Real Analysis II Homework 3

Due Date: April 17

Solve the following problems.

Problem 1 Let $f\colon \mathbb{R} \to \mathbb{C}$. Prove that f satisfies the Lipschitz condition

$$|f(x) - f(y)| \leq M|x - y|$$

for some $M > 0$ and for all $x, y \in \mathbb{R}$, if and only if f satisfies the following two properties:

  (i) f is absolutely continuous.

  (ii) $|f'(x)| \leq M$ for a.e. x.

Problem 2 Prove that f is convex on (a, b) if and only if it is continuous and

$$f\left(\frac{x + y}{2}\right) \leq \frac{f(x) + f(y)}{2}$$

for all $x, y \in (a, b)$.