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Assignment 3 *1910

1) a) {2}
b) {3,4,5,6,7}
c) {1,2,4,8,16,32}

2) a) yes
b) No because the square of a number is never 2, thus the element 2 does not exist in the set
c) yes
d) NO because 2 & of {{2},{{2}}}
e) NO because 2 & of {{2},{2,{2}}}
f) NO because 2 & of {{{2}}}

3) P(X) = {{1,2,3,4},{1,2,3},{1,2,4},{1,3,4},{2,3,4},{1,2},{1,3},{1,4},{2,3},{2,4},{3,4},{1},{2},{3},{4},{ }}
→ cardinality of P(X) = 2^4 = 16

4) A: {{1,2},{1,3},{1,4},{2,3},{2,4},{3,4}}, where
P(X): {{1,2,3,4},{1,2,3},{1,2,4},{1,3,4},{2,3,4},
{1,2},{1,3},{1,4},{2,3},{2,4},{3,4},{1},{2},{3},
{4},{ } }

5) A = {a,b,c,d,e}, B = {a,c,f}
a) {a,b,c,d,e,f}
b) {a,c}
c) A = {a,b,c,d,e}
B = {a,c,f}
∴ = {b,d,e}
d) B = {a,c,f}
A = {a,b,c,d,e}
∴ = {f}

6) $A=\{1,2,3\}$, $B=\{a,b\}$

LS	RS
$A\times B=\{(1,a)(1,b),(2,a),(2,b),(3,a)(3,b)\}$	$\lvert A\rvert=3$
	$\lvert B\rvert=2$
$\lvert A\times B\rvert=6$	$3+2=5$

$\therefore\ \lvert A\times B\rvert \neq \lvert A\rvert+\lvert B\rvert$

7) $A=\{a,b,c,d\}$ $A^3=\{a,b,c,d\}\{a,b,c,d\}\{a,b,c,d\}$ (expand)

$\{a,b,c,d\}\{a,b,c,d\}(A^2)$ (expand)

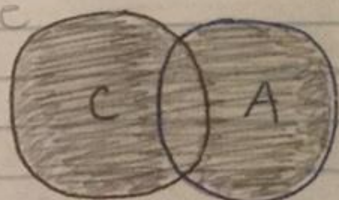
$\{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d)(c,a)$
$(c,b),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d)\}\{a,b,c,d\}$ (A^3)

$\{(a,a,a),(a,b,a),(a,c,a),(a,d,a),(b,a,a),(b,b,a)\ldots\}$

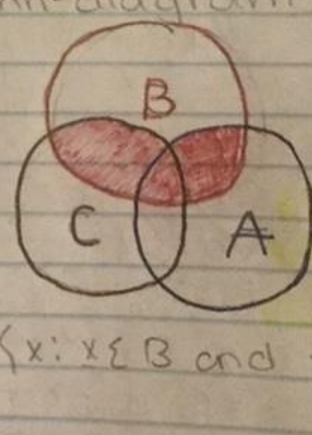
Thus, ex of 3 tuples are (a,a,a), (b,c,a), and (a,c,a) because each element is in the set of A^3, that is $(a,a,a)\in A^3$ and $(b,c,a)\in A^3$ and $(a,c,a)\in A^3$.

8)

set is $B\cap(C\cup A)$ because ex $(C\cup A)$ is evaluated first thus the venn-diagram will look like



Then:
$B\cap(C\cup A)$ $\rightarrow$ $\therefore$

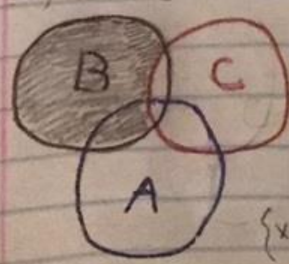


$\{x: x\in C \text{ or } x\in A \text{ or both}\}$

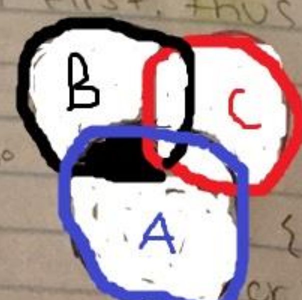
$\{x: x\in B \text{ and } x\in(C\cup A)\}$

a)

9) $A\cup(B-C)$, $(B-C)$ is evaluated first, thus



$\rightarrow$ then $A\cap(B-C)$ $\therefore$



$\{x: x\in B \text{ and } x\notin C\}$

$\{x: x\in A \text{ or } x\in(B-C) \text{ or both}\}$

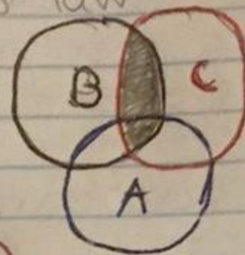
b) $(A \cap \overline{B}) \cup (A \cap \overline{C})$

$A \cap (\overline{B} \cup \overline{C})$ distributive law

$A \cap (\overline{B \cap C})$ De Morgan's law

evaluate $B \cap C$ first →

then $\overline{B \cap C}$



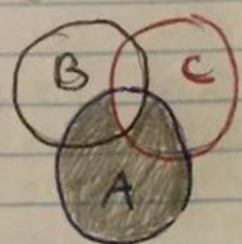
$\{x : x \in B$ and $x \in C\}$

↳

$A \cap (\overline{B \cap C})$

↳

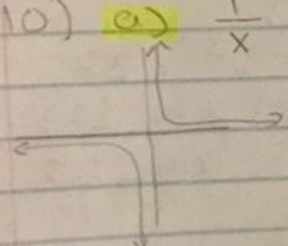
$\therefore$



$\{x : x \in A$ and $x \in (\overline{B \cap C})\}$

$\{x : x \notin (B \cap C)\}$

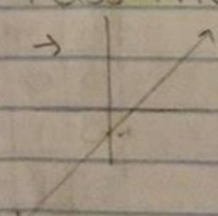
10) a) $\frac{1}{x}$ NO, because $f(x)$ is not well defined at $x=0$ b/c $\frac{1}{0}$ is undefined where VA=0 as $x \to 0^-$, $y \to -\infty$, and $x \to 0^+$, $y \to \infty$. thus $\pm\infty$ are not real numb



b) $\sqrt{x}$. NO because if x is neg, then f(x) is an imaginary number i, not a real numb

11) a) $f(x) = x-1$, is one to one b/c if $x \neq y$, then $x-1 \neq y-1$

aswell you can also determine if a f(x) is one-to-one by checking if the f(x) passes the Horozintal and vertical line test, if it doesn't pass the Horozintal line test, then the f(x) has no inverse if the f(x) doesn't pass the vertical line test, then it's not a f(x) →



passes both horozintal and vertical line test

↳ $(x-1)$ is onto, such that for every y, there is an int x such that $x-1=y$

→ $f(x) = x-1$ is also bijective! inverse: $y = x+1$

b) $F(x) = x^2+1$ is not onto such that let y be a real num. such that $y < 1$, then there is no x value such that $x^2+1 = y$. ex no x value such that $x^2+1 = -1$ or $x^2+1 = 0$

↳ not one-to-one ex $F(1) = 2$
$F(-1) = 2$ plus $F(x)$ doesn't pass the horizontal line test

c) $\left\lceil \frac{x}{2} \right\rceil$ $F(x)$ is not one to one because $F(3) = 1$, $F(2) = 1$
↳ onto because for every y value there is an x such that $F(x) = y$.

12 a) $F(x) = x^2$
$g(x) = 2^x$
$h(x) = \log_2(x)$

$F \circ g(0)$
$g(0) = 1$
$F(g(0)) = F(1)$
$F(1) = 1$
$= 1$

b) $h \circ g(5)$
$g(5) = 2^5 = 32$
$h(g(5)) = h(32)$
$h(32) = \log_2 32$
$= \frac{\log 32}{\log 2}$
$= 5$

c) $h(g(x))$
$\log_2(2^x)$

d) $h(g(F(x)))$
$\log_2(2^{x^2})$

13) a)
↳ $\bar{x}\bar{y}z$
b) $x\bar{y}z$
c) $\bar{x}\bar{y}z$
d) $F(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + xyz$

e)

x	y	z	F(x,y,z)	$\overline{F(x,y,z)}$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

F) $\overline{F(x,y,z)} = \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xy\bar{z}$

g) → next page

9)

$$\overline{\overline{(\bar x \bar y \bar z)}\,\overline{(\bar x \bar y z)}\,\overline{(x \bar y \bar z)}\,\overline{(xyz)}}$$ generalized version of De Morgan's law

$$\overline{\overline{(\bar x \bar y \bar z)}\,\overline{(\bar x \bar y z)}} + \overline{\overline{(x \bar y \bar z)}\,\overline{(xyz)}}$$ De Morgan's law

$$\overline{(\bar x \bar y \bar z)}\,\overline{(\bar x \bar y z)} + \overline{(x \bar y \bar z)}\,\overline{(xyz)}$$ double neg law

$\overline{(\bar x \bar y)(\bar z)}$ $\quad$ $\overline{(\bar x \bar y)(z)}$ $\quad$ $\overline{x(\bar y \bar z)}$ $\quad$ $\overline{x(yz)}$

$(\overline{x+y})(\bar z)$ $\quad$ $\overline{\bar x \bar y} + \bar z$ demorgan $\quad$ $\bar x + \overline{\bar y \bar z}$ demorgan $\quad$ $\bar x + \overline{yz}$ demorgan

$\overline{(x+y+z)}$ $\quad$ $\bar{\bar x} + \bar{\bar y} + z$ demorgan $\quad$ $\bar x + \bar{\bar y} + \bar{\bar z}$ demorgan $\quad$ $(\bar x + \bar y + \bar z)$ demorgan's

double neg $\quad$ double neg

$(x+y+z)$ $\quad$ $(x+y+\bar z)$ $\quad$ $(\bar x + y + z)$

$= (x+y+z)(x+y+\bar z)(\bar x+y+z)(\bar x+\bar y+\bar z)$

14)

x	y	$x \to y$	$x \oplus y$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	1	0

→ the set {addition, multiplication, complement} is functionally complete because any Boolean function can be expressed in disjunctive norm which only uses the operations of complement, addition, and multiplication. Thus the set {$\oplus$, $\to$} can be proven to be functionally complete is to first create a Boolean expression in disjunctive normal of the expression $(x \to y)$ and second create a Boolean expression in disjunctive form of the expression

$(x \oplus y)$ ↳ $x \oplus y = (x \vee y) \wedge \neg(x \wedge y)$

$(x \vee y) \wedge (\neg x \vee \neg y)$ De Morgan's law

$\therefore (x+y)(\bar x + \bar y)$, thus the operator ($\oplus$) is functionally complete b/c it can be expressed in disjunctive normal form. Next $x \to y = \neg x \vee y$ $= \bar{x} + y$, thus, the operator $(\to)$ is functionaly complete b/c it can be expressed in disjunctive norm form. ∴ set $\{\oplus, \to\}$ is functionally complete

15) circuit:
↳ next page (pg 1)

16) $(x+z)(xy+z)+(xy)$

$(x+z)(z+xy)+(xy)$ communitive law.

$(x+z)(z+x)(z+y)+(xy)$ Distributive law

$(z+x)(z+x)(z+y)+(xy)$ communitive law

$(z+x)(z+y)+(xy)$ idempotent law

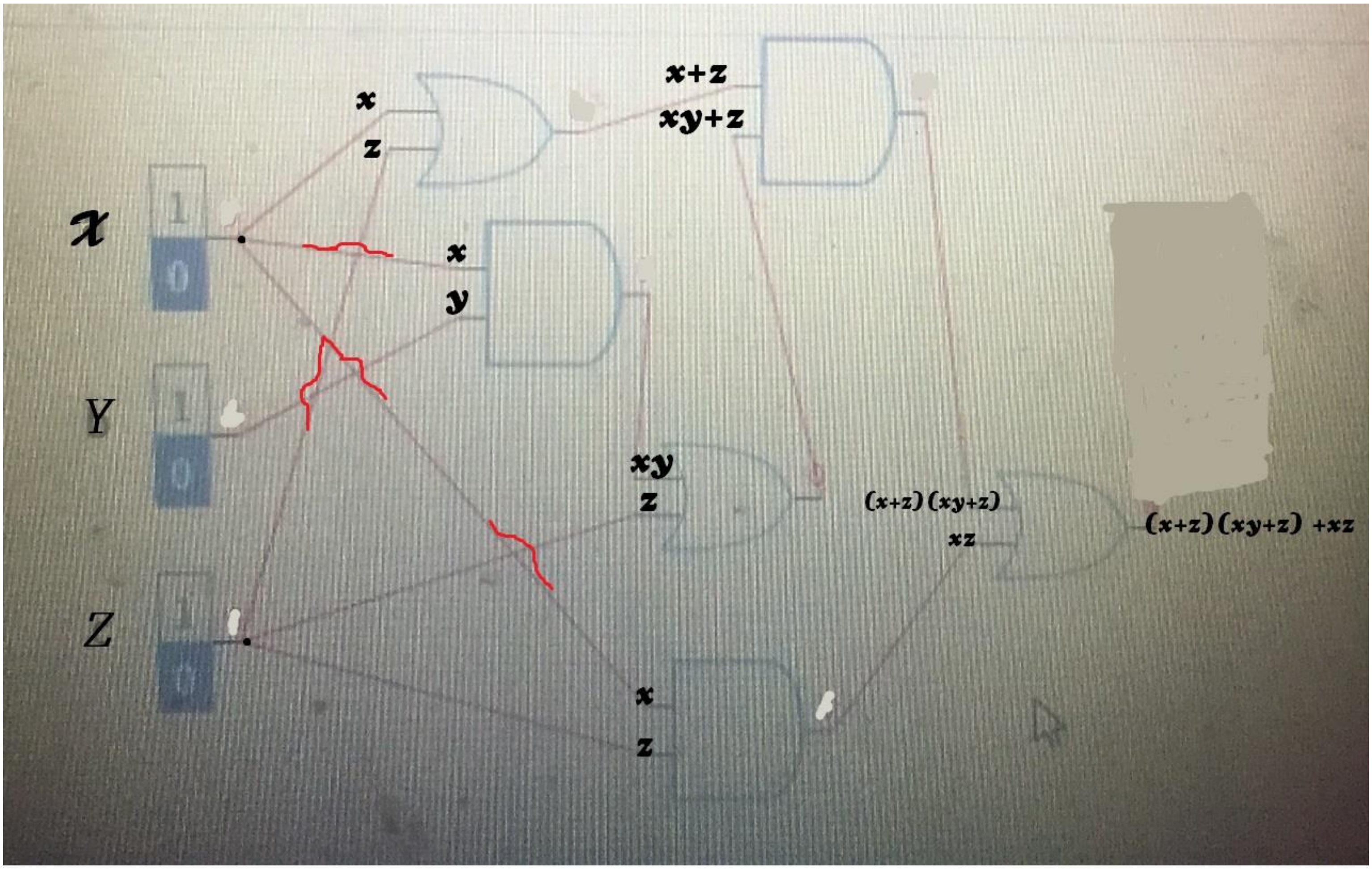
$(z+xy)+(xy)$ distributive law

$z+(xy+xy)$ Associative law

$z+(xy)$ idompotent law

circuit:
↳ next page (pg 2)

Pg. 1



Pg. 2