2_distances

April 10, 2023

1 Cosmology Homework 2

Note: Adapted directly from the `2_distances.ipynb` notebook

[ ]:
```
import astropy.units as u
import numpy as np
import pylab as plt
from sympy import *
from scipy import integrate
%matplotlib inline
```

[ ]:
```
#cosmological parameters
h = 0.7
OmegaM = 0.3
OmegaR = 0.0
OmegaL = 1- OmegaM  #assumes flat

lightspeed = 3e5 * u.km/u.s

#Hubble function
def Hz(z):
    H = 100*h* (
        OmegaM*(1+z)**3 +
        OmegaR*(1+z)**4 +
        OmegaL +
        (1 - OmegaM - OmegaL - OmegaR)*(1+z)**2 )**.5
    return H

#conformal distance
def Dconf(z, **kwargs):
    return lightspeed * np.array([integrate.romberg(dConfDistdz, 0, zi,␣
 ↪divmax=15) for zi in z]) * (u.Mpc * u.s / u.km)

def dConfDistdz(z):
    return 1.0 / Hz(z)

#angular diameter distance
def DA(z):
```

```python
    return Dconf(z)/(1+z)

def DL(z):
    return Dconf(z)*(1+z)

from astropy.coordinates import Angle
# angle subtended via angular diameter distance
def AngleSize(z, length):
    # Since DA = l/theta => theta = l/DA
    return Angle( (length / DA(z)) * u.rad, unit=u.arcsec)
```

2 Problem 1

We get a minimum angular size at some redshift because there are two competing effects. An object far away is smaller, but also an object that subtends some large amount of the universe will be bigger. As you go back these competing effects balance at some point. At this critical redshift (or time) going back in time a little bit means when observed from present day earth the 1kpc object is in a smaller universe and takes a larger observation angle. At the same time that object is moving further away and is therefore smaller. At the critical redshift these two effects are the same size; however, as you go further back in time the effect of expansion on the angle becomes the larger effect by far.

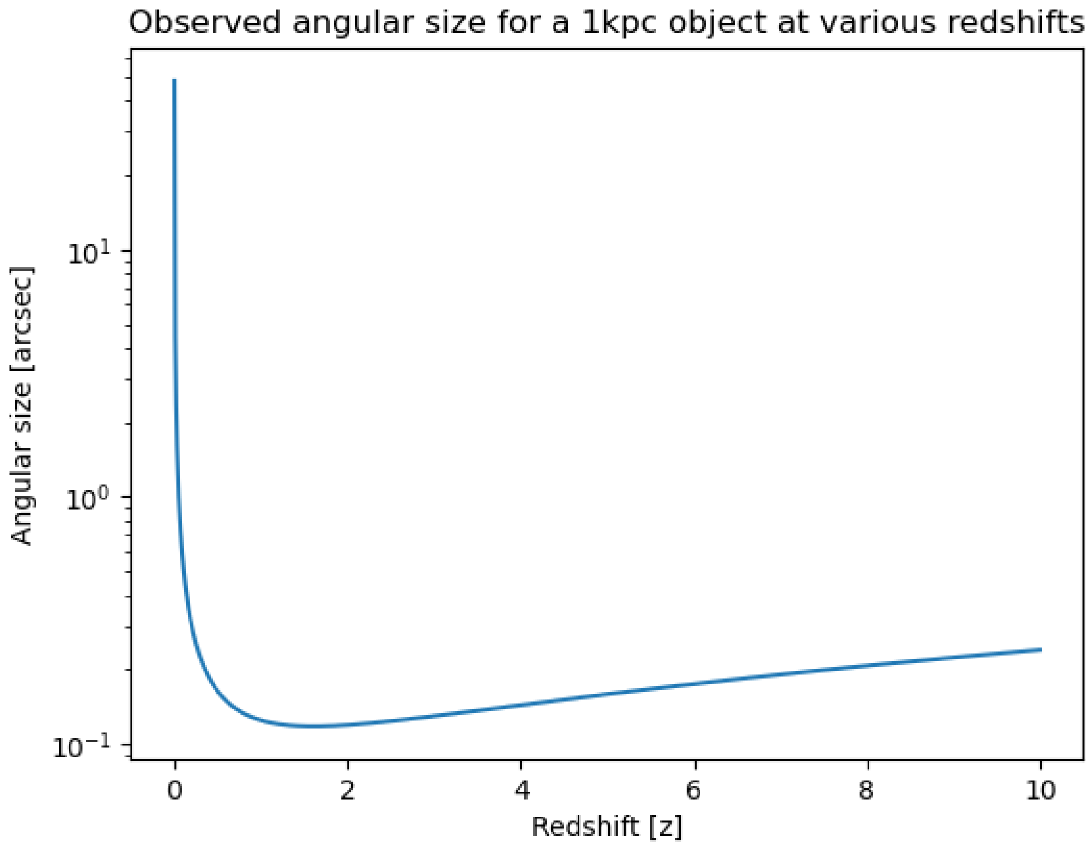
Values for parts a, b, and c are all printed below

[ ]:
```python
redshift = np.linspace(0.001, 10, num=1000)
angular_size = AngleSize(redshift, 1* u.kpc).to(u.arcsec)

fig, ax = plt.subplots(1,1)
ax.plot(redshift, angular_size)
ax.set_title("Observed angular size for a 1kpc object at various redshifts")
ax.set_xlabel("Redshift [z]")
ax.set_ylabel("Angular size [arcsec]")
ax.set_yscale("log")

print(f"Minimum angular size of {np.min(angular_size):.3} achieved at redshift␣
  ↪z={redshift[np.argmin(angular_size)]:.4}")
```

```
Minimum angular size of 0.118 arcsec achieved at redshift z=1.602
```

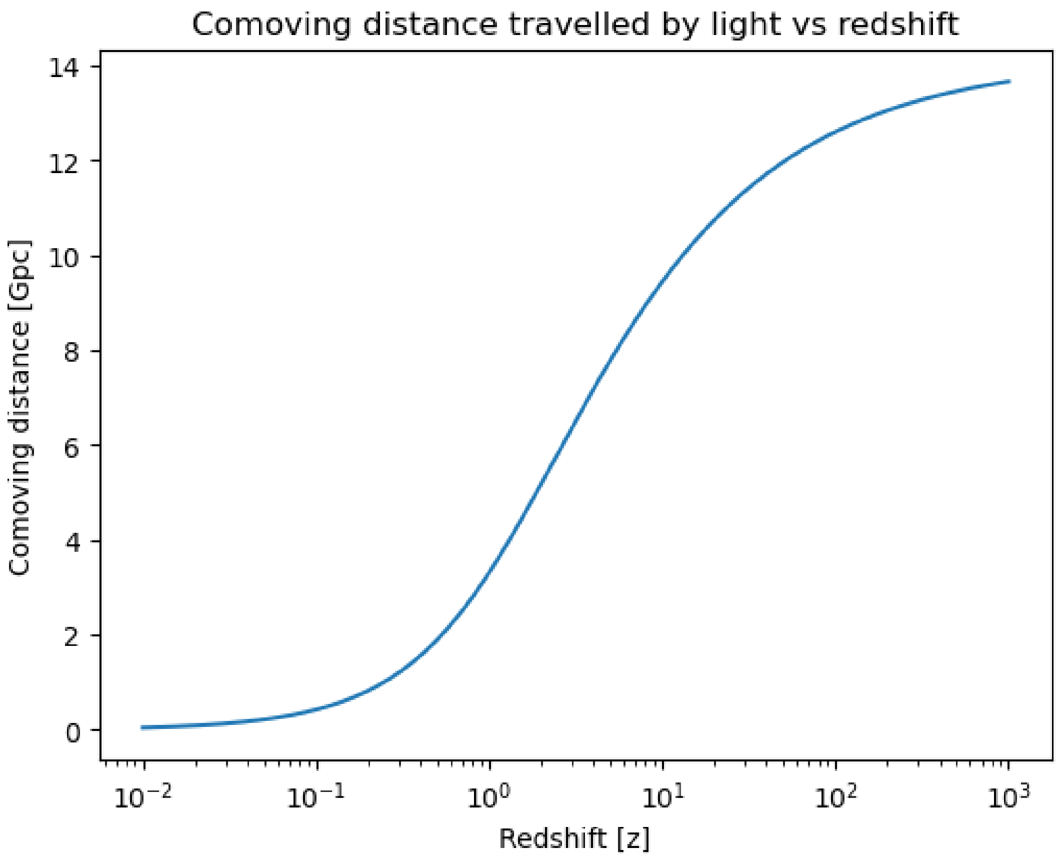


[ ]:
```
redshift = np.logspace(-2, 3, num=1000, base=10)
comoving_distance = Dconf(redshift).to(u.Gpc)

fig, ax = plt.subplots(1,1)
ax.plot(redshift, comoving_distance)
ax.set_title("Comoving distance travelled by light vs redshift")
ax.set_xlabel("Redshift [z]")
ax.set_xscale("log")
ax.set_ylabel("Comoving distance [Gpc]")

print(f"Comoving distance at z={redshift[-1]:.5} is {comoving_distance[-1]:.4}")
```

```
Comoving distance at z=1000.0 is 13.67 Gpc
```



[ ]:
```
# Angle subtended by the comoving horizon
z = 1000
horizon_distance = comoving_distance[-1]
horizon_size_past = (horizon_distance/(1+z))
print(f"Size of the Comoving horizon at z=0: {horizon_distance:.5}")
print(f"Size of the Comoving horizon at z=1000: {horizon_size_past.to(u.Mpc):.
  ↪5}")
print(f"Angle subtended by the Comoving horizon at z=1000 is␣
  ↪{Angle(horizon_size_past/horizon_distance * u.rad, u.arcmin):.4}")
```

```
Size of the Comoving horizon at z=0: 13.67 Gpc
Size of the Comoving horizon at z=1000: 13.656 Mpc
Angle subtended by the Comoving horizon at z=1000 is 3.434 arcmin
```

3 Problem 2

Reproduce the diagram !

I had fun with this and tracked down the original diagram and the original paper. There's a little conversion needed to get it ship-shape for the plot.

[ ]:
```
# Original paper: https://arxiv.org/pdf/1401.4064.pdf
# From Table F.1
z_data = np.array([
    0.010, 0.012, 0.014, 0.016, 0.019, 0.023, 0.026, 0.031, 0.037, 0.043,
    0.051, 0.060, 0.070, 0.082, 0.097, 0.114, 0.134, 0.158, 0.186, 0.218,
    0.257, 0.302, 0.355, 0.418, 0.491, 0.578, 0.679, 0.799, 0.940, 1.105,
    1.300])
u_b_data = np.array([
    32.9538, 33.8790, 33.8421, 34.1185, 34.5934, 34.9390, 35.2520, 35.7485, 36.
 ↪0697, 36.4345,
    36.6511, 37.1580, 37.4301, 37.9566, 38.2532, 38.6128, 39.0678, 39.3414, 39.
 ↪7921, 40.1565,
    40.5649, 40.9052, 41.4214, 41.7909, 42.2314, 42.6170, 43.0527, 43.5041, 43.
 ↪9725, 44.5140,
    44.8218])
# From Table F.2 Diagonals of the covariance matrix are the error squared
u_b_err_data = np.sqrt( np.array([
    21282, 28155, 6162, 5235, 7303, 3150, 3729, 3222, 3225, 5646,
    8630,  3855,  4340, 2986, 3592, 1401, 1491, 1203, 1032, 1086,
    1006,  1541,  1127, 1723, 1550, 1292, 3941, 2980, 4465, 23902,
    19169
]) * 10**(-6) )

# Get from z to c*z
cz_data = lightspeed * z_data

# Get from distance modulus u_b to distances
dist_data = 10**(1 + u_b_data/5) * u.pc

# Derivative of distance above times the error to find error bars
dist_err_data = np.log(10)/5 * 10**(1 + u_b_data/5) * u_b_err_data * u.pc
```

Next I'm plotting each of these universes. I ran into some issues exactly matching the diagram, see comments

[ ]:
```
fig, ax = plt.subplots(1,1)
fig.set_figwidth(7)
fig.set_figheight(7)
ax.set_xlim(0,11)
ax.set_ylim(0,500000)
ax.set_xlabel("Luminosity Distance [Gpc]")
ax.set_ylabel("cz [km/sec]")
ax.grid(True)
ax.set_xticks(np.arange(0,11,1))
# Plot the data with error bars
ax.errorbar(dist_data.to(u.Gpc), cz_data, xerr = dist_err_data.to(u.Gpc), fmt =␣
 ↪'.')
```

```python
redshift = np.linspace(0, 2, num=1000)

def plot_universe(ax, redshift, **kwargs):
    luminosity_distance = DL(redshift).to(u.Gpc)
    ax.plot(luminosity_distance, redshift*lightspeed, **kwargs)

h = 0.71
OmegaR = 0
# EdS universe (only matter)
OmegaM = 1
OmegaL = 1 - OmegaM  #assumes flat
plot_universe(ax, redshift, color="black")

# dS universe (only cosmological constant)
OmegaM = 0
OmegaL = 1 - OmegaM  #assumes flat
plot_universe(ax, redshift, color="blue")

# Flat Dark Energy Model
OmegaM = 0.27
OmegaL = 0.73  #assumes flat
plot_universe(ax, redshift, color="magenta")

# These two don't match the parameters in the problem

# Closed matter only
#
# "Closed Matter only" says matter only only on the plot
# In the original text it's just "A closed universe" with Omega = 2
# Setting OmegaR to 0.3 seems to put the line in the right place
OmegaM = 1.7  # Ideally 2.0 for matter only
OmegaL = 0.0
OmegaR = 0.3  # Ideally 0.0 for matter only
plot_universe(ax, redshift, color="red")

# Empty Universe
# Without OmegaL=0.4 it is in very much the wrong place on the diagram
# It still looks like its not separated enough from our universe in this view
# Something still may be wrong, or the Friedman equation Ned Wright used here
# May have a different form.
OmegaM = 0
OmegaL = 0.4  # Ideally 0.0 so we are empty of cosmological constant as well
OmegaR = 0
plot_universe(ax, redshift, color="green")

# Put these ugly global variables back to their correct values!
```

```python
h = 0.7
OmegaM = 0.3
OmegaR = 0.0
OmegaL = 1- OmegaM  #assumes flat
```