Astro 513A; Question set 2: distances

There is an ipython notebook file on Canvas '2_distances.ipynb' that may be of aid for the numerical calculations. (PS I may go around room and look at everyone's results this time.)

1. Angular Diameter Distances

Please assume the concordance cosmology for this problem: $(\Omega_{m,0}, \Omega_{\Lambda,0}, h) = (0.3, 0.7, 0.7)$. Calculations in this cosmology generally require numerical evaluation. Feel free to check any numerical calculation using a cosmology calculator, but you should also code up the calculation yourself (using Mathematica is also fine).

a) Consider a galaxy-sized object of fixed radius with $R = 1$ kpc. Plot its angular size in arc-seconds if it is observed at $z = 0.001 - 10$. What is the redshift at which the object attains its minimum angular size? What is that size? Why does this peak at a given redshift?

b) Calculate the comoving distance traveled from the beginning of time to $z = 1000$ assuming a velocity equal to c. This distance is called the comoving "horizon".

[You do not have to include radiation in $H(z)$ even though this component becomes dominant at $z > 3000$ and would need to be included for $\lesssim 10\%$ accuracy.]

c) Observing from the present, what is the angle subtended by the comoving horizon at $z = 1000$?

Patches of the CMB larger than this angular scale appear as if they should not have been in casual contact, but nonetheless the CMB is observed to be smooth across the entire sky. This is the famous "horizon problem".

2. Hubble diagram

Making a Hubble diagram using Type 1a supernova, where their absolute luminosity has been calibrated with Cepheid variable stars, is one of our gold-standard methods to measure H_0 and the energy constituents of the Universe, as we think we can calibrate the luminosity of each Type 1a to 10% accuracy and thereby infer the luminosity distance from the observed flux (e.g. $f = L/[4\pi d_L^2]$). Type 1a's are also so bright that we can see them literally across the Universe, often more luminous than their entire host galaxy.

Using e.g. a python notebook, reproduce at least four of the curves in the following Hubble diagram made in 2015. The 'Binned SNe' points are measurements of the luminosity distances using Type 1a's; you do not need to include these points. (If someone grabs them from the plot using one of the many software packages that can do this [you will find you often need to do this in your research], it might be fun to plot them too. It you do grab them, please share with us!).

All the curves assume $H_0 = 71\,\text{km/s/Mpc}$, which sets the slope at small distances. This is the linear relation that Hubble measured, but a linear relation no longer holds once $z \gtrsim 0.5$. Most of the curves you probably can identify, but ignore 'Dusty EdS Model' and the 'Closed Dark Energy

Model', as I don't know what was assumed there. Einstein-de Sitter means a flat universe with only non-relativistic matter ($\Omega_{m,0} = 1$), which is not to be confused with 'de Sitter' which is a flat universe with ($\Omega_{\Lambda,0} = 1$), the 'Closed Matter Only Model' assumes $\Omega_0 = 2$ whereas the empty universe is $\Omega_0 = 0$ (Ω_0 is the sum of all the Omegas today [excluding curvature]).

The 'Flat Dark Energy Model' is actually what we call the ΛCDM model, and this curve adopts values consistent with the accepted values today of $(\Omega_{m,0}, \Omega_{\Lambda,0}) = (0.27, 0.73)$. This curve goes nicely through the measurements, whereas most of the others do not!

I haven't checked these curves myself, so no guarantees that what's plotted is exactly what they stated.

The maximum of the y-axis, $cz = 5 \times 10^5$km/s, corresponds to $z = 1.7$.