ml

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(data + learning algorithm) $\rightarrow$ function

1 Supervised learning

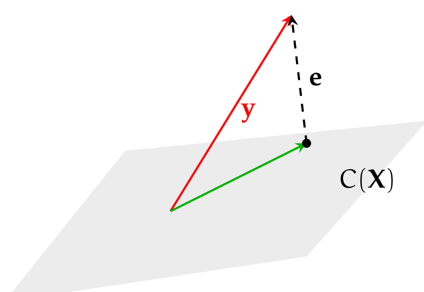
1.1 Regression

model, parameters, cost function, objective

1.1.1 Linear (in w) Regression

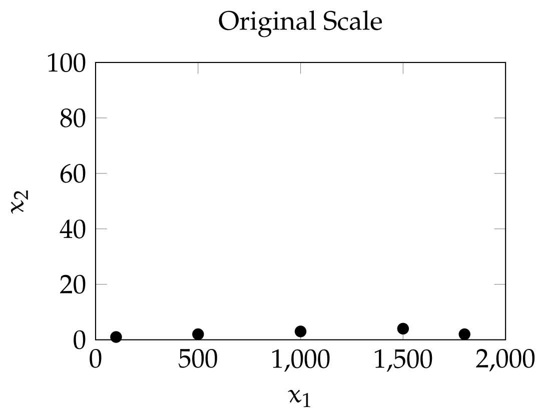
$\mathbf{X}^{m\times n}$ has m examples, each having n features. Usually $m >> n$. $\mathbf{y}^{m\times 1}$ are the corresponding outputs.

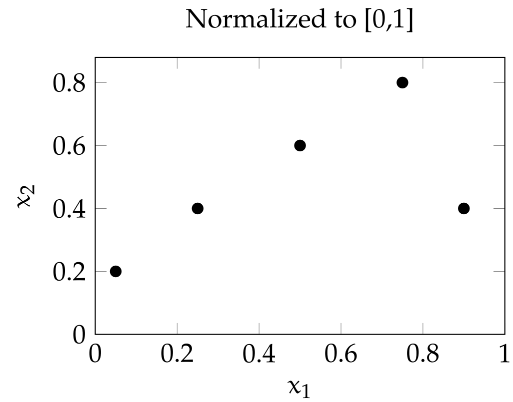
$$\begin{aligned}
\mathbf{Xw} &\approx \mathbf{y} \\
\mathbf{e} &= \mathbf{Xw} - \mathbf{y} \\
\underset{\mathbf{w}}{\text{minimize}}\ J(\mathbf{w}) &= \|\mathbf{e}\|^2 = (\mathbf{Xw} - \mathbf{y})^T(\mathbf{Xw} - \mathbf{y}) \\
\text{gradient descent: } \mathbf{w} &= \mathbf{w} - 2\alpha\mathbf{X}^T(\mathbf{Xw} - \mathbf{y})
\end{aligned}$$

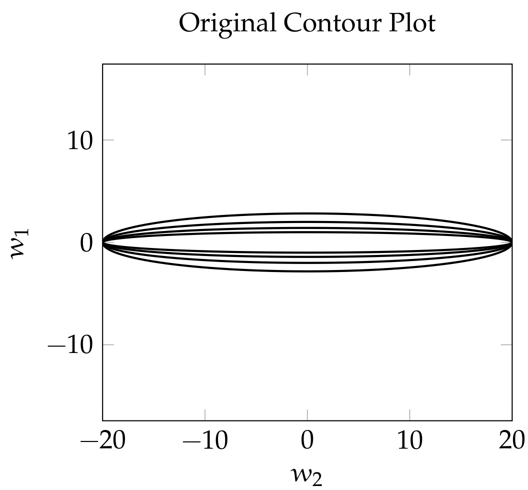


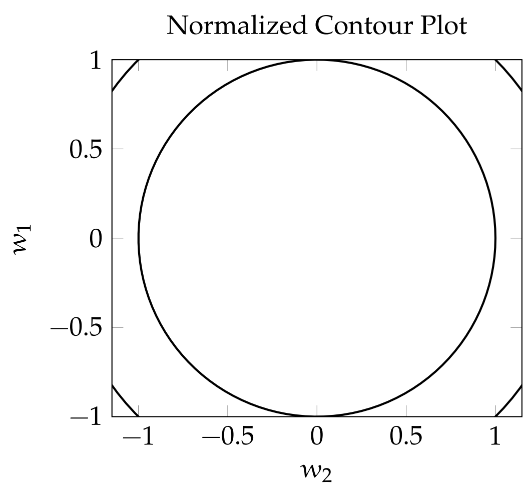
1.2 Normalization

Changes scale keeping the shape of distribution same. Gradient descent now works, with more ease, in the world of concentric circular contours.

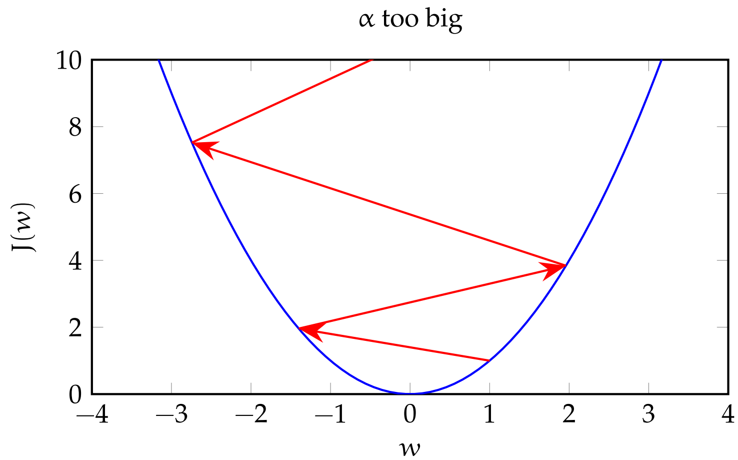








1.3 Learning rate, α



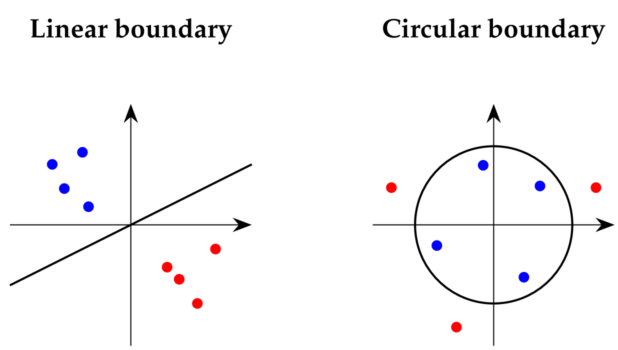
1.4 Classification

1.4.1 Logistic Regression

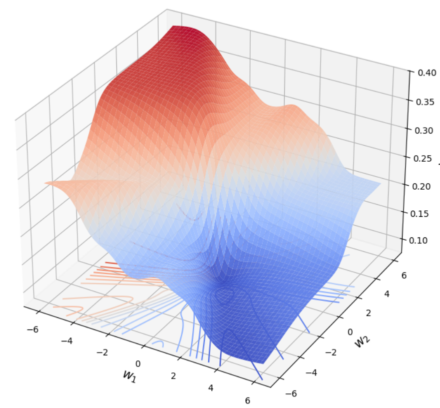
$$\mathbf{z} = \mathbf{Xw}, \qquad \texttt{// linear regression}$$

$$\sigma(\mathbf{z}) = \frac{1}{1 + e^{-\mathbf{z}}} \qquad \texttt{// maps z to probability-like (0,1)}$$

At $\mathbf{z} = 0$, $\sigma(\mathbf{z}) = 0.5$. So, with threshold 0.5, $\mathbf{z} = 0$ is the decision boundary. As linear regression doesn't have to be straight line, logistic regression's decision boundary does not have to be a straight line.



The squared error loss function $L(\mathbf{w}, y_i) = (\sigma(\mathbf{w}^T \mathbf{x}_i) - y_i)^2$, makes the cost function non-convex.

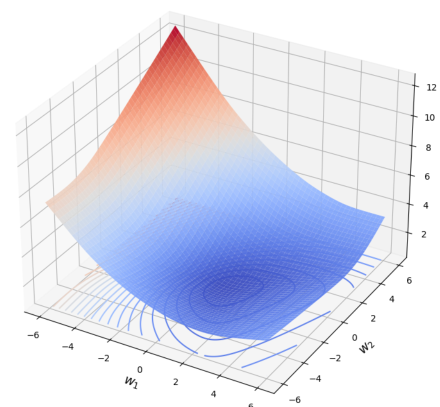


Negative log likelihood loss:

$$\hat{y}_i = \sigma\left(\mathbf{w}^T \mathbf{x}_i\right) \in (0, 1)$$

$$J(\mathbf{w}) = \frac{1}{m} \sum_{i=1}^{m} \begin{cases} -\log(\hat{y}_i), & \text{if } y_i = 1, \\ -\log(1 - \hat{y}_i), & \text{if } y_i = 0 \end{cases}$$

The negative log likelihood loss function with $\sigma(z)$ makes the cost function convex.



Negative log likelihood also incentivizes appropriately: big mismatch $\implies$ big loss, small mismatch $\implies$ small loss.