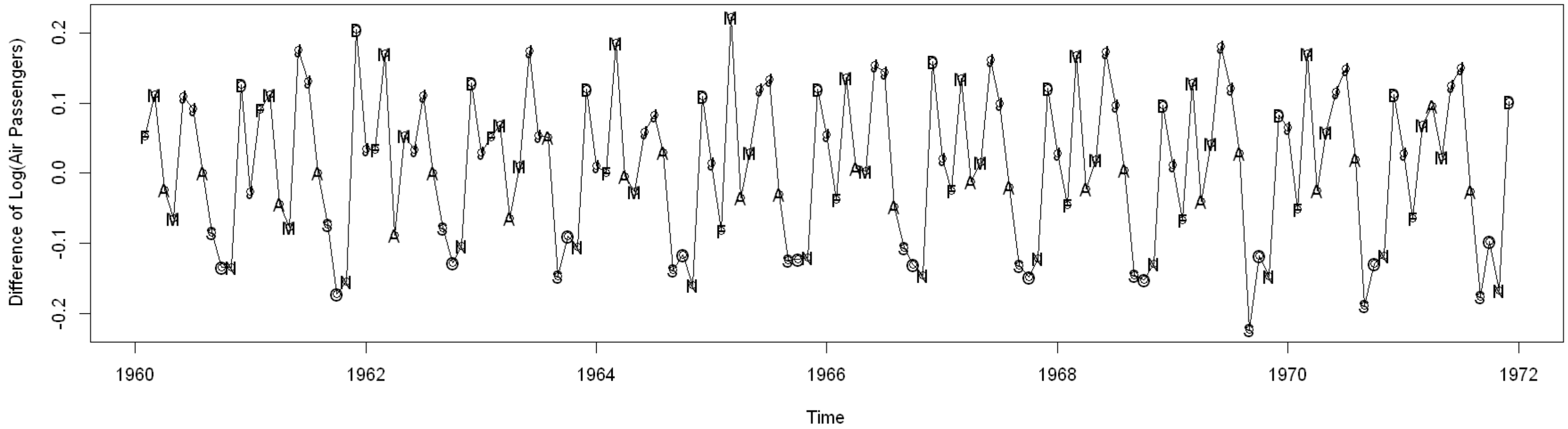




The graph of the logarithms displays a much more constant variation around the upward “trend.”

(b) Make the first-order difference over the “log-transformed” data. What do you observe?

In [ ]:
```
plot(diff(log(airpass)),type='o',ylab='Difference of Log(Air Passengers)')
points(diff(log(airpass)),x=time(diff(log(airpass))),
pch=as.vector(season(diff(log(airpass)))))
```

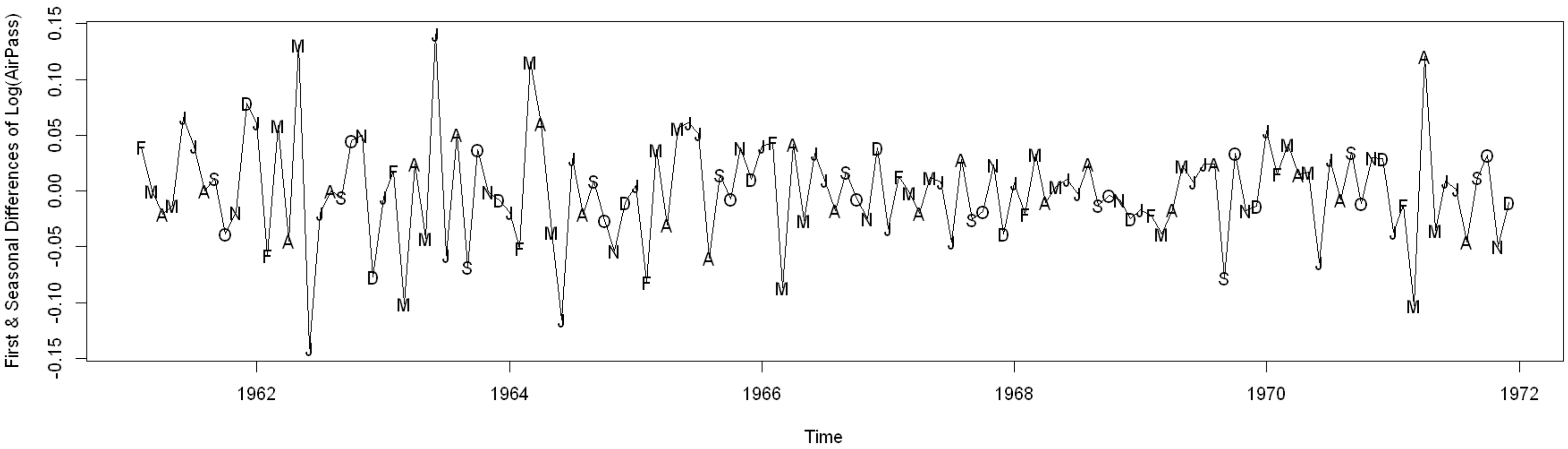


This series appears to be stationary.

The seasonality can be observed by looking at the plotting symbols carefully. Septembers, Octobers, and Novembers are mostly at the low points and Decembers mostly at the high points.

(c) Make a **seasonal** difference of the resulted series in (b), what do you observe?

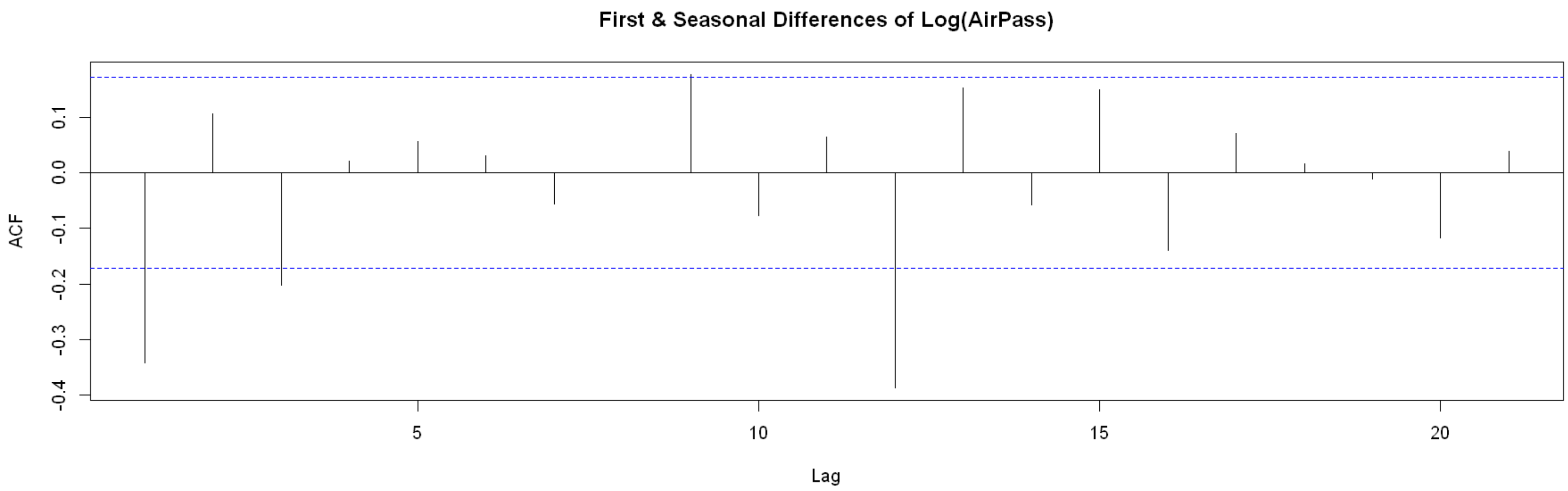
In [ ]:
```
plot(diff(diff(log(airpass)),lag=12),type='l',
ylab='First & Seasonal Differences of Log(AirPass)')
points(diff(diff(log(airpass)),lag=12),x=time(diff(diff(log(airpass)),lag=12)), pch=as.vector(season(diff(diff(log(airpass)),lag=12))))
```



The seasonality is much less obvious now

(d) Plot the sample ACF of the resulted series in (c), explain what you see.

In [ ]:
```
acf(as.vector(diff(diff(log(airpass)),lag=12)),
# ci.type='ma',
main='First & Seasonal Differences of Log(AirPass)')
```



Although there is a “significant” autocorrelation at lag 3, the most prominent autocorrelations are at lags 1 and 12.

(e) Fit an ARIMA $(0,1,1) \times (0,1,1)_{12}$ model to the log-transformed series. Diagnose the residuals of this model, including the sample ACF and the normality test.

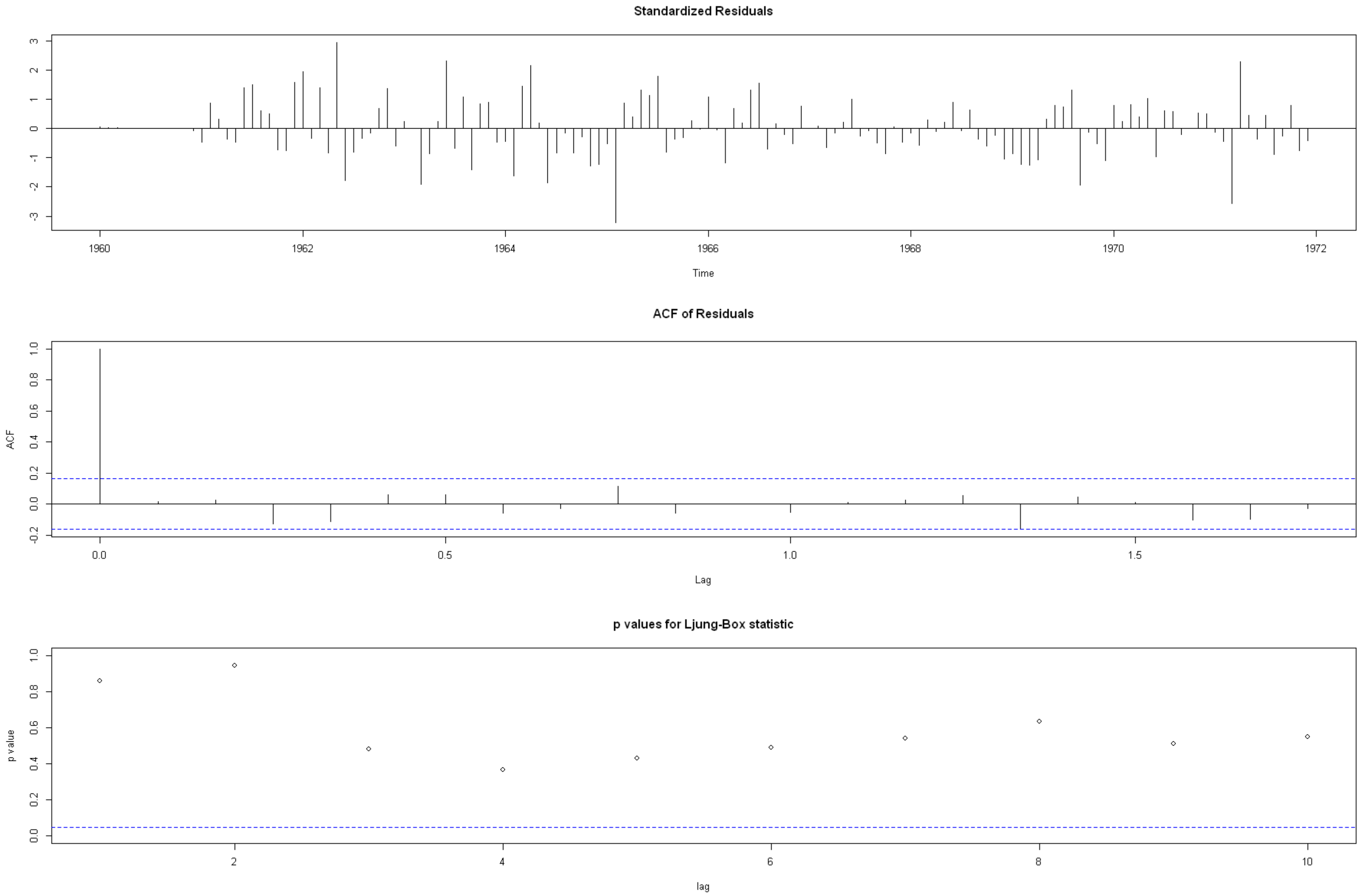
In [ ]:
```
model=arima(log(airpass),order=c(0,1,1),seasonal=list(order=c(0,1,1),period=12))
model
```

```
Call:
arima(x = log(airpass), order = c(0, 1, 1), seasonal = list(order = c(0, 1,
    1), period = 12))

Coefficients:
          ma1     sma1
      -0.4018  -0.5569
s.e.   0.0896   0.0731

sigma^2 estimated as 0.001348:  log likelihood = 244.7,  aic = -485.4
```

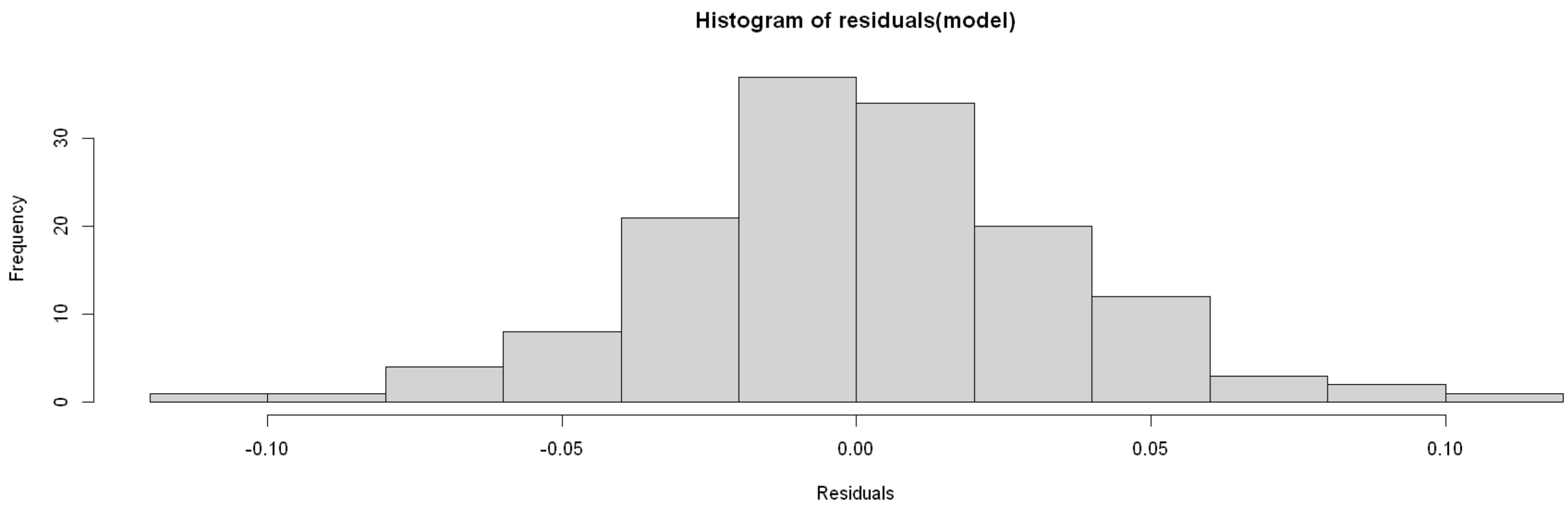
In [ ]:
```
options(repr.plot.width=15, repr.plot.height=10) # modify the plot size
tsdiag(model)
```



For residuals:

There is no significant autocorrelation. The hypothesis of randomness is not rejected.

In [ ]:
```
options(repr.plot.width=15, repr.plot.height=5)
hist(residuals(model),xlab='Residuals')
```



In [ ]:
```
shapiro.test(residuals(model))
```

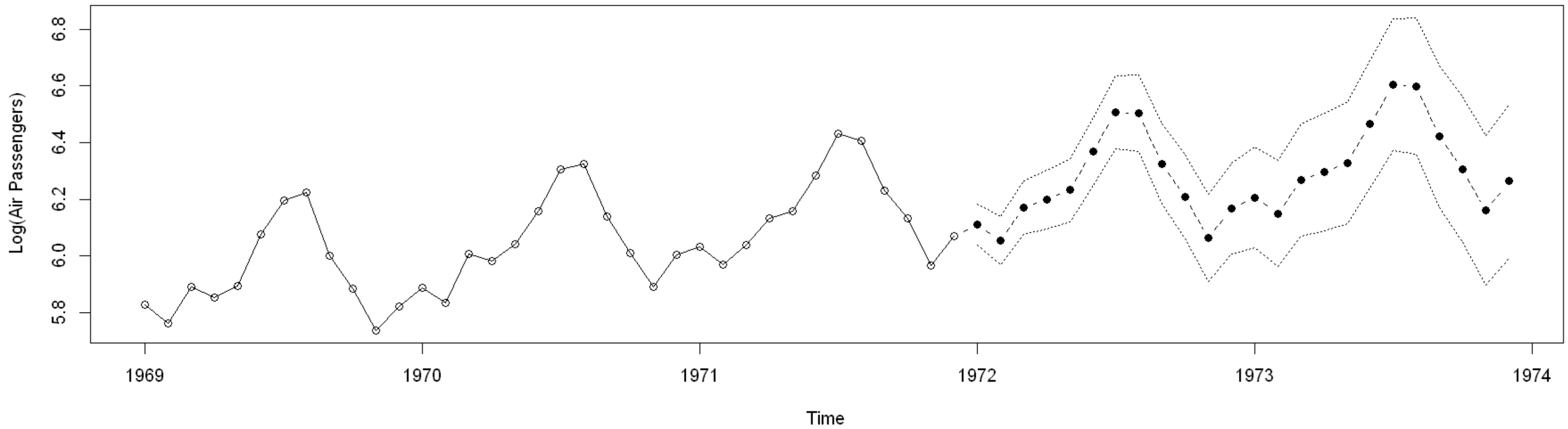
```
	Shapiro-Wilk normality test

data:  residuals(model)
W = 0.98637, p-value = 0.1674
```

The Shapiro-Wilk test does not reject normality of the error terms at any of the usual significance levels.

(f) Make forecasts for “two” years based on the model in (e). The confidence intervals shall be included

In [ ]:
```
plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Log(Air Passengers)')
```




```
In [ ]: plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Air Passengers',transform=exp)$pred # plot in orginal terms
        plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Air Passengers',transform=exp)$lpi # Lower bound
        plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Air Passengers',transform=exp)$upi # upper bound
```

A Time Series: 2 × 12

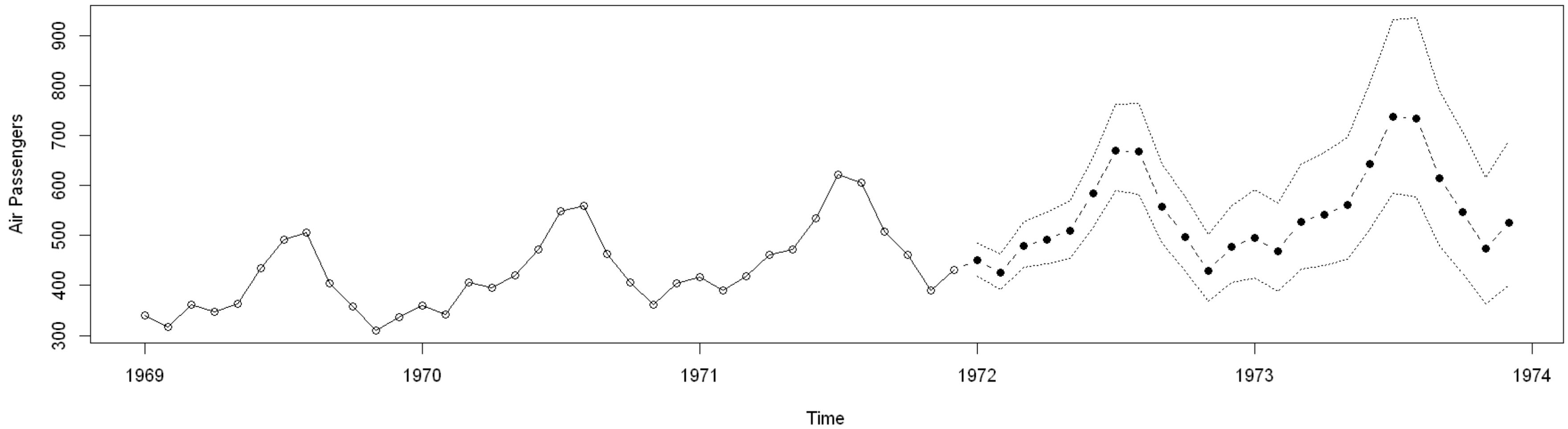
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1972	450.4224	425.7172	479.0068	492.4045	509.0550	583.3449	670.0108	667.0776	558.1894	497.2078	429.8720	477.2426
1973	495.9301	468.7289	527.4025	542.1538	560.4865	642.2823	737.7043	734.4748	614.5852	547.4424	473.3034	525.4600

A Time Series: 2 × 12

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1972	419.1476	391.4747	435.9193	443.9352	455.0234	517.2870	589.7133	582.9999	484.5733	428.8781	368.5266	406.7287
1973	415.6605	388.7165	433.0060	440.8856	451.6518	513.0512	584.3271	577.0553	479.0764	423.4948	363.4393	400.5920

A Time Series: 2 × 12

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1972	484.0307	462.9550	526.3533	546.1657	569.5025	657.8385	761.2418	763.2807	642.9890	576.4238	501.4290	559.9813
1973	591.7008	565.2108	642.3779	666.6825	695.5472	804.0651	931.3407	934.8380	788.4231	707.6668	616.3785	689.2506



In [ ]:
```
# pred = data.frame(plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Air Passengers',transform=exp)$pred,
# plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Air Passengers',transform=exp)$lpi,
# plot(model,n1=c(1969,1),n.ahead=24,pch=19,ylab='Air Passengers',transform=exp)$upi)
# colnames(pred)=c('point','lpi','upi')
# pred
```

In [ ]:
```
```