Problem 1. Zygmund p58 exercise 01

(a) There is an analogue for bases different from 10 of usual decimal expansion of number. If b is an integer larger than 1 and $0 < x < 1$, show that there exist integral coefficient c_k, $0 \leq c_k < b$, such that $x = \sum_{k=1}^{\infty} c_k b^{-k}$. Furthermore, show that expansion is unique unless $x = cb^{-k}$, in which case there are two expansions.
(b) When $b = 3$, the expansion is called the triadic or ternary expansion of x. Show that Canter set consist of point in $[0, 1]$ which has triadic representation such that c_k is either 0 or 2, namely,

$$\mathcal{C} = \{x \in [0, 1] : x = \sum_{k=1}^{\infty} c_k 3^{-k}, c_k \in \{0, 2\}\}$$

Random Walks on Graphs

We define the following terms[a]:

Definition 0.1 (Hitting time)**.** The hitting time h_{uv} (sometimes called the mean first passage time) is the expected number of steps in a random walk that starts at u and ends upon first reaching v.

Definition 0.2 (Commute time)**.** We define C_{uv}, the commute time between u and v, to be $C_{uv} = h_{uv} + h_{vu} = C_{vu}$. This is the expected time for a random walk starting at u to return to u after at least one visit to v.

Definition 0.3 (Cover time)**.** Let $C_u(G)$ denote the expected length of a walk that starts at u and ends upon visiting every vertex in G at least once. The cover time of G, denoted $C(G)$, is defined by $C(G) = \max_u C_u(G)$.

[a] **motwani95**.

Solution.

$$mR(G) \leq C(G) \leq 32mR(G) \log_2 n + 1.$$

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Problem 2. Zygmund p58 exercise 03

Construct a two-dimensional Cantor set in the unit square $\{(x, y) : 0 \leq x, y \leq 1\}$ as follows. Subdivide the square into nine equal parts and keep only the four closed corner squares, removing the remaining region (which forms a cross). Then repeat this process in a suitably scaled version for the remaining squares, ad infinitum. Show that the resulting set is perfect, has plane measure zero, and equals $\mathcal{C} \times \mathcal{C}$

Problem 3. Zygmund p59 exercise 04

Construct a subset of $[0, 1]$ in the same manner as the Cantor set by removing from each remaining interval a subinterval of relative length $, 0 < \ < 1$. Show that the resulting set is perfect and has measure zero.