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Assignment 1 *1910

#1. (10 marks) Which of the following are propositions? What are the truth values of those that are propositions?

(a) (2 + 2 = 4) is a proposition because the truth value can be either true or false and is typically a declarative sentences thus, in this case the proposition is true because 2 +2 does equal 4.

(b) (Thursday is a day of the week) is a proposition because the truth value can be either true or false and in this case the proposition is true because in fact Thursday is a day of the week.

(c) [$(x)^{1/2}$ + 5= 10] is not a proposition because the value of x is not known therefore since the value of x is not known , x would be considered as a 'free variable' which means that the statement's truth value cannot be determined.

(d) (If pigs fly, then I can fly) is a proposition because the statement is either true or false even though the statement seems absurd , however in this case the statement is false because pigs can't fly (x) therefore it will be impossible for you (y) to fly .

(e) (All students have finished all the textbook activities) is a proposition because the statement has a true and false value , even though the value is a matter of opinion. In this case, the proposition is true because perhaps all students have finished all the textbooks activities.

(f) (Would you like some chocolate?) is not a proposition because the statement is a question .

(g) (House prices will fall this year) is a proposition because the statement has a truth value and can be either true or false, although nobody knows which. In this case, the proposition is true because house prices usually rise higher every year.

#2. (10 marks) Let p be the proposition "She was a tennis player", let q be the proposition "She practiced every day" and let r be "She won many tennis tournaments". Translate the following into propositional expressions:

(a) $p \wedge q$

(b) $r \leftrightarrow q$

(c) $r \wedge \neg q$

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(d) $r \rightarrow q$

(e) $(p \land \neg q) \land r$

#3. (10 marks) How many rows appear in the truth table for each of the following compound propositions?

(a) 4 ($2^2 = 4$)

(b) 16 ($2^4=16$)

(c) 8 ($2^3=8$)

#4. (10 marks) Construct truth tables for each of the following compound propositions:

(a) $p \vee (p \rightarrow q)$

p	q	p→q	p v (p→ q)
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

(b) $(p \land q) \leftrightarrow (\neg q \vee r)$

p	q	r	¬q	(p ∧ q)	(¬q v r)	(p ∧ q) ←→ (¬q v r)
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	T	F	F	F	F	T
F	F	T	T	F	T	F

#5. (10 marks) Using the laws of propositional logic, show that the following are logically equivalent:

(a) $(p \rightarrow q) \vee (p \rightarrow r) \equiv (p \rightarrow (q \vee r))$

Martina Tawedrous
0977473

(¬p v q) v (¬p v r) Conditional identities
(¬p v (q v ¬p) v r Associative law
(¬p v (¬p v q) v r Communitive law
(¬p v ¬p) v (q v r) Associative law
¬p v (q v r) Idempotent law
p → (q v r) Conditional identities

(b) (p ∧ q) → (p → q) Ξ T
¬(p ∧ q) v (¬p v q) Conditional identities
(¬p v ¬q) v (¬p v q) De Morgan's Law
¬p v (¬p v ¬q) v q Associative law
(¬p v ¬p) v (¬q v q) Associative law
¬p v T Idempotent law
T Domination law

#6. (10 marks) The domain for the variable x is the set of all positive integers. The predicates P and Q are defined as follows:
P(x): x is odd
Q(x): x is divisible by 4
Indicate whether each quantified statement is true or false and explain:

(a) ∃ × (P(x) ∧ Q(x)) is false because when one were to translate this statement into English, at least there is an x value that is odd and divisible by 4, therefore there isn't an x value that is odd and divisible by 4 (ex, 7, 3, 13, 21 etc.).

(b) ∃ × (¬P(×) ∧ ¬Q(x)) is true because if one were to translate this statement to English, at least there is an x value that is not odd (even) and not divisible by 4, therefore there exists an x value that would make this statement true (ex, 6).

(c) ∀ × (P(×) → ¬Q(x)) is true because this statement means that all for all x values, if x is an odd number then x is not divisible by 4, (ex, 7, 41, 9 etc.).

#7. (10 marks) Which of the following are propositions? Explain why.

Martina Tawedrous
0977473

(a) ∀ × P(×) is a proposition because there are no free variables, and only the universal quantifier is bounded by x, which therefore means that the statement's truth value can be determined.

(b) ∀ × P(×) v Q(×) is not a proposition because a free variable exits and can take on any value in the domain, the universal quantifier is bounded with x and P(×), however Q(**x**) is still remaining.

(c) ∃ × P(x) v Q(x) is not a proposition because a free variable exits and can take on any value in the domain, the existential quantifier is bounded with x and P(×), however Q(**x**) is still remaining.

(d) ∀ × ∃ y P(×, y) is a proposition because there are no free variables due to the fact that the universal quantifier and the existential quantifier are both bounded by × and y, which therefore means that the statement's truth value can be determined.

#8. (10 marks) Express the following in predicate logic using predicates, quantifiers and logical operators. State the domain of values and the predicates.

(a) Everybody plays Super Mario.
x is the domain of all people
M(x): x plays Super Mario

∀ × M(×)

(b) Someone finished all the levels of the game.
x is the domain of all players
y is the domain of levels
G(x, y) : x finished y Levels

∃ × ∀ y G(×, y)

(c) At least two players passed level 5.

Martina Tawedrous
0977473

x is the domain of all players passed
y is the domain of players passed
P(x) : player x passed level 5

$\exists x \exists y (P(x) \wedge P(y) \wedge (x \neq y))$

(d) There is exactly one level that all players cannot pass.
x is the domain of levels
y is the domain of all players
L(x, y): x level that y player pass

$\exists x \forall y (\neg L(x, y))$

#9. (10 marks) Use De Morgan's law for quantified statements and the laws of propositional logic to show the following equivalences

(a) $\neg \forall x (P(x) \wedge \neg Q(x)) \equiv \exists x (\neg P(x) \vee Q(x))$

$\exists x \neg(P(x) \wedge \neg Q(x))$	De Morgan's Law of Quantified Statements
$\exists x (\neg P(x) \vee \neg\neg Q(x))$	De Morgan's law
$\exists x (\neg P(x) \vee Q(x))$	Double Negation law

(b) $\neg \forall x (\neg P(x) \rightarrow Q(x)) \equiv \exists x (\neg P(x) \wedge \neg Q(x))$

$\exists x \neg(\neg P(x) \rightarrow Q(x))$	De Morgan's Law of Quantified Statements
$\exists x \neg(\neg\neg P(x) \vee Q(x))$	Conditional identities
$\exists x \neg(P(x) \vee Q(x))$	Double Negation law
$\exists x (\neg P(x) \wedge \neg Q(x))$	De Morgan's law

(c) $\neg \exists x(\neg P(x) \vee (Q(x) \wedge \neg R(x)) \equiv \forall x (P(x) \wedge (\neg Q(x) \vee R(x)))$

$\forall x \neg(\neg P(x) \vee (Q(x) \wedge \neg R(x)))$	De Morgan's Law of Quantified Statements
$\forall x (\neg\neg P(x) \wedge \neg(Q(x) \wedge \neg R(x)))$	De Morgan's law
$\forall x (P(x) \wedge (\neg Q(x) \vee \neg\neg R(x)))$	De Morgan's law and Double Negation law
$\forall x (P(x) \wedge (\neg Q(x) \vee R(x)))$	Double Negation law

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#10. (10 marks) The domain for the variables x and y are the set of musicians in an orchestra. The predicates S, B, and P are defined as:
S(x): x plays a string instrument
B(x): x plays a brass instrument
P(x, y): x practices more than y
Give a quantified expression that is equivalent to the following English statements:

(a) Someone in the orchestra plays a string instrument and a brass instrument.
$\exists x \; (S(x) \wedge B(x))$

(b) There is a brass player who practices more than all the string players.
$\exists x \; [B(x) \wedge \forall y \; (S(y) \rightarrow P(x, y))]$

(c) All the string players practice more than all the brass players.
$\forall x \; \forall y \; [S(x) \wedge B(y) \rightarrow P(x, y)]$

(d) Exactly one person practices more than Sam.
$\exists x \; \forall y \; [P(x, sam) \wedge (P(y), sam) \rightarrow (y = x)]$

(e) Sam practices more than anyone else in the orchestra.
$\forall x \; [P(sam, x) \wedge \neg P(sam, sam)]$