Problem Set 2

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1 Problem 1: Stock & Watson

1.1 Exercise 3.2

(a)
$\overline{Y} = E\left[Y_1, ..., Y_n\right] = \hat{p} * 1 + (1 - \hat{p}) * 0$
$\overline{Y} = \hat{p}$
(b)
$E\left[\overline{Y}\right] = E\left[\hat{p}\right] = \frac{1}{n}\sum_{i=1}^{n} E\left(Y_i\right) = \frac{1}{n}\sum_{i=1}^{n} p = p$
Therefore, $\hat{p}$ is an unbiased estimator of p.
(c)
$var\left(\hat{p}\right) = var\left(\overline{Y}\right) = var\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} var\left(Y_i\right)$
$var\left(\hat{p}\right) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_Y^2$, $\sigma_Y = p\left(1 - p\right)$ for Bernoulli rv.
$var\left(\hat{p}\right) = \frac{p(1-p)}{n}$ $\square$

1.2 Exercise 3.3

(a)
$\hat{p} = 215/400 = .538$
(b)
$SE\left(\hat{p}\right) = \sqrt{var\left(\hat{p}\right)} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.538 * .462}{400}} = .025$
(c)
By the Central Limit Theorem, $\left|\frac{\hat{p} - \mu_0}{SE(\hat{p})}\right|$ has approximate distribution $N\left(0, 1\right)$.

$$
\begin{aligned}
p - value &= P_{H_0}\left\{\left|\frac{\hat{p} - \mu_0}{SE\left(\hat{p}\right)}\right| > \left|\frac{\hat{p}^{act} - \mu_0}{SE\left(\hat{p}^{act}\right)}\right|\right\} \\
&= 2\Phi\left(-\left|\frac{.538 - .5}{.025}\right|\right) = 2\Phi\left(-1.52\right) = 2\left(.0643\right) \\
&= .1286
\end{aligned}
$$

(d)

$$
\begin{aligned}
p - value &= P_{H_0}\left\{\frac{\hat{p}-\mu_0}{SE\left(\hat{p}\right)} > \frac{\hat{p}^{act}-\mu_0}{SE\left(\hat{p}^{act}\right)}\right\} \\
&= \Phi\left(\frac{.038}{.025}\right) = \Phi\left(-1.52\right) \\
&= .0643
\end{aligned}
$$

(e)
p-value of (d) is one half that of (c) because it only counts one direction of inequality and not two.
(f)
Depends on the significance level one wants to take. Assuming a significance level $\alpha = .05$, both p-values show that we do not have significant enough evidence to suggest that the incumbent is ahead of the challenger.

1.3 Exercise 3.4

(a)
95% confidence for $p = \{\hat{p} \pm 1.96 * SE\left(\hat{p}\right)\} = \{.538 \pm .049\} = \{.489, .587\}$
(b)
99% confidence for $p = \{.538 \pm 2.58 * .025\} = \{.4735, .6025\}$
(c)
The interval in (b) is wider because to be able to capture the population fraction in a larger percentage of samples one has to expand the accepted range of values around the sample mean.
(d)
We cannot reject the null hypothesis at the 5% significance level; see part (f) of Exercise 3.3.

1.4 Exercise 3.15

1.5 Exercise 3.16

1.6 Exercise 3.17

1.7 Exercise 3.18

2 Problem 2

(a)
(b)
(c)
(d)
(e)
(f)

3 Problem 3

(a)
(b)
(c)

4 Problem 4

(a)
(b)
(c)

5 Problem 5

(a)
(b)