CTA200 2020 Assignment 2

DUE: Friday May 8th by 11:59PM

[4] Question 1

For each point in the complex plane $c = x + iy$, with $-2 < x < 2$ and $-2 < y < 2$, set $z_0 = 0$ and iterate the equation $z_{i+1} = z_i^2 + c$. Note what happens to the z_i's: some points will remain bounded in absolute value $|z|^2 = \Re(z)^2 + \Im(z)^2$, while others will run off to infinity. Make an image in which your points c that diverge are given one color and those that stay bounded are given another. (Once you have done this, you can try coloring the points that diverge using a colorscale that indicates the iteration number at which the given point diverged.) Try zooming in on a portion of the image and trying again.

[4] Question 2

The SIR model is a simple mathematical model of disease spread in a population. The model divides a fixed population of size N into three groups, which vary as a function of time, t:

- $S(t)$ is those that are susceptible but not yet infected
- $I(t)$ is the number of infected individuals
- $R(t)$ is those individuals that have recovered and are now immune

The model can be described by a set of 3 first order differential equations for each of the variables as

$$\frac{dS}{dt} = -\frac{\beta SI}{N}, \tag{1}$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I, \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

Using the ODE integrator of your choice (must be callable in Python, we recommend using Scipy as will be covered in lecture on Friday), integrate the equations with $N = 1000$ from $t = 0$ to $t = 200$ for various values of γ and β (at least 3-4 values, justify your choices physically).

Use the initial condition $I(0) = 1, S(0) = 999, R(0) = 0$ (you can also experiment with other initial conditions if you wish). Plot the curves for S, I, R on the same figure with a legend. make separate plots for each choice of the parameters.

Bonus: Add a 4th parameter D for deaths and justify the addition of a 4th differential equation as well as any RHS terms that must be changed on the initial 3 equations. Integrate the new set of equations for some choice of parameters and comment on the results compared to the SIR model.

[2] Question 3

To practice using the `LaTex`, markup language, we would like you to writeup your results in a .tex file and submit both the `LaTex` code and the associated PDF. For each of the questions, save and insert your figures into a tex file and write a 1 paragraph methods section, which briefly describes what you did as well as a 1 paragraph analysis section which describes what you see in the results.