Problem 1. The Prékopa-Leindler Inequality

Let $0 < \lambda < 1$ and let f, g and h be non-negative integrable functions on $\mathbb{R}^n$ satisfying for every $x, y \in \mathbb{R}^n$, one has

$$h\left((1-\lambda)x + \lambda y\right) \geq f(x)^{1-\lambda} g(y)^{\lambda}.$$

Show that

$$\int_{\mathbb{R}^n} h(x)\,dx \geq \left(\int_{\mathbb{R}^n} f(x)\,dx\right)^{1-\lambda} \left(\int_{\mathbb{R}^n} g(x)\,dx\right)^{\lambda}.$$

Problem 2. The Brunn-Minkowski Inequality

Let $0 < \lambda < 1$ and let A and B be non-empty bounded measurable sets in $\mathbb{R}^n$ such that $(1-\lambda)A + \lambda B$ is also measurable. Show that

$$|A+B|^{\frac{1}{n}} \geq |A|^{\frac{1}{n}} + |B|^{\frac{1}{n}}.$$