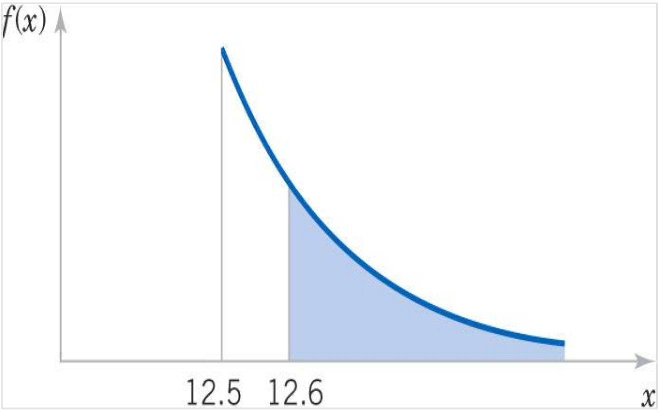
Problem Statement 1: [150 marks]

Let the continuous random variable D denote the diameter of the hole drilled in an aluminium sheet. The target diameter to be achieved is 12.5mm. Random disturbances in the process often result in inaccuracy. Historical data shows that the distribution of D can be modelled by the PDF $f(d) = 20e^{-20(d-12.5)}, d \geq 12.5$. *If a part with diameter > 12.6 mm needs to be scrapped, what is the proportion of those parts? What is the CDF when the diameter is of 11 mm? What is the conclusion of this experiment?*

Solution 1:

If the PDF is given by $f(d) = 20e^{-20(d-12.5)}, d \geq 12.5$, then if a part of diameter > 12.6 needs to be scrapped, it means we are looking for the area under the graph of $f(d)$ with the boundaries $d = 12.6$, and the x-axis i.e.



$$P(D > 12.6) = \int_{12.6}^{\infty} 20e^{-20(D-12.5)}\, dD$$
$$= 20\int_{12.6}^{\infty} e^{-20(D-12.5)}\, dD$$
$$= \frac{20\left[e^{-20(D-12.5)}\right]}{-20}\Big|_{12.6}^{\infty}$$
$$= -\left[e^{-20(D-12.5)}\right]\Big|_{12.6}^{\infty}$$
$$= -0 + e^{-2} = \mathbf{0.135}$$

The proportion of those parts is **13.5%**

The CDF is given by

$$F(x) = 0, \qquad for\; x < 12.5$$
$$F(x) = \int_{12.6}^{x} 20e^{-20(D-12.5)}\, dD$$
$$= 1 - 20e^{-20(x-12.5)}, \quad for\; x \geq 12.5$$

Hence the CDF is given as $F(x) = \begin{cases} 0 & x < 12.5 \\ 1 - 20e^{-20(x-12.5)}, & x \geq 12.5 \end{cases}$

When the diameter is 11mm, $F(x) = 0$

Problem Statement 2: [100 marks]

Please compute the following:

a) $P(Z > 1.26), P(Z < -0.86), P(Z > -1.37),\; P(-1.25 < Z < 0.37),\; P(Z \leq -4.6)$

b) Find the value such that $P(Z > z) = 0.05$

c) Find the value of such that $P(-z < Z < z) = 0.99$

Solution 2:

(a) $(i) P(Z > 1.26) = 1 - P(Z \leq 1.26) = 1 - 0.8962 = \mathbf{0.1038}$

$$(ii)\; P(Z < -0.86) = \mathbf{0.1949}$$

$$(iii)\ P(Z > -1.37) = 1 - P(Z < -1.37) = 1 - 0.0853 = \mathbf{0.9147}$$

$$(iv)\ P(-1.25 < Z < 0.37) = \varphi(0.37) + \varphi(1.25) - 1 = \mathbf{0.5387}$$

$$(v) P(Z \le -4.6) = \varphi(-4.6) = \mathbf{0.00000211}$$

(b) $P(Z > z) = 0.05$

$$P(Z > z) = 0.05$$
$$1 - P(Z \le z) = 0.05$$
$$P(Z \le z) = 0.95$$
$$\varphi(z) = 0.95$$
$$z = \varphi^{-1}(0.95)$$
$$= \mathbf{1.645}$$

(c) $P(-z < Z < z) = 0.99$

$$P(-z < Z < z) = 0.99$$

$$\varphi(z) - \varphi(-z) = 0.99$$
$$2\varphi(z) - 1 = 0.99$$
$$\varphi(z) = \frac{1.99}{2}$$
$$z = \varphi^{-1}(0.995)$$
$$= \mathbf{2.576}$$

Problem Statement 3: [100 marks]

The current flow in a copper wire follow a normal distribution with a mean of 10 A and a variance of 4 ( )2. What is the probability that a current measurement will exceed 13? What is the probability that a current measurement is between 9 and 11mA? Determine the current measurement which has a probability of 0.98.

Solution 3: Let X be the random variable that represents the current flow in a copper wire $X \sim N(10, 4)$

a) $P(X > 13) = P\left(\frac{X-\mu}{\sigma} > \frac{13-10}{2}\right) = P\left(Z > \frac{3}{2}\right)$

$$= 1 - P(Z \le 1.5) = 1 - \mathbf{0.9332}$$

$$= \mathbf{0.0668}$$

b) $P(9 < X < 11) = P\left(\frac{9-10}{2} < \frac{X-\mu}{\sigma} < \frac{11-10}{2}\right) = P(-0.5 < Z < 0.5)$

$$= 2\varphi(0.5) - 1 = 2(0.6915) - 1$$
$$= \mathbf{0.3829}$$

c) $P(X < x) = 0.98$

$$P(Z < z) = 0.98$$
$$\varphi(z) = 0.98$$
$$z = \varphi^{-1}(0.98)$$
$$= \mathbf{2.054}$$

Problem Statement 4: [150 marks]

The shaft in a piston has its diameter normally distributed with a mean of 0.2508 inch and a standard deviation of 0.0005 inch. The specifications of the shaft are 0.2500 ∓0.0015 inch. What proportion of shafts are in sync with the specifications? If the process is centred so that the mean is equal to the target value of 0.2500, what proportion of shafts conform to the new specifications? What is your conclusion from this experiment?

Solution 4:

The proportion of shafts that conform to the new specifications if given by

$$P(0.2485 < X < 0.2515) = P\left(\frac{0.2485 - 0.2508}{0.0005} < Z < \frac{0.2515 - 0.2508}{0.0005}\right)$$

$$= P(-4.6 < Z < 1.4)$$

$$= P(Z < 1.4) - P(Z < -4.6)$$

$$= 0.91924 - 0.0000 = \mathbf{0.91924}$$

If the process is centered so that the mean is equal to the target value of 0.2500, then the proportion of conforming shafts will be

$$P(0.2485 < X < 0.2515) = P\left(\frac{0.2485 - 0.2500}{0.0005} < Z < \frac{0.2515 - 0.2500}{0.0005}\right)$$

$$= P(-3 < Z < 3)$$

$$= P(Z < 3) - P(Z < -3)$$

$$= 0.99865 - 0.00135 = \mathbf{0.99730}$$

Conclusion: By centering the process, a 7.806% increase in yield is realised from 91.924% to 99.730%.