Time Series Analysics

111-1 Homework #01

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> - I use R in Jupyter Notebook

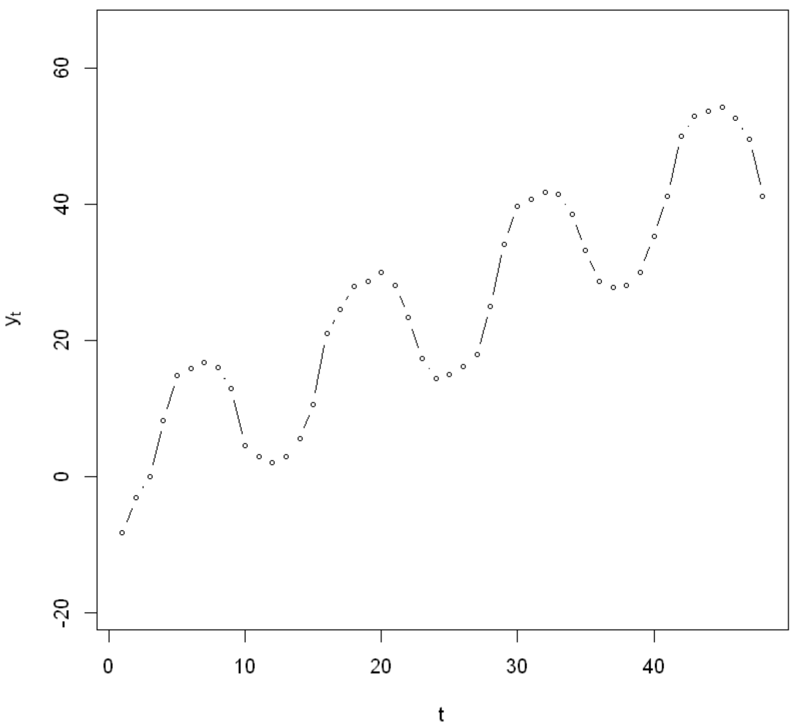
1.

In [ ]:
```
library(latex2exp)
```

$$Y_t = t + 10\sin\left[\pi\left(\frac{1}{2}\cos\left(\frac{t}{2}\right) + 1 + \Phi\right)\right] \quad \text{for } t = 1, 2, \ldots, 48,$$

where Φ is generated from a uniform distribution on the interval $[-0.1, 0.1]$.

In [ ]:
```
y_t = array(48)
for (t in 1:48){
    y_t[t] = t + 10*sinpi(cos(t/2)/2+1+0.1*runif(1,-1,1))
    }
plot(y_t,xlab = "t",ylab = TeX(r'($y_t$)'),type = 'b',asp = 0.5,cex = 0.5)
```



2.

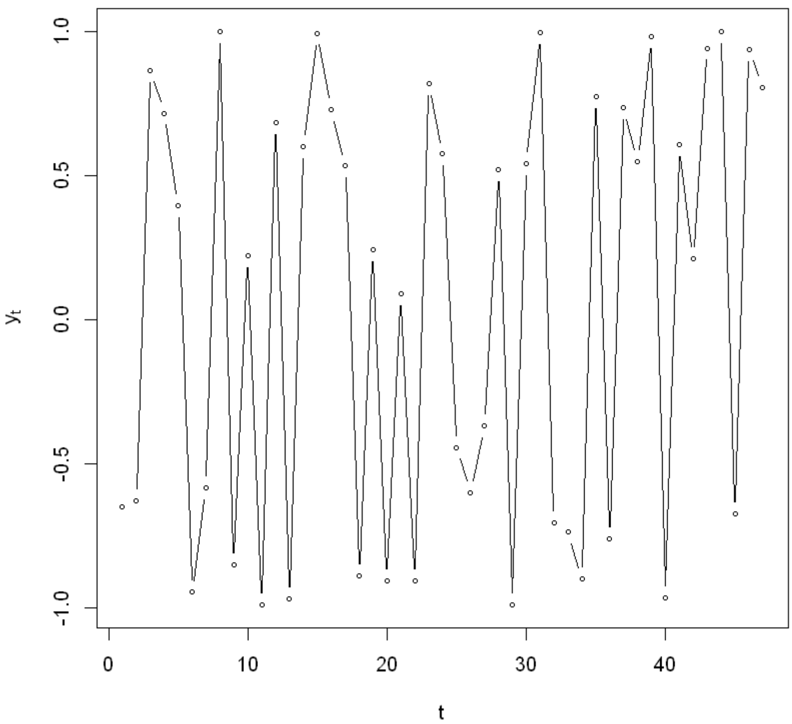
Simulate a time series of length 48 following the settings below

$$Y_t = \cos\left[2\pi\left(\frac{t}{12} + \Phi\right)\right] \quad \text{for } t = 0, 1, 2, \ldots, 47,$$

where Φ is selected from a uniform distribution on the interval $[0, 1]$.

In [ ]:
```
y_t2 = array(48)

for (t in 0:47){
    y_t2[t] = cos(2*pi*(t/12+runif(1)))
    }
plot(y_t2,xlab = 't',ylab = TeX(r'($y_t$)'),type = 'b',cex = 0.5)
```



3.

X and Y are two dependent random variables and $\text{Var}[X] = \text{Var}[Y]$, find $\text{Cov}[X+Y, X-Y]$

$$\begin{aligned} \text{Cov}[X+Y, X-Y] &= \text{Cov}(X, X-Y) + \text{Cov}(Y, X-Y) \quad \text{(Bi-linear Property of Covariance)} \\ &= \text{Cov}(X, X) - \text{Cov}(X, Y) + \text{Cov}(Y, X) - \text{Cov}(Y, Y) \\ &= \text{Var}(X) - \text{Var}(Y) \\ &= 0 \end{aligned}$$

4.

Suppose $E[X]=3, \operatorname{Var}[x]=9, E[Y]=4, \operatorname{Var}[Y]=16$, and $\operatorname{Corr}[X, Y]=0.25$. Find:
a. $\operatorname{Var}[X+Y]$
b. $\operatorname{Cov}[X, X+Y]$
c. $\operatorname{Corr}(X+Y, X-Y)$

a.

$$\operatorname{Corr}[X, Y]=\frac{\operatorname{Cov}[X, Y]}{\operatorname{std}[X] \operatorname{std}[Y]}$$

$$\begin{aligned} \operatorname{Cov}[X, Y] & =\operatorname{Corr}[X, Y] \sqrt{\operatorname{Var}[X] \operatorname{Var}[Y]} \\ & =0.25 * \sqrt{9 * 16} \\ & =3 \end{aligned}$$

$$\begin{aligned} \operatorname{Var}[X+Y] & =\operatorname{Var}[X]+\operatorname{Var}[Y]+2 \operatorname{Cov}[X, Y] \\ & =9+16+2 * 3 \\ & =31 \end{aligned}$$

b.

$$\begin{aligned} \operatorname{Cov}[X, X+Y] & =\operatorname{Cov}[X, X]+\operatorname{Cov}[X, Y] \\ & =\operatorname{Var}[X]+\operatorname{Cov}[X, Y] \\ & =9+3 \\ & =12 \end{aligned}$$

c.

$$\begin{aligned} \operatorname{Var}[X-Y] & =\operatorname{Var}[X]+\operatorname{Var}[Y]-2 \operatorname{Cov}[X, Y] \\ & =9+16-2 * 3 \\ & =19 \end{aligned}$$

$$\begin{aligned} \operatorname{Corr}(X+Y, X-Y) & =\frac{\operatorname{Cov}[X+Y, X-Y]}{\operatorname{std}[X+Y] \operatorname{std}[X-Y]} \\ & =\frac{\operatorname{Var}[X]-\operatorname{Var}[Y]}{\sqrt{\operatorname{Var}[X+Y] \operatorname{Var}[X-Y]}} \\ & =\frac{9-16}{\sqrt{31 * 19}} \\ & =-0.2884 \end{aligned}$$