CSC 2504 Assignment 1

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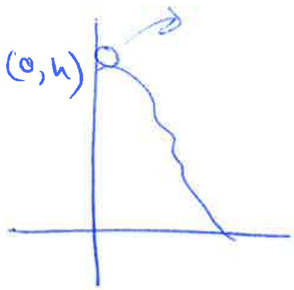
1]



$x(t) = at$

$y(t) = -\frac{1}{2}gt^2 + bt + h$

$v_0 = (a, b)$

$t = [0, t_i]$ ← time of impact $y = 0$

a) tangent

$$J(t) = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} a \\ -gt + b \end{bmatrix}$$

The normal is the negative reciprocal

Tangent $= \dfrac{a}{-gt+b}$

normal $= \dfrac{gt-b}{a}$

$$\therefore n(t) = \begin{bmatrix} gt-b \\ a \end{bmatrix}$$

2]

Transformations can be represented as H matrices. To prove $f_1 \cdot f_2 = f_2 \cdot f_1$, we must show

$$H_1 H_2 = H_2 H_1$$

a)

8/

b) Location @ t_i

$$x = a t_i$$

$$y = 0$$

Using $y = 0$

$$0 = -\frac{1}{2} g t^2 + bt + h$$

we can solve the above quadratic equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t_i = \frac{-b \pm \sqrt{b^2 - 4\left(-\frac{1}{2}\right)(g)(h)}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-b \pm \sqrt{b^2 + (2) g h}}{-g}$$

$$t_i = \frac{-b + \sqrt{b^2 + 2gh}}{-g} \quad \text{or} \quad \frac{-b - \sqrt{b^2 + 2gh}}{-g}$$

since $b > \sqrt{b^2 + 2gh}$ (g is positive, h is positive)

$t_i = \frac{b + \sqrt{b^2 + 2gh}}{g}$ is the only feasible solution (since $t > 0$)

$$\therefore \begin{bmatrix} x(t_i) \\ y(t_i) \end{bmatrix} = \begin{bmatrix} \frac{a\left(b + \sqrt{b^2 + 2gh}\right)}{g} \\ 0 \end{bmatrix}$$

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$$v(t) = \begin{bmatrix} \frac{d x(t_i)}{dt} \\ \frac{d}{dt} y(t_i) \end{bmatrix} = \begin{bmatrix} a \\ -g t_i + b \end{bmatrix} = \begin{bmatrix} a \\ \left(-b - \sqrt{b^2 + 2gh}\right) + b \end{bmatrix} = \begin{bmatrix} a \\ -\sqrt{b^2 + 2gh} \end{bmatrix}$$

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2)

Transformations can be represented by H matrices.
Thus we simple need to prove $H_1 H_2 = H_2 H_1$
or disprove

a) translation

$$H_1 = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

shear in x

$$H_2 = \begin{bmatrix} 1 & h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using MATLAB

$$H_1 H_2 = \begin{bmatrix} 1 & h & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_2 H_1 = \begin{bmatrix} 1 & h & t_1 + ht_2 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1 H_2 \neq H_2 H_1$$

$\therefore$ They are not commutative

b) Rotation 1

$$H_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation 2

$$H_2 = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & \end{bmatrix}$$

Block multiplication gives us

$$H_1 H_2 = \left[\begin{array}{c|c} A_1 A_2 & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hline 0 \quad 0 & 1 \end{array}\right]$$

$\therefore$ we prove whether $A_1 A_2 = A_2 A_1$

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$$A_1A_2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\varphi - \sin\theta\sin\varphi & -\cos\theta\sin\varphi - \sin\theta\cos\varphi \\ \sin\theta\cos\varphi + \cos\theta\sin\varphi & -\sin\theta\sin\varphi + \cos\theta\cos\varphi \end{bmatrix}$$

$$A_2A_1 = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\varphi\cos\theta - \sin\theta\sin\varphi & -\sin\theta\cos\varphi - \cos\theta\sin\varphi \\ \sin\varphi\cos\theta + \sin\theta\cos\varphi & -\sin\theta\sin\varphi + \cos\theta\cos\varphi \end{bmatrix}$$

$A_1A_2 = A_2A_1$ $\therefore$ 2 seperate rotations are commutative

c) Using block multiplication

$$A_1 = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \qquad A_2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$A_1 = aI$

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$$aIA_2 = aA_2I$$

$\therefore$ A rotation & uniform scaling are commutative

2| d) non-uniform scaling

rotation

$$A_1 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \qquad A_2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

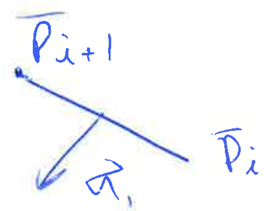
$$A_1A_2 = \begin{bmatrix} a\cos\theta & -a\sin\theta \\ b\cos\theta & b\cos\theta \end{bmatrix} \qquad A_2A_1 = \begin{bmatrix} a\cos\theta & -b\sin\theta \\ \cdots & \cdots \end{bmatrix}$$

Since $A_1A_2 \neq A_2A_1$ a rotation & a non-uniform scaling are not commutative

Summary

a) not commutative
b) commutative
c) Commutative
d) not commutative

3| a)



Let $\bar{a} = \bar{P}_i - \bar{P}_{i+1}$

$$m_{\vec{a}} = \frac{\bar{y}_i - \bar{y}_{i+1}}{x_i - x_{i+1}}$$

$\therefore$ the ~~midpoint is~~

$$\cancel{\frac{1}{2}\left[x_{i+1} + x_i\right.}$$

$$\vec{m}_n = \frac{x_{i+1} - x_i}{y_i - y_{i+1}}$$

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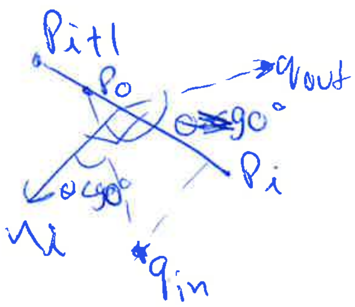
$$\therefore n_i = \begin{bmatrix} y_i - y_{i+1} \\ x_{i+1} - x_i \end{bmatrix}$$

3b)



assume $\bar{q} - \bar{P}_o$, where P_o is any point along $\bar{P}_i - \bar{P}_{i+1}$, and $\bar{n}$ is an arbitrary point. The test is to ~~find~~ evaluate the below dot product

$$\underbrace{\cancel{(\bar{P}_i - \bar{P}_{i+1})}}_{\bar{n}_i} \cdot (\bar{q} - P_o) = 0 \quad \bar{q} \text{ is on the line}$$

> 0 inside

< 0 outside

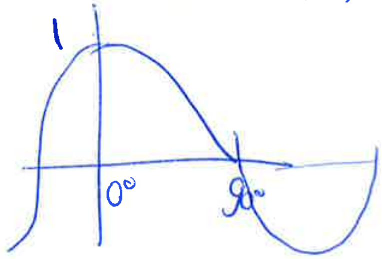
Note this is due to

$a \cdot b = |a||b| \cos\theta$

$\cos\theta = 1$

$\cos\theta < 0$ if $\theta > 90°$

$\cos\theta > 0$ if $\theta < 90°$



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c) Each point $\bar{q}$ must meet the below conditions

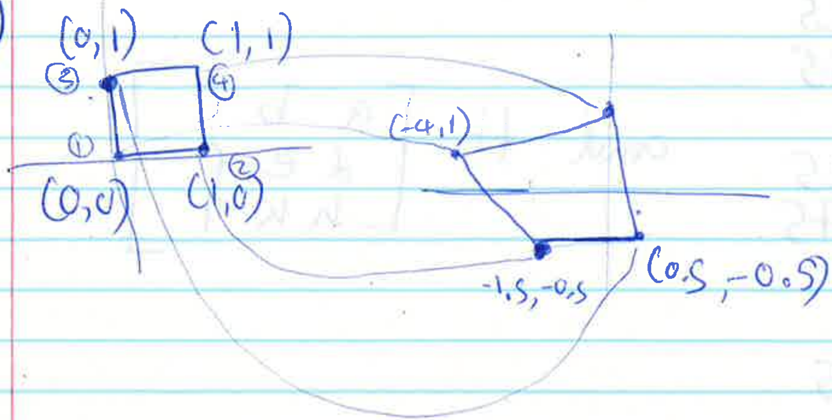
1) q is inside the outer edge

2) q is outside the inner edge

∴ the algorithm may be

- find the normals of all the outside edges
- ensure the point q lies inside w/ the dot product test: $\bar{n}_i \cdot (P_i - \bar{q}) > 0$, where P_i is any point along the current edge
- find the normals of all the outside edges & confirm $n_i \cdot (\bar{P}_s - \bar{q}) < 0$

4.4)



Transform to homographic coordinates

①

$$\begin{bmatrix} a & b & c \\ d & e & f \\ h & k & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cong \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$$

$$c - (-4)(1) = 0$$

$$\boxed{\begin{aligned} c &= -4 \\ f &= 1 \end{aligned}}$$

②

$$\begin{bmatrix} a & b & c \\ d & e & f \\ h & k & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cong \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix}$$

$$a + c - (-1.5)(h+1) = 0$$

$$\boxed{a + c + 1.5(h+1) = 0}$$

$$\boxed{d + f + 0.5(h+1) = 0}$$

③

$$\begin{bmatrix} a & b & c \\ d & e & f \\ h & k & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cong \begin{bmatrix} 0.5 \\ -0.5 \\ 1 \end{bmatrix}$$

$$\boxed{b + c - 0.5(k+1) = 0}$$

$$\boxed{e + f + 0.5(k+1) = 0}$$

④

$$\begin{bmatrix} a & b & c \\ d & e & f \\ h & k & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cong \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{a + b + c = 0}$$

$$\boxed{d + e + f - (h + k + 1) = 0}$$

Inputting these into Wolfram Alpha, we get

$a = 0.625$
$b = 3.375$
$c = -4$
$d = -2.125$
$e = -0.375$
$f = 1$
$h = 1.25$
$k = -2.25$

and $H = \begin{bmatrix} a & b & c \\ d & e & f \\ h & k & 1 \end{bmatrix}$

b) Point → $\overset{x,\ y}{(1, 2)}$
find (u, v)

$$ax_k + by_k + c = u_k(hx_k + ky_k + 1)$$

$$u_k = \frac{a(x_k) + by_k + c}{hx_k + ky_k + 1}$$

$$u_k = \frac{3.375}{-2.25} = -1.5$$

It maps to

$$\therefore \begin{bmatrix} -1.5 \\ 0.83 \end{bmatrix}$$

$$v_k = \frac{dx_k + ey_k + f}{hx_k + ky_k + 1} = \frac{-1.875}{-2.25} = 0.83$$

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c) No, it doesn't maintain parallelism & H isn't of the following form:

$$\left[\begin{array}{cc|c} \multicolumn{2}{c|}{A^{2\times2}} & \begin{matrix} t_1 \\ t_2 \end{matrix} \\ \hline 0 & 0 & 1 \end{array}\right]$$

Q5 again H is made from

translate, scale, rotate

$$H = RST = \begin{bmatrix} 8 & 3 & -7 \\ 6 & -4 & -24 \\ 0 & 0 & 1 \end{bmatrix}$$

$t_1 = -7$ since $T = \begin{bmatrix} I & \begin{matrix} t_1 \\ t_2 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix}$

$t_2 = -24$

$$\therefore \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ 6 & -4 \end{bmatrix}$$

$a\cos\theta = 8$ $\qquad -b\sin\theta = 3$

$a\sin\theta = 6$ $\qquad b\cos\theta = -4$

$\tan\theta = \dfrac{6}{8}$

$\underline{\theta = 0.64 \text{ rad}}$

$a\cos\theta = 8$

$a = \dfrac{8}{\cos\theta}$

$\underline{a = 9.97}$

~~ta~~

$b = \dfrac{-4}{\cos\theta}$

$\underline{= -4.99}$

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$$\therefore H = \overset{R}{\begin{bmatrix} \cos 0.64 & -\sin 0.64 & 0 \\ \sin 0.64 & \cos 0.64 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{S}{\begin{bmatrix} 9.97 & 0 & 0 \\ 0 & -4.99 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{T}{\begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & -24 \\ 0 & 0 & 1 \end{bmatrix}}$$