Real Analysis

Homework 3

Deadline: 4 October 2023

Choose two of the five exercise below, and hand in you homework in Room 554 before 5 P.M. September 20.

Exercise 1

(a) Show that the Borel σ-algebra $\mathcal{B}$ in $\mathbb{R}^n$ is the smallest σ-algebra containing the close set in $\mathbb{R}^n$.

[Zygmund p48 exercise 8]

Exercise 2

If $\{E_k\}_{k=1}^{\infty}$ sequence of sets with $\Sigma_{k=1}^{\infty}|E_k|_e < +\infty$, show that $\limsup_{k\to\infty} E_k$ has measure zero.

[Zygmund p48 exercise 9]

Exercise 3

Show that there exist sets E_i such that $E_i \searrow E$ and $|E_i|_e < +\infty$, yet $\lim_{i\to\infty} |E_i|_e > |E|_e$.

[Zygmund p48 exercise 21]

Exercise 4

Let f be a simple funtion, taking distinct value on disjoint sets $E_1, ..., E_N$. Show that f is measuable if and only if E_i is measurable for $1 \leq i \leq k$.

[Zygmund p61 exercise 2]

Exercise 5

Given an example to show that $\phi(f(x))$ may not be measurable if ϕ and f are measurable.

[Zygmund p61 exercise 5]