Real Analysis II Homework 2

Due Date: April 3

Solve the following problems.

Problem 1   Let $0 < \lambda < 1$ and let f, g and h be non-negative integrable functions on $\mathbb{R}^n$ satisfying for every $x, y \in \mathbb{R}^n$, one has

$$h\big((1 - \lambda)x + \lambda y\big) \geq f(x)^{1-\lambda} g(y)^{\lambda}$$

Show that

$$\int_{\mathbb{R}^n} h(x)dx \geq \left(\int_{\mathbb{R}^n} f(x)dx \right)^{1-\lambda} \left(\int_{\mathbb{R}^n} g(x)dx \right)^{\lambda}$$

Problem 2   Let $0 < \lambda < 1$ and let A and B be non-empty bounded measurable sets in $\mathbb{R}^n$ such that $(1 - \lambda)A + \lambda B$ is also measurable. Show that

$$|A + B|^{\frac{1}{n}} \geq |A|^{\frac{1}{n}} + |B|^{\frac{1}{n}}$$