Analysis II, 2024
Midterm Exam

DEP. ________ **NAME** ______________ **ID NUMBER** ________

1. (a) 15% Let f be an absolutely continuous and singular function on $[a, b]$. Show that f is a constant function.

   (b) 15% Let f be a measurable function on $[a, b]$. Show that f is absolutely continuous if and only if there is an integrable function $g : [a, b] \to \mathbb{R}$ such that
$$f(x) = f(a) + \int_a^x g(t)dt.$$

2. (a) 15% Suppose f is an integrable function with compact support in $\mathbb{R}^n$. Show that when $|x|$ is large
$$\frac{c_1}{|x|^n} \leq f^*(x) \leq \frac{c_2}{|x|^n}$$
for some $0 < c_1 < c_2$.

3. (a) 20% Show that $f : (a, b) \to \mathbb{R}$ is a convex function if and only if
$$f\left(\frac{x+y}{2}\right) \leq \frac{f(x) + f(y)}{2}$$
for all $a < x < y < b$ and f is continuous.

   (b) 15% State the Jensen's inequality.

4. (a) 15% Suppose E and H are measurable subsets of $\mathbb{R}^n$. Show that almost every point in E is a point of density of E.

   (b) 15% Suppose x is point of density of both E and H. Show that x is a point of density of $E \cap H$.