Real Analysis

Homework 1

Deadline: 20 September 2023

Choose two of the three exercise below, and hand in you homework in Room 554 before 5 P.M. September 20.

Exercise 1

(a) There is an analogue for bases different from 10 of usual decimal expansion of number. If b is an integer larger than 1 and $0 < x < 1$, show that there exist integral coefficient c_k, $0 \leq c_k < b$, such that $x = \Sigma_{k=1}^{\infty} c_k b^{-k}$. Furthermore, show that expansion is unique unless $x = cb^{-k}$, in which case there are two expansions.

[Zygmund p47 exercise 1]

(b) Show that Canter set consist of point in [0,1] which has triadic representation such that c_k is either 0 or 2, namely,

$$\mathcal{C} = \{x \in [0,1] : x = \Sigma_{i=1}^{\infty} c_k 3^{-k}, c_k \in \{0,2\}\}$$

[Zygmund p47 exercise 2]

Exercise 2

Construct a two-dimension al Cantor set in the unit square $\{(x,y) : 0 \leq (x,y) \leq 1\}$ as follow: subdivide the square into nine equal parts and keep only the four closed corner squares(which forms the cross). Then repeat this process in a suitably scaled version for the remaining squares for ever. Show that the resulting set is perfect, measure zero, and equal to $\mathcal{C} \times \mathcal{C}$.

[Zygmund p47 exercise 3]

Exercise 3

Construct a subset of [0,1] in the same manner as the Canter set by removing from each remaining inteval a subinteval of relative length θ, $0 < \theta < 1$, show that the resulting set E is perfect and measure zero, i.e., $\mathbf{m}_e(E) = 0$.

[Zygmund p47 exercise 4]