

Demostración de Igualdad:

$$2 \cdot \sin(\alpha) \cdot \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$\rightarrow \sin(\alpha) = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \quad \text{y} \quad \sin(\beta) = \frac{e^{i\beta} - e^{-i\beta}}{2i}$$

$$\boxed{2 \cdot \sin(\alpha) \cdot \sin(\beta)} = 2 \cdot \frac{(e^{i\alpha} - e^{-i\alpha})}{2i} \cdot \frac{(e^{i\beta} - e^{-i\beta})}{2i} = -\frac{1}{2} (e^{i\alpha} - e^{-i\alpha}) \dots$$

→ hago distributiva.

$$= -\frac{1}{2} \left[e^{i\alpha} \cdot e^{i\beta} - e^{i\alpha} \cdot e^{-i\beta} - e^{-i\alpha} \cdot e^{i\beta} + e^{-i\alpha} \cdot e^{-i\beta} \right] =$$

$$= -\frac{1}{2} \left[\underbrace{e^{i(\alpha+\beta)} + e^{-i(\alpha+\beta)}}_a - \underbrace{(e^{i(\alpha-\beta)} + e^{-i(\alpha-\beta)})}_b \right]$$

→ desarrollo los $e^{i\theta} = \cos(\theta) + i \sin(\theta)$
 $e^{-i\theta} = \cos(\theta) - i \sin(\theta)$

$$\begin{array}{l} \cdot e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i \sin(\alpha+\beta) \\ \cdot e^{-i(\alpha+\beta)} = \cos(\alpha+\beta) - i \sin(\alpha+\beta) \\ \cdot e^{i(\alpha-\beta)} = \cos(\alpha-\beta) + i \sin(\alpha-\beta) \\ \cdot e^{-i(\alpha-\beta)} = \cos(\alpha-\beta) - i \sin(\alpha-\beta) \end{array} \quad \left. \begin{array}{l} \text{sumo y obtengo } a \\ \text{sumo y obtengo } b. \end{array} \right\}$$

→ reemplazo en la ecuación

$$= -\frac{1}{2} [2 \cos(\alpha+\beta) - 2 \cos(\alpha-\beta)] = \boxed{\cos(\alpha-\beta) - \cos(\alpha+\beta)}$$

⇒ podemos decir que la igualdad se cumple.