Restaurant Prep-time Analysis

A Project for the UCSC Baysian Statistics Course on Coursera

Mike O'Dell

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Executive summary

This report is intended to satisfy of the project requirement of the UCSC *Bayesian Statistics: Techniques and Models* course offered on Coursera. The data and analysis is intended to determine if restaurant chain stores see an improvement in order prep-time over the long term. The analysis uses a simple hierarchical linear regression using two covariates, and performs reasonably well showing that store age has a minor influence on order prep-time (order volume is much more important) and the model reasonably predicts data from a test store.

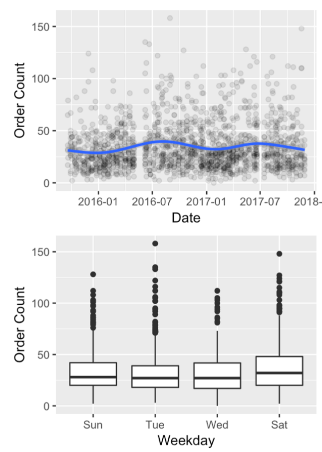
Introduction

New stores often see significant improvement in prep-time as new staff become more practiced, but it is not clear whether or improvements continue after a few years. This project atempts to answer the question using data collected from stores in a single restaurant chain.

The data

The data is obseravational data collected in a sample of stores from a single restaurant chain. The data was time-stamped allowing the calculation of the store `age` at time of collection and order counts per 10 minutes (`orders`) and prep-times for orders (`prep`) were recorded. The data was collected every other week at lunchtime over two years but suffers from a few missing values and a 4-week gap in coverage. The data was cleaned for typos and other typical data entry errors.

An initial examination of the data shows that there is is some regular seasonality, little to no long-term (annualized) trend, and perhaps some change in demand by weekday. However, it is unlikely that the weekday or month would direclty affect the prep time (as opposed to the actual order rate for any given day).



Seasonal and weekday fluxuation in orders

The dependent variable `prep` and the two continuous variables `orders` and `age` are all non-negative suggesting that a poisson or a normal (of the variables' log) might be appropriate model distribution.

Comparing the means and variance of the these variables, however, show that they are not the same and thus a poisson distribution is not appropriate. Also, the means and variances differ by stores, suggesting a hierarchical model may be appropriate.

Modeling

Given that prep-times, order counts, and store age are always positive, and given that the mean and variance differ signficantly (ruling out a poisson distribution), we will use a normal distributions and the `log` of each variable in our model.

Since the data exploration showed that means and variance for orders and prep-time vary by store, we will use a hierarchical linear regression and start with only a random intercept:

$$y|store_i, \tilde{x}_i, \tilde{\alpha}, \tilde{\beta}, \sigma^2 \overset{ind}{\sim} \mathcal{N}(\alpha_{store\ i} + \beta_{1\ i} x_{age\ i} + \beta_{1\ i} x_{orders\ i}, \sigma^2), store_i \in \{1, \ldots, S\}, i = 1, \ldots,$$

$$\alpha_{store} \,|\mu, \tau \overset{iid}{\sim} \mathcal{N}(\mu, \tau^2), store = 1, \ldots, S$$

Since it is a linear regression, β_{age} will tell us the effect of age on prep-time (our question) and can be tested for significance. The model will use non-informative priors given that we come to the question without prior knowledge.

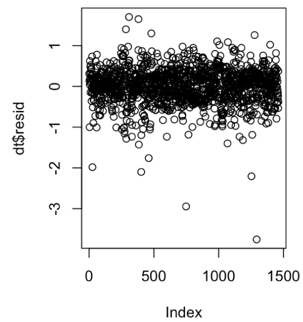
With the training data, the model will produce 26 parameters:

- `a0`—the overall store intercept,
- 21 `a[j]` individual store intercepts,
- two `b[k]` covariate coefficients,
- a standard deviation τ for the overall store intercept distribution, and
- a standard deviation σ for the prep-time response variable.

To start, we initialize the model with three chains, run a burn-in of 1,000 iterations, and then run the model for 5,000 iterations.

A plot of the parameters shows that the MCMC has not converged for most of the parameters. Furthermore, autocorrelation is high for the alpha and beta coefficents and the effective sample size is low.

Rerunning the model, this time for 500,000 iterations and thining to every 100th iteration yields convergence, low autocorrelation, scale reduction factor upper C.I.'s are nearly 1.0, and effective sample size is ~1750.



Residuals for random intercept model.

Residuals look good with a few outliers. The DIC Penalized deviance for this model is 1754. A second model with β_1 and β_2 also parameterized by `store` produced a DIC Penalized deviance of 1760. Thus the random intercept only model is preferred.

Results/Conclusion

The model coefficients β_{age} is -0.086 which does indeed indicate that older stores do continue to see decreasing prep-times. However, order volume has a greater impact as β_{orders} is almost twice as large in magnitude at 0.141.

Posterior predictive simulation drawing an α_* distribution using mean a0 and variance τ^2 and then computing yhat values for a test store with observed age and orders yields decent results as well.