National Taiwan University
Institute of Industrial Engineering
Jakey BLUE

Time Series Analytics

111-1 Homework #08

Due at 23h59, December 04, 2022; files uploaded to NTU-COOL

1. (20%) Suppose the annual sales (in millions) of company A follow an AR(2) model:
$$y_t = 5 + 1.1y_{t-1} - 0.5y_{t-2} + a_t, \text{where } \sigma_a^2 = 2.$$
   (a) Show that the ψ_1 in the random shock form is also 1.1.
   (b) If the sales for 2005, 2006, and 2007 were 9, 11, and 10, respectively, forecast the sales for 2008 and 2009.
   (c) Calculate the 95% confidence interval of the 2008 forecast in (b).
   (d) If we now know the real sales of 2008 is 12, update your forecast for 2009.

2. (15%) Recall the dataset "robot" firstly introduced in TSA HW06.
   (a) Use IMA(1, 1) to forecast five values ahead and calculate the 95% confidence intervals.
   (b) Display the actual values, the five forecasts and the 95% confidence intervals of the five forecasts, all in one graph. What do you observe?
   (c) Use ARMA(1, 1) to forecast five values ahead and calculate the 95% confidence intervals. Compare the results with those in (a), what do you observe?

3. (15%) The dataset "boardings" contains the monthly number of passengers who boarded light rail trains and buses in Denver, Colorado, from August 2000 to March 2006.
   (a) Plot the time series and tell your observation if there exists seasonality and if the series is stationary.
   (b) Plot the sample ACF and see what are the significant lags?
   (c) Fit the data with ARMA$(0,3) \times (1,0)_{12}$, evaluate if the estimated coefficients $\{\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\phi}_{12}\}$ are significant. Hint: you need to check the associated standard errors "s.e." to the estimated coefficients to know if the coefficients are significant, via hypothesis testing.

4. (30%) The monthly airline passengers, first investigated by Box and Jenkins in 1976, is considered as the classic time series dataset (see "TSA HW08.airpass.csv").
   (a) Plot the time series in its original scale and the log-transformed scale. Do you think making the log-transformation is appropriate?
   (b) Make the first-order difference over the "log-transformed" data. What do you observe?
   (c) Make a seasonal difference of the resulted series in (b), what do you observe?
   (d) Plot the sample ACF of the resulted series in (c), explain what you see.
   (e) Fit an ARIMA$(0,1,1) \times (0,1,1)_{12}$ model to the log-transformed series. Diagnose the residuals of this model, including the sample ACF and the normality test.
   (f) Make forecasts for "two" years based on the model in (e). The confidence intervals shall be included.