Real Analysis

Homework 5

Deadline: 18 October 2023

Choose two of the five exercise below, and hand in you homework in Room 554 before 5 P.M. October 18.

Exercise 1

(a)Construct a subset of $[0, 1]$ in the same manner as Canter set, except that at the k- stage, each interval removed has length $\delta 3^{-k}$, $0 < \delta < 1$, show that the set has measure $1 - \delta$.

[Zygmund p48 exercise 5]

(b) Construct a measurable subset in $[0, 1]$ such that for every interval in $[0, 1]$, both $E \cap I$ and $E^c \cap I$

[Zygmund p49 exercise 25]

Exercise 2

Five $E \subset \mathbb{R}^3$,we define the inner measure of E as $|E|_{\mathbf{i}} := \sup |F|$, where supremum is taking over all closed subset of F. Show that if $|E|_e < \infty$, then E is measurable if and only if $|E|_{\mathbf{i}} = |E|_e$.

[Zygmund p48 exercise 13]

Exercise 3

Construct a continuous function f such that f on $[0, 1]$ is not of bounded variation on any interval.(Hint: Modify the Cantor-Lebesgue function)

[Zygmund p49 exercise 26]

Exercise 4

Show that there are disjoint $E_i \subset \mathbb{R}$, $i = 1, 2, ...$ such that $|\cup_{i=1}^{\infty} E|_e < \Sigma_{i=1}^{\infty} |E_i|_e$.

[Zygmund p48 exercise 20]