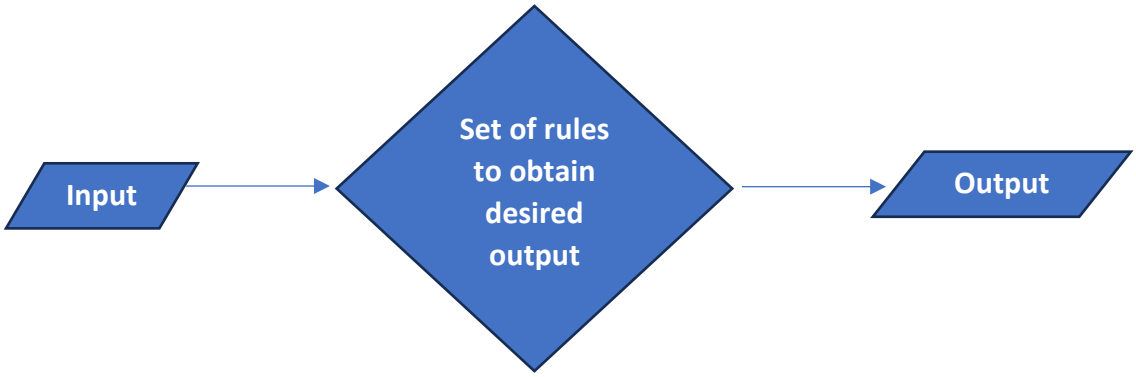
Introduction to Algorithms

- An **algorithm** is a finite set of instructions or logic, written in order, to accomplish a certain predefined task.



Algorithm

- **Algorithm** is not the complete code or program, it is just the core logic(solution) of a problem, which can be expressed either as an informal high level description as pseudocode or using a flowchart.

Every Algorithm must satisfy the following properties:

- **Input-** There should be 0 or more inputs supplied externally to the algorithm.
- **Output-** There should be atleast 1 output obtained.
- **Definiteness-** Every step of the algorithm should be clear and well defined.
- **Finiteness-** The algorithm should have finite number of steps.

- **Correctness-** Every step of the algorithm must generate a correct output.

- An algorithm is said to be efficient and fast, if it takes less time to execute and consumes less memory space. The performance of an algorithm is measured on the basis of following properties :

  - **Time Complexity**
  - **Space Complexity**

➔ Space Complexity

Space complexity is the amount of memory used by the algorithm (including the input values to the algorithm) to execute and produce the result.

Sometime Auxiliary Space is confused with Space Complexity. But Auxiliary Space is the extra space or the temporary space used by the algorithm during it's execution.

Space Complexity = Auxiliary Space + Input space

Memory Usage while Execution

While executing, algorithm uses memory space for three reasons:

1. **Instruction Space**

   It's the amount of memory used to save the compiled version of instructions.

2. **Environmental Stack**

   Sometimes an algorithm(function) may be called inside another algorithm(function). In such a situation, the current variables are pushed onto the system stack, where they wait for further execution and then the call to the inside algorithm(function) is made.

For example, If a function A() calls function B() inside it, then all th variables of the function A() will get stored on the system stack temporarily, while the function B() is called and executed inside the funciton A().

3. **Data Space**

   Amount of space used by the variables and constants.

But while calculating the **Space Complexity** of any algorithm, we usually consider only **Data Space** and we neglect the **Instruction Space** and **Environmental Stack**.

Calculating the Space Complexity

For calculating the space complexity, we need to know the value of memory used by different type of datatype variables, which generally varies for different operating systems, but the method for calculating the space complexity remains the same.

Type	**Size**
bool, char, unsigned char, signed char, __int8	1 byte
__int16, short, unsigned short, wchar_t, __wchar_t	2 bytes
float, __int32, int, unsigned int, long, unsigned long	4 bytes
double, __int64, long double, long long	8 bytes

Now let's learn how to compute space complexity by taking a few examples:

```
{

    int z = a + b + c;

    return(z);

}
```

In the above expression, variables a, b, c and z are all integer types, hence they will take up 4 bytes each, so total memory requirement will be (4(4) + 4) = 20 bytes, this additional 4 bytes is for **return value**. And because this space requirement is fixed for the above example, hence it is called **Constant Space Complexity**.

Let's have another example, this time a bit complex one,

```
// n is the length of array a[]

int sum(int a[], int n)

{

      int x = 0;      // 4 bytes for x

      for(int i = 0; i < n; i++)     // 4 bytes for
i

      {

          x  = x + a[i];

      }

      return(x);

}
```

- In the above code, 4*n bytes of space is required for the array a[] elements.
- 4 bytes each for x, n, i and the return value.

Hence the total memory requirement will be (4n + 12), which is increasing linearly with the increase in the input value n, hence it is called as **Linear Space Complexity.**

Similarly, we can have quadratic and other complex space complexity as well, as the complexity of an algorithm increases.

But we should always focus on writing algorithm code in such a way that we keep the space complexity **minimum**.

➔ Time Complexity

Time complexity of an algorithm signifies the total time required by the program to run till its completion.

The time complexity of algorithms is most commonly expressed using the **big O notation**. It's an asymptotic notation to represent the time complexity. We will study about it in detail in the next tutorial.

Time Complexity is most commonly estimated by counting the number of elementary steps performed by any algorithm to finish execution. Like in the example above, for the first code the loop will run n number of times, so the time complexity will be n atleast and as the value of n will increase the time taken will also increase. While for the second code, time complexity is constant, because it will never be dependent on the value of n, it will always give the result in 1 step.

And since the algorithm's performance may vary with different types of input data, hence for an algorithm we usually use the **worst-case Time complexity** of an algorithm because that is the maximum time taken for any input size.

Calculating Time Complexity

Now lets tap onto the next big topic related to Time complexity, which is How to Calculate Time Complexity. It becomes very confusing some times, but we will try to explain it in the simplest way.

Now the most common metric for calculating time complexity is Big O notation. This removes all constant factors so that the running time can be estimated in relation to **N**, as N approaches infinity. In general you can think of it like this :

```
statement;
```

- Above we have a single statement. Its Time Complexity will be **Constant**. The running time of the statement will not change in relation to N.

```
for(i=0; i < N; i++)
{
    statement;
}
```

- The time complexity for the above algorithm will be **Linear**. The running time of the loop is directly proportional to N. When N doubles, so does the running time.

```
for(i=0; i < N; i++)
{
    for(j=0; j < N;j++)
    {
```

```
    statement;

    }

}
```

- This time, the time complexity for the above code will be **Quadratic**. The running time of the two loops is proportional to the square of N. When N doubles, the running time increases by N * N.

Asymptotic Analysis

The efficiency of an algorithm depends on the amount of time, storage and other resources required to execute the algorithm. **The efficiency is measured with the help of asymptotic notations**.

An algorithm may not have the same performance for different types of inputs. With the increase in the input size, the performance will change.

The study of change in performance of the algorithm with the change in the order of the input size is defined as asymptotic analysis.

Asymptotic Notations

Asymptotic notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.

- For example: In bubble sort, when the input array is already sorted, the time taken by the algorithm is linear i.e. the best case.

But, when the input array is in reverse condition, the algorithm takes the maximum time (quadratic) to sort the elements i.e. the worst case.

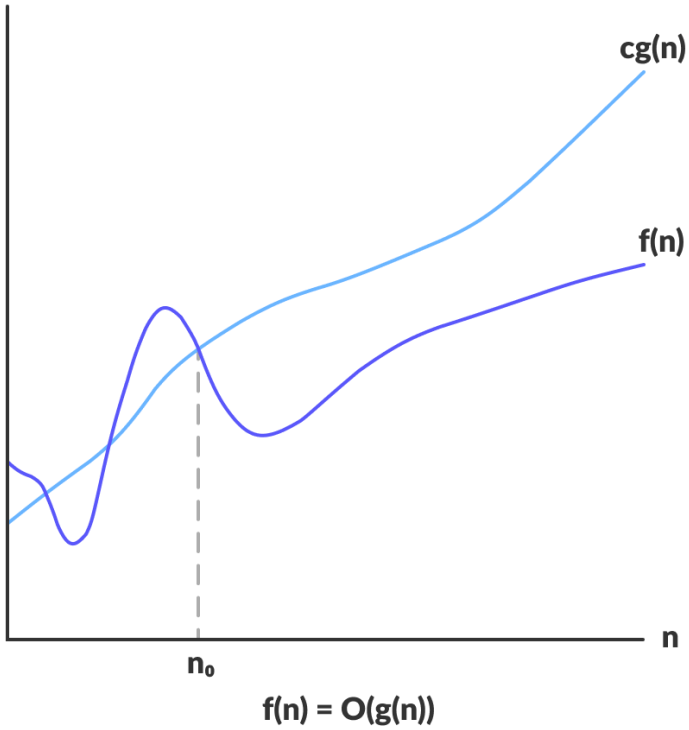
When the input array is neither sorted nor in reverse order, then it takes average time. These durations are denoted using asymptotic notations.

There are mainly three asymptotic notations:

- **Big-O notation**
- **Omega notation**
- **Theta notation**

➔ Big-O Notation (O-notation)

Big-O notation represents the upper bound of the running time of an algorithm. Thus, it gives the worst-case complexity of an algorithm.



Big-O gives the upper bound of a function

$O(g(n)) = \{ f(n)$: there exist positive constants c and n_0 such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0 \}$

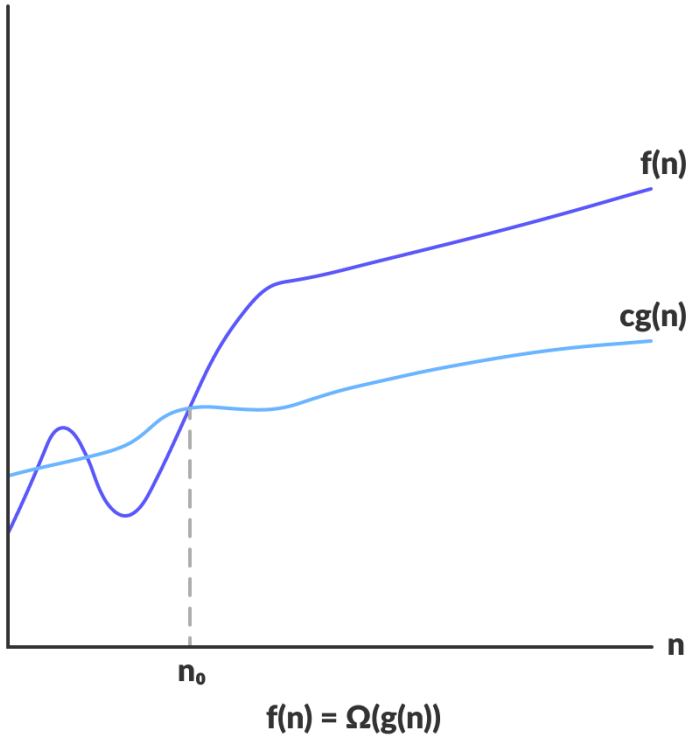
- The above expression can be described as a function f(n) belongs to the set O(g(n)) if there exists a positive constant c such that it lies between 0 and cg(n), for sufficiently large n.

  For any value of n, the running time of an algorithm does not cross the time provided by O(g(n)).

  Since it gives the worst-case running time of an algorithm, it is widely used to analyze an algorithm as we are always interested in the worst-case scenario.

➔ Omega Notation (Ω-notation)

Omega notation represents the lower bound of the running time of an algorithm. Thus, it provides the best case complexity of an algorithm.



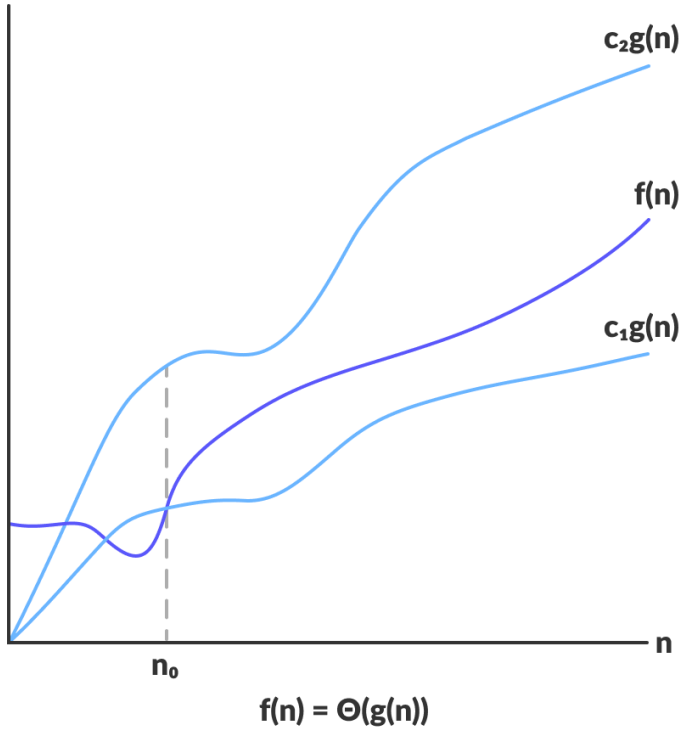
Omega gives the lower bound of a function

$\Omega(g(n)) = \{ f(n)$: there exist positive constants c and n_0 such that $0 \leq cg(n) \leq f(n)$ for all $n \geq n_0 \}$

- The above expression can be described as a function f(n) belongs to the set Ω(g(n)) if there exists a positive constant c such that it lies above cg(n), for sufficiently large n.
  For any value of n, the minimum time required by the algorithm is given by Omega Ω(g(n)).

➔ Theta Notation (Θ-notation)

Theta notation encloses the function from above and below. Since it represents the upper and the lower bound of the running time of an algorithm, it is used for analyzing the average-case complexity of an algorithm.



Theta bounds the function within constants factors

For a function $g(n)$, $\Theta(g(n))$ is given by the relation:

$$\Theta(g(n)) = \{ f(n): \text{there exist positive constants } c_1, c_2 \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \}$$

- The above expression can be described as a function $f(n)$ belongs to the set $\Theta(g(n))$ if there exist positive constants c_1 and c_2 such that it can be sandwiched between $c_1 g(n)$ and $c_2 g(n)$, for sufficiently large n.