Real Analysis

Homework 9

Deadline: 23 November 2023

Choose two of the four exercise below, and hand in you homework in Room 554 before 5 P.M. November 23.

Exercise 1

Given $f \in L(0,1)$ show that $\int_0^1 x^k f(x)dx \to 0$

[Zygmund p85 exercise 4]

Exercise 2

Use Egorov's theorem to prove the bounded convergence theorem.

[Zygmund p85 exercise 5]

Exercise 3

Let $f(x,y)$, $0 \leq x, y \leq 1$, satisfy the following condition: for each x, $f(x,y)$ is an integrable function of y, and $\frac{\partial f}{\partial x}$ is a bounded function of (x,y). Show that $\frac{\partial f}{\partial x}$ is a measurable function of y for each x and

$$\int_0^1 \frac{\partial}{\partial x} f(x,y)dy = \frac{d}{\partial x} \int_0^1 f(x,y)dy$$

[Zygmund p85 exercise 6]

Exercise 4

Given $p > 0$ and $\int_E |f - f_k|^p \to 0$ as $k \to \infty$, (a)
show that $f_k \to f$ in measure.

[Zygmund p85 exercise 9]

(b) If in addition, $\int_E |f_k|^p \leq M$ for all $k \in \mathbb{N}$ show that $\int_E |f|^p \leq M$.

[Zygmund p85 exercise 10]