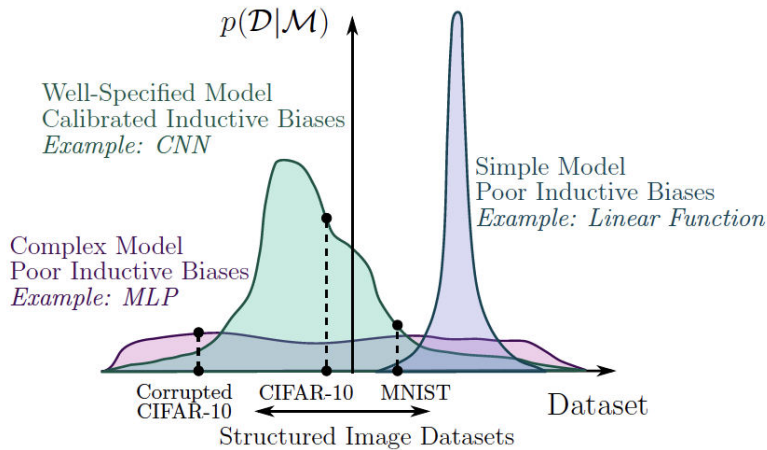


**Bayesian Probability** - the belief that something will happen, which is a belief we update given new information

- **Prior** - Our old belief of seeing something
- **Likelihood** - evidence, something that we have observed and its associated probability distribution
- **Posterior** - our new belief, which is the old belief updated with the new evidence that we have gained

**Support** - range of dataset classes that a model can support

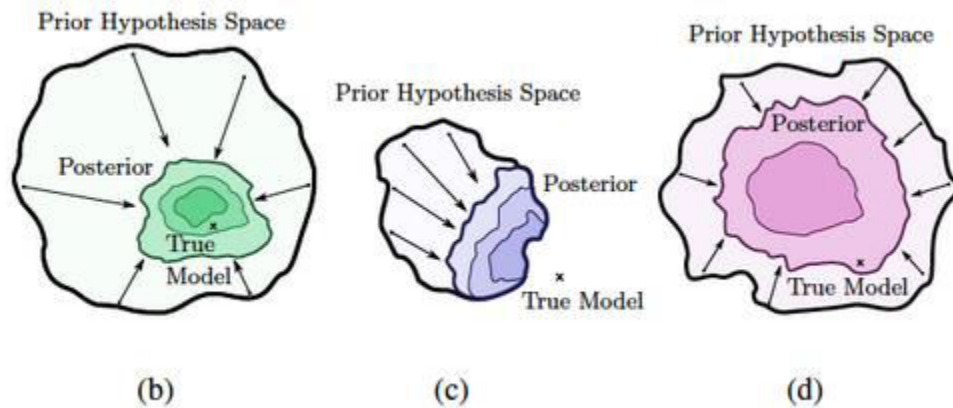
- Range of functions a model can represent
- **Inductive Bias** - how good a model class is at fitting a specific dataset
- **Generalisation of a model** - depends on two properties
  - **Support** - how many functions a model can support
  - **Inductive Bias** - how good is the performance



**The Marginal Likelihood (Bayesian Evidence)** - performance of our model, how well it fits a dataset

**The Bayesian Posterior** - Our model, which should get the right solution with the right inductive bias

- **Prior hypothesis space should have a broad support**



**Key Idea** - We use marginalization instead of optimisation

**Frequentist Approach** - it is to optimise a loss function to obtain optimal parameters

- We try to get a parameter **point estimate**
- e.g. using SGD with cross entropy and backpropagation
- We try to **maximise** the **likelihood**  $p(D|w, M)$ 
  - **w** - parameters (weights)
  - **D** - Dataset
  - **M** - Model (often left out)
  - We try to pick **w** such that we maximise the probability of observed dataset **D** given our model **M**
  - Called **Maximum Likelihood Estimation**, a special case of **Maximum a posteriori (MAP)** estimation with a uniform prior

## Bayesian Approach - It is to quantify uncertainty

- We get a **full probability distribution** over parameters, called a **posterior distribution**
- Represents how much uncertainty we have about each parameter
- We use **Bayes' Theorem** to compute the **posterior distribution**

$$\text{posterior} = \frac{\text{likelihood} * \text{prior}}{\text{evidence}}, \text{ or } p(w|\mathcal{D}) = \frac{p(\mathcal{D}|w)p(w)}{p(\mathcal{D})}$$

$$\text{evidence} = p(\mathcal{D}) = \int p(\mathcal{D}|w)p(w)dw$$

- **Posterior  $p(w|\mathcal{D})$**  - represents our **belief/hypothesis/uncertainty** about parameters **after** getting data
- **Prior Distribution  $p(w)$**  - Belief about what our model parameters are **before (prior)** to getting data
- **Likelihood  $p(\mathcal{D}|w)$**  - How well our data is explained by our parameters
  - This is the same as thing as in the frequentist approach
  - **Loss function** of our parameters
- **Marginalisation** - use **marginal likelihood  $p(\mathcal{D})$  (Bayesian (model) evidence)** to get **probability distribution**
  - This normalises the data
  - Marginalisation = **summing and integrating** over all parameter settings
  - Tells us **how likely the data is** over **all possible parameter settings**
  - Provides **evidence** for how good our model is
  - *Helps us get new probabilities given existing probabilities*

Bayesian approach is about **marginalization** rather than **optimization**

- It is hard to find  $p(\mathcal{D})$  exactly
- Therefore instead of optimising **w**, we find the best **distribution** for it
- We do this by using:
  - **Sampling methods** (Markov Chain Monte Carlo MCMC)
  - **Variational Inference** -
  - **Normalizing flows** -
- For **generative** models, we use:
  - **VAE** (similar to variational inference) -