Assignment 2

1. Camera Transformation

Camera position $(1,2,2) \leftarrow \vec{e}$, origin of camera
viewing direction $(1,1,3) \leftarrow \vec{g}$
up vector $(0,1,0) \leftarrow \vec{r}$

Compute world to camera transformation matrix:

$Mwc = ?$

$\therefore \bar{s} = -\bar{g}$

$$= \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$$

$$\vec{u} = \frac{\vec{r} \times \vec{s}}{\|\vec{r} \times \vec{s}\|} = \begin{bmatrix} -3/\sqrt{10} \\ 0 \\ 1/\sqrt{10} \end{bmatrix}$$

$$r \times s = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ -1 & -1 & -3 \end{vmatrix}$$

$= i(-3) - j(0) + k(1)$

$$\therefore \bar{r} \times \bar{s} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\|\bar{r} \times \bar{s}\| = \sqrt{3^2 + 0^2 + 1^2} = \sqrt{10}$$

$$v = \bar{s} \times \bar{u} / \|\bar{r} \times \bar{s}\|$$

$$= \frac{-1}{\sqrt{10}} \begin{vmatrix} i & j & k \\ +1 & +1 & +3 \\ -3 & 0 & 1 \end{vmatrix}$$

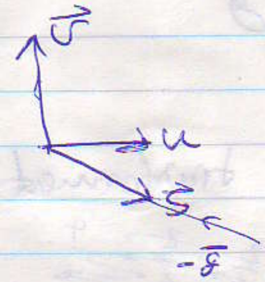
$$= \frac{-1}{\sqrt{10}} \left[i(1) - j(+8) + k(-3) \right]$$

$$= \frac{-1}{\sqrt{10}} \begin{bmatrix} 1 \\ -8 \\ +3 \end{bmatrix}$$

$$\|v\| = \frac{\sqrt{1^2 + 8^2 + 3^2}}{\sqrt{10}} = \frac{\sqrt{1+64+9}}{\sqrt{10}} = \sqrt{\frac{74}{10}}$$

1

$$\therefore \; v = \frac{\frac{-1}{\cancel{\sqrt{10}}}\begin{bmatrix}1\\8\\3\end{bmatrix}}{\sqrt{\frac{74}{\cancel{10}}}} = \begin{bmatrix}-1/\sqrt{74}\\8/\sqrt{74}\\3/\sqrt{74}\end{bmatrix}$$



Compute the camera to world transform

$$M_{cw} = \begin{bmatrix} -3/\sqrt{10} & -1/\sqrt{74} & -1 & 0 \\ 0 & 8/\sqrt{74} & -1 & 0 \\ 1/\sqrt{10} & -3/\sqrt{74} & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_{wc} = R_{cw}^{-1} = R_{cw}^{T}$, since $\vec{u}, \vec{v}, \vec{w}$ are orthogonal

$$M_{wc} = \left[\begin{array}{c|c} [u \; v \; s]^T & -[\vec{u} \; \vec{v} \; \vec{s}]^T \vec{e} \\ \hline 0 \; 0 \; 0 & 1 \end{array}\right]$$

$$[-\vec{u} \; \vec{v} \; \vec{s}]^T e = \begin{bmatrix} -\frac{3}{\sqrt{10}} & -1/\sqrt{74} & -1 \\ 0 & 8/\sqrt{74} & -1 \\ 1/\sqrt{10} & 3/\sqrt{74} & -3 \end{bmatrix}^T \begin{bmatrix}1\\2\\2\end{bmatrix}$$

$$\cancel{\frac{8}{\sqrt{740}}}\begin{bmatrix} -3/\sqrt{10} & 0 & 1/\sqrt{10} \\ -1/\sqrt{74} & 8/\sqrt{74} & 3/\sqrt{74} \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3}{\sqrt{10}} + \frac{2}{\sqrt{10}} \\ \frac{-1+16+6}{\sqrt{74}} \\ -1-2-6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{10}} \\ \frac{21}{\sqrt{74}} \\ -9 \end{bmatrix}$$

$$M_{wc} = \begin{bmatrix} -3/\sqrt{10} & 0 & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ -1/\sqrt{74} & 8/\sqrt{74} & -3/\sqrt{74} & \frac{4}{\sqrt{74}} \\ -1 & -1 & 3 & -9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. $P = [P_x, P_y, P_z]$ ← Camera coordinate

$q = [q_x, q_y, q_z]$

$m = 0.5(P+q)$

focal length is $-f$

$p', q', m' = ?$

$$\begin{bmatrix} P'_x \\ P'_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P'_x = P_x \\ P'_y = P_y \\ = -\frac{1}{f} P_z \end{bmatrix}$$

$$m = \frac{1}{2} \left\{ \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} \right.$$

$$\bar{P}' = \begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} P_x / -\frac{1}{f} P_z \\ P_y / -\frac{1}{f} P_z \end{bmatrix} = \frac{-f}{P_z} \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

$$\bar{q}' = \begin{bmatrix} q_x \\ q_y \end{bmatrix} \frac{-f}{q_z}$$

$\cancel{m' = \frac{1}{2}\left(\frac{-1}{f}\right) q_z \left\{ \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \end{bmatrix} \right\}}$

∴ ~~The midpoint is not the same~~

Calculated from p', q'

$$m' = \frac{1}{2}(-f)\left\{ \frac{1}{q_z}\begin{bmatrix} q_x \\ q_y \end{bmatrix} + \frac{1}{p_z}\begin{bmatrix} p_x \\ p_y \end{bmatrix} \right\}$$

Now project m directly

$$m' = \frac{f}{m_z}\begin{bmatrix} m_x \\ m_y \end{bmatrix}$$

$$= \frac{f}{m_z}\frac{1}{2}\begin{bmatrix} p_x + q_x \\ p_y + q_y \end{bmatrix}$$

$$= \frac{f}{\frac{(q_z + p_z)}{2}} \cdot \frac{1}{\cancel{2}}\begin{bmatrix} p_x + q_x \\ p_y + q_y \end{bmatrix}$$

$$m' = \frac{f}{q_z + p_z}\begin{bmatrix} p_x + q_x \\ p_y + q_y \end{bmatrix}$$

Since the 2 m' don't yield the same value, they are not the same.

Orthographic Case

$$\begin{bmatrix} p'_x \\ p'_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} p'_x \\ p'_y \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} q'_x \\ q'_y \\ 1 \end{bmatrix} = \begin{bmatrix} q_z \\ q_y \\ 1 \end{bmatrix}$$

$$\bar{m}' = \frac{\bar{p}' + \bar{q}}{2}$$

$$\begin{bmatrix} m'_x \\ m'_y \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} p_x + q_x \\ p_y + q_y \\ 1+1 \end{bmatrix}$$

Calculating the other way

$$m = \frac{1}{2} \begin{bmatrix} p_x + q_x \\ p_y + q_y \\ p_z + q_z \\ 1 + 1 \end{bmatrix}$$

$$m' = \frac{1}{2} \begin{bmatrix} p_x + q_x \\ p_y + q_y \\ \cancel{p_z + q_z} \\ 2 \end{bmatrix}$$

$\therefore$ In orthographi projection, the midpoint is the same

3) $l_1^{(u)} = \begin{bmatrix} P_{1x} + u d_x \\ P_{1y} + u d_y \\ P_{1z} + u d_z \end{bmatrix}$

Doing the perspective projection

$$l_1'(u) = \frac{f}{P_{1z} + u d_z} \begin{bmatrix} P_{1x} + u d_x \\ P_{1y} + u d_y \\ 1 \end{bmatrix}$$

$$l_2'(u) = \frac{f}{P_{2z} + u d_z} \begin{bmatrix} P_{2x} + u d_x \\ P_{2y} + u d_y \\ 1 \end{bmatrix}$$

The intersection point is the cross product of l_1 & l_2

Let $\overline{K}$ be the intersection

$$\overline{K} = \overline{l_1} \times \overline{l_2}$$

$$= \frac{f^2}{(P_1 z + u d_z)(P_2 z + u d_z)} \begin{vmatrix} i & j & k \\ P_{1x} + u d_x & P_{1y} + u d_y & 1 \\ P_{2x} + u d_x & P_{2y} + u d_y & 1 \end{vmatrix}$$

$$= \frac{f^2}{(P_1 z + u d_z)(P_2 z + u d_z)} \begin{bmatrix} P_{1y} + \cancel{u d_y} - P_{2y} - \cancel{u d_y} \\ -(P_{1x} + \cancel{u d_x} - P_{2x} - \cancel{u d_x}) \\ (P_{1x} + u d_x)(P_{2y} + u d_y) - (P_2 x + u d_x)(P_{1y} + u d_y) \end{bmatrix}$$

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2nd point

$$\begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ 0 & 0 & 1.002 & -2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -10 \\ 1 \end{bmatrix} \quad \begin{matrix} 0 & 0 \\ 0 & 0 \\ -100 & -1000 \\ 1 & 1 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ -10.02 - 2.002 \\ -10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -12.202 \\ -10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ +1.2002 \\ 1 \end{bmatrix}$$

3rd Point

$$\begin{bmatrix} \\ \\ -100.2 - 2.002 \\ -100 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1004.002 \\ -100 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ +10.04002 \\ 1 \end{bmatrix}$$

4th point

$$\begin{bmatrix} 0 \\ 0 \\ -1002 - 2.002 \\ -1000 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1004.002 \\ -1000 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ +1.004002 \\ 1 \end{bmatrix}$$

4.5

$$\bar{K} = \begin{bmatrix} P_{1y} - P_{2y} \\ -P_{1x} + P_{2x} \\ (P_{1x}+ud_x)(P_{2y}+ud_y) - (P_{2x}+ud_x)(P_{1y}+ud_y) \end{bmatrix} \frac{f^2}{(P_{1z}+ud_z)(P_{2z}+ud_z)}$$

4]

4×4

$$\begin{bmatrix} \frac{2f}{R-L} & 0 & \frac{R+L}{R-L} & 0 \\ 0 & \frac{2f}{T-B} & \frac{T+B}{T-B} & 0 \\ 0 & 0 & -\left(\frac{f+F}{f-F}\right) & \frac{2fF}{f+F} \\ 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\frac{2(1)(1001)}{-1000} = -2.002$$

$$-\left(\frac{1+1001}{1-1001}\right) = \frac{1002}{1000} = 1.002$$

$$= \begin{bmatrix} 0 \\ 0 \\ +\frac{(f+F)}{f-F} + \frac{2fF}{f-F} \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1.002 - 2.002 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 3.004 \\ 1 \end{bmatrix}$$

It is ~~not~~ a linear relationship.

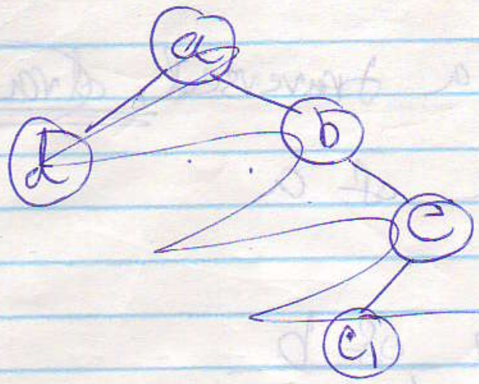
As depth increases, pseudodepth increases by a much lower factor. The change eventually becomes insignificant.

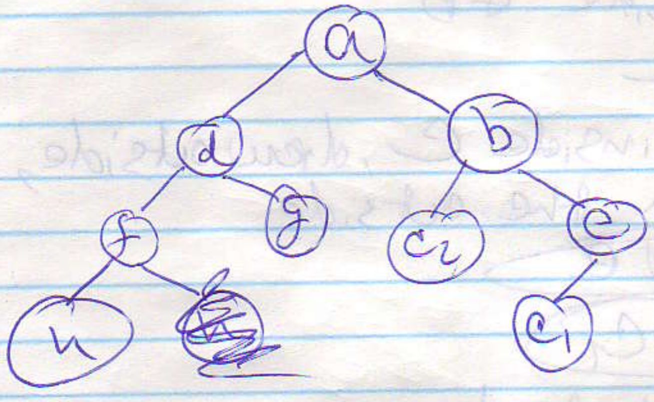
* pseudo depth is monotonic with real depth

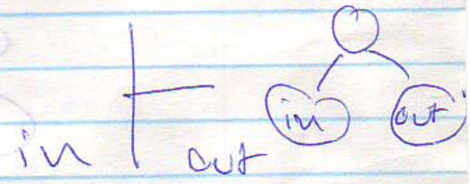
strictly non-increasing
or
non-decreasing

6)

a)







b) e_i is outside of a

∴ draw everything inside of a (the root
draw a
draw everything outside of i
e_1 outside of a
d is ~~outside~~ inside of a
e_1 is inside of d ~~draw~~ go to g, g has no more branches draw g
came back to d, draw d
go inide d
go to f
e_i is inside f, go outside f
nothing is on the outside, draw f

~~draw h~~

inside of a traversed, draw a

draw outside of a
go to b

e_1 is inside of b
draw outside of b
go to e
e_1 is inside e, draw outside, nothing
is on the outside
draw e
draw c_1
go back to b
draw b
draw c_2

$\boxed{g, d, f, h, a, e, c_1, b, c_2}$ ← final answer

e_2 is ~~inside~~ inside a
draw outside a
go to b
e_2 is outside b, draw inside of b
draw c_2
draw b
go to e

e_2 is outside of e
draw inside of e
draw c_1
draw e

go back to b
go back to a
draw a

draw inside a
go to d
e_2 is outside of d
draw inside d
go to f
e_2 is inside of f
draw outside f
~~nothing~~
draw f
draw h
draw d
draw g

$\boxed{c_2, b, c_1, e, a, f, h, d, g}$ ← Final answer

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Assignment 2

Ellipsoid, 3 radii

$$x(t,u) = a\cos(2\pi t)\sin(2\pi u)$$
$$y(t,u) = b\sin(2\pi t)\sin(2\pi u)$$
$$z(t,u) = c\cos(2\pi u)$$

2(a) Setting $z = 0$

$c\cos(u 2\pi) = 0$

$\cos(2\pi u) = 0$

$2\pi u = \frac{\pi}{2}$

$4u = 1$

$\boxed{u = \frac{1}{4}}$

$\therefore x(t,u) = a\cos(2\pi t)\sin\left[\frac{2\pi}{4}\right]$

$= a\cos(2\pi t)\,1$

$x(t,u) = a\cos(2\pi t)$

$$\boxed{\begin{aligned} x(t) &= a\cos(2\pi t) \\ y(t) &= b\sin(2\pi t) \end{aligned}}$$

2a) Set $\cancel{x} = \frac{c}{2}$

$\frac{\cancel{c}}{2} = \cancel{c}\cos(u2\pi)$

$1 = 2\cos(2\pi u)$

$\cos(2\pi u) = \frac{1}{2}$

$\cancel{u = 1.0479 \text{ rad} = \frac{\pi}{3}}$

$2\pi u = 60°$

$2\pi u = \frac{\pi}{3}$

$\boxed{u = \frac{1}{6}}$

$x(t) = a\cos(2\pi t)\sin(2\pi(\tfrac{1}{6}))$

$$\boxed{\begin{aligned} x(t) &= a\cos(2\pi t)\sin(\tfrac{\pi}{3}) \\ y(t) &= b\sin(2\pi t)\sin(\tfrac{\pi}{3}) \end{aligned}}$$

$$\boxed{\begin{aligned} x(t) &= \frac{\sqrt{3}}{4} a\cos(2\pi t) \\ y(t) &= \frac{\sqrt{3}}{4} b\sin(2\pi t) \end{aligned}}$$

~~b) for parametric curves, $\tau = \frac{dx(t)}{d(t)}$~~

~~1st ellipse~~

ellipse @ $z=0$

2 h) $x'(t) = -a\sin(2\pi t)(2\pi)$

$y'(t) = b\cos(2\pi t)(2\pi)$

$z'(t) = 0$

ellipse @ $z = \frac{c}{2}$

$x'(t) = \frac{-\sqrt{3}}{4} a\sin(2\pi t)(2\pi)$

$y'(t) = \frac{\sqrt{3}}{4} b\cos(2\pi t)(2\pi)$

$z'(t) = 0$

Computin the normal

assume $t = \frac{1}{4}$

@ ellipse $z=0$

$x'(\frac{1}{4}) = -a\sin(\frac{2\pi}{4})\ 2\pi$

$= -2a\pi$

$y'(\frac{1}{4}) = 0$

@ ellipse $z = \frac{c}{2}$

~~$x'(\frac{1}{4}) = -a\frac{\sqrt{3}}{4} 2\pi$~~

let $t=0$

$x'(0) = -a\sin(0)\ 2\pi$

$= 0$

$y'(0) = 2b\pi$

Find normal

$$\begin{vmatrix} i & j & k \\ -2a\pi & 0 & 0 \\ 0 & 2b\pi & 0 \end{vmatrix}$$

$i(0) + j(0) + k(-4ab\pi^2)$

The normal is

$(0, 0, -4\pi^2 ab)$

3. c) $z^2 = c^2 \cos^2(2\pi u)$

Try again

$z^2 = c^2 \left[1 - \sin^2(2\pi u)\right]$ $\qquad \frac{z^2}{c^2} = 1 - \sin^2(2\pi u)$

$y^2 = b^2 \sin^2(2\pi t) \sin^2(2\pi u)$ $\qquad \frac{z^2}{c^2} - 1 = \sin^2(2\pi u)$

$1 - \frac{z^2}{c^2} = \sin^2(2\pi u)$

$y^2 = b^2 \sin^2(2\pi t)\left(1 - \frac{z^2}{c^2}\right)$

$x^2 = a^2 \cos^2(2\pi t)\left(1 - \frac{z^2}{c^2}\right)$

$\cancel{x^2 + y^2} =$

$a + b = c$
$d + e = f$
$c + f = a + b + d + e$

① $\frac{x^2}{a^2} = \cos^2(2\pi t)\left(1 - \frac{z^2}{c^2}\right)$

② $\frac{y^2}{b^2} = \sin^2(2\pi t)\left(1 - \frac{z^2}{c^2}\right)$

$① + ② \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \left(1 - \frac{z^2}{c^2}\right)\Big[\underbrace{\cancel{\sin^2(2\pi t) + \cos^2(2\pi t)}}^{1}\Big]$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{z^2}{c^2}$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1}$$

3.d) The normal of the implicit form is the gradient.

$$\therefore \vec{n} = \begin{bmatrix} \frac{2x}{a^2} \\ \frac{2y}{b^2} \\ \frac{2z}{c^2} \end{bmatrix}$$

Let's compare to part b, where we found the normal for the crosssectional ellipses @ $z=0$ & $z = \frac{c}{2}$

@ $z=0$, $t = \frac{1}{4}$; $u = \frac{1}{4}$

$$\cancel{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

~~The normal is~~

$$\therefore x = a\cos(2\pi t) = a\cos\left(\frac{\pi}{2}\right) = 0$$

$$y = b\sin(2\pi t) = b\sin\left(\frac{\pi}{2}\right) = b$$

$$z = 0$$

$$\therefore \vec{n} = \begin{bmatrix} 0 \\ \frac{2b}{b^2} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2}{b} \\ 0 \end{bmatrix}$$