Faster Hash-based Multi-valued Validated Asynchronous Byzantine Agreement

(Regular Paper)

***Abstract*—Multi-valued Validated Asynchronous Byzantine Agreement (MVBA) is vital for asynchronous distributed protocols like asynchronous BFT consensus and distributed key generation, making performance improvements a long-standing goal. Existing communication-optimal MVBA protocols rely on computationally intensive public-key cryptographic tools, such as non-interactive threshold signatures, which are also vulnerable to quantum attacks. While hash-based MVBA protocols have been proposed to address these challenges, their higher communication overhead has raised concerns about practical performance. We present a novel MVBA protocol with adaptive security, relying exclusively on hash functions to achieve post-quantum security. Our protocol delivers near-optimal communication, constant round complexity, and significantly reduced latency compared to existing schemes. For example, with $n = 201$ and input size 1.75 MB, it reduces latency by 81% over previous hash-based approaches. Despite tolerating 20% Byzantine faults (compared to the typical 33%), it can integrate into existing frameworks as a fast path to complement full-resilience solutions when needed.**

I. Introduction

Byzantine fault-tolerant (BFT) consensus is the foundation of distributed computing, providing a means for individuals to establish a consistent view in a distributed environment, and facilitating the execution of higher-level functionalities. With the surge of distributed applications over the global Internet in the last decade, asynchronous BFT protocols are gaining resurged attention and substantial progress in recent years, due to their resilience to network churns and ease of implementation.

Multi-Valued Validated Byzantine Agreement (MVBA), introduced in the seminal work of Cachin et al. [1], stands out as one of the most critical tools for asynchronous distributed protocols, particularly the recent *practical* ones. In an MVBA protocol, each node provides a multi-bit value as input, collectively deciding on one of the input values to output, which has to satisfy a pre-defined condition. MVBA remained as theoretical study, [1]–[3], until Dumbo [4] re-established its critical importance to construct practical asynchronous BFT protocols. Since then, it has started to play a pivotal role in many *practical* distributed protocols, including asynchronous distributed key generation [5]–[7], dynamic-committee proactive secret sharing [8], [9], optimistic asynchronous consensus protocols [10], [11], and network agnostic distributed protocols [12]. Moreover, MVBA plays a crucial role in achieving an efficient Asynchronous Common Subset (ACS), vital for Asynchronous Multi-party Computation to agree on sets of inputs [13]–[21]. Notably, as observed in [4], [22], [23], MVBA usually constitutes the bottleneck of its applications, and high communication complexity is the main reason when the system scales up to a moderate size, thus reducing the communication of MVBA is greatly desired.

The original MVBA of Cachin et al. [1] is with $\mathcal{O}(\ell n^2 + \lambda n^2 + n^3)$ bits of communication. Here, ℓ is the bit length of input, n is the number of participants, and λ is the security parameter that captures the signature size etc. Its large communication complexity becomes the major bottleneck of its practical use. After 20 years, the communication complexity was reduced by Abraham et al. [2] to be $\mathcal{O}(\ell n^2 + \lambda n^2)$. However, when the input size is moderate, such as $\mathcal{O}(n)$ (e.g., a vector of input bits), the ℓn^2 term becomes n^3 and dominates again. For this reason, Lu et al. [3] gave an extension framework that finally led to an optimal MVBA in Dumbo-MVBA* [3], with $\mathcal{O}(\ell n + \lambda n^2)$ bits of communication, that matches the lower bound [2], [24]. Subsequently, Guo et al. [22] gave further concrete optimizations on rounds, and constructed Speeding MVBA, which eventually led to Dumbo-NG [25], a fast asynchronous BFT with very high throughput. While those recent communication efficient MVBA protocols made use of some "heavy-weight" cryptography, particularly non-interactive threshold signatures like BLS [26]–[28]. Relying on those tools raises *both* security and performance (particularly computation) concerns. Specifically, the underlying algebraic assumptions are vulnerable to quantum attackers, the pairing operations are considerably expensive (e.g., 10^5 times slower than computing hash), and they may require a private setup for decentralized application scenarios like blockchains. Despite the trusted setup possibly being eliminated by using distributed key generation [29] or recently introduced transparent threshold signatures [30], [31], more communication and computation will be incurred, further hindering the performance.

Considering practical concerns associated with using threshold signatures and motivated by a desire to minimize cryptographic assumptions for enhanced security (e.g., plausible post-quantum security), there is renewed interest in exploring MVBA in the information-theoretical (IT) setting [29], [32], [33], or in solely using collision-resistant hash functions for better performance than their IT-secure analogs. In the literature, the IT setting is sometimes referred to as the signature-free setting [34], [35], which subsumes the error-free setting [36], [37]. This is in contrast with the above "classical" MVBA protocols. However, while enjoying the obvious benefits of using hash functions rather than heavy cryptographic tools like threshold signatures, the state-of-the-art design, FIN-MVBA by Duan et al. [32], suffers from high communication complexity $\mathcal{O}(\ell n^2 + \lambda n^3)$, which is even

TABLE I: Comparison of the Multi-valued Validated BA protocols [1]

Protocol	Resilience	Adaptive? [2]	Communication	Message	#coin	Cryptographic Tools	Trivial Hash-based [3]
CKPS01-MVBA [1]	$f < n/3$	✓	$\mathcal{O}(\ell n^2 + \lambda n^2 + n^3)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	threshold signature	$\mathcal{O}(\ell n^2 + \lambda n^3)$
VABA [2]	$f < n/3$	✓	$\mathcal{O}(\ell n^2 + \lambda n^2)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	threshold signature	$\mathcal{O}(\ell n^2 + \lambda n^3)$
Dumbo-MVBA [3]	$f < n/3$	✓	$\mathcal{O}(\ell n + \lambda n^2)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	threshold signature	$\mathcal{O}(\ell n + \lambda n^3)$
sMVBA [22]	$f < n/3$	✓	$\mathcal{O}(\ell n^2 + \lambda n^2)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	threshold signature	$\mathcal{O}(\ell n^2 + \lambda n^3)$
sMVBA⋆-BLS [22] [5]	$f < n/3$	✓	$\mathcal{O}(\ell n + \lambda n^2)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	threshold signature	$\mathcal{O}(\ell n + \lambda n^3)$
sMVBA⋆-ECDSA [22] [6]	$f < n/3$	✓	$\mathcal{O}(\ell n + \lambda n^3)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	ECDSA	$\mathcal{O}(\ell n + \lambda n^3)$
FIN-MVBA [32]	$f < n/3$	✓	$\mathcal{O}(\ell n^2 + \lambda n^3)$	$\mathcal{O}(n^3)$	$\mathcal{O}(1)$	hash	-
ELV-HMVBA (sketched in Sec.I-B)	$f < n/3$	✗	$\mathcal{O}(\ell \kappa n + \lambda \kappa n^2 \log n)$ [4]	$\mathcal{O}(n^2 \kappa)$	$\mathcal{O}(\kappa)$	hash	-
Our HMVBA (Sect. V)	$f < n/5$	✓	$\mathcal{O}(\ell n + \lambda n^2 \log n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(1)$	hash	-

[1] Following the standard practice in asynchronous consensus literature, we assume a common coin and consistently omit its cost. Throughout this paper, We use f to denote the maximal number of nodes that an adversary can corrupt.
[2] The "classical" MVBA schemes [1]–[3], [22], [32] were not paired with detailed security proofs for adaptive security, which do hold if their all components are adaptively secure, particularly, as BLS [27] has been proved to be adaptively secure in a recent work [28].
[3] The trivial hash-based version means the hash-based MVBA constructions obtained by naively replacing the threshold signature used in the corresponding MVBA scheme by a concatenation of $n - f$ hash-based signatures. The asymptotic communication complexity may only get slightly worse, but the actual cubic term gets a larger coefficient too for much worse concrete complexity.
[4] κ is the statistical security parameter, which is usually chosen as a few tens.
[5] sMVBA⋆-BLS is the MVBA scheme obtained by plugging sMVBA into Dumbo-MVBA⋆'s framework to reduce $\mathcal{O}(\ell n^2)$ term.
[6] sMVBA⋆-ECDSA replaces the threshold signature in sMVBA⋆-BLS with the catenation of $n - f$ ECDSA signatures.

asymptotically worse than the early construction from Cachin et al.'s [1]. It follows that current hash-based MVBA achieves post-quantum security, but at the cost of larger communication (thus inferior performance), which did not realize its full power. This leads to the natural question:

Can we develop an MVBA *that is free of heavy cryptographic operations (using collision resistant hash functions only), while at the same time, demonstrates performance benefits?*

A. *Our Results*

Hash-based MVBA **with (Nearly) Optimal Communication and Adaptive Security.** In this paper, we answer the question affirmatively by repeating the successful developments in "classical" MVBA protocols, i.e., match the complexity, and solely making blackbox use of conventional hash functions. Specifically, we present the first hash based MVBA protocol HMVBA with adaptive security, $\mathcal{O}(1)$ rounds, and $\mathcal{O}(\ell n + \lambda n^2 \log n)$ communication. This is achieved via a new "Dispersal-Elect-Agree" paradigm, which makes use of an overlooked primitive of asynchronous multi-valued byzantine agreement with weak validity (see Sect.I-B for details). We compare our results with existing ones in Table I. A caveat is that our scheme tolerates up to 20% Byzantine nodes, rather than being optimally resilient against 33% corrupted nodes. Nevertheless, it is still practically useful and can be extended to handle more corruptions by leveraging common fallback mechanisms (see Sect.I-C for more discussions).

We also observe a simple hash-based MVBA construction (dubbed ELV-HMVBA in Table I) that enjoys optimal resilience and quadratic communication, under static corruption. While the conventional committee-based approach for reducing communication complexity usually has to sacrifice resilience, this result can be interesting for some applications.

Immediate applications. We can just plug-in our new HMVBA protocol to existing frameworks such as Dumbo-NG [25] and Jumbo [38] to get a better asynchronous BFT protocol, and many more. Notably, our HMVBA also directly implies a hash-based ACS with quadratic communication complexity, specifically $\mathcal{O}(\ell n^2 + \lambda n^2 \log n)$, by instantiating the framework of Cachin et al. [1] with hash-based signatures. In contrast, all existing hash-based ACS schemes [32], [39] incur cubic communication complexity.

TABLE II: Improvements of latency with mini-payloads

Scale (N)	Latency (milisec)			Improvement	
	FIN-MVBA	sMVBA⋆-BLS	Our HMVBA	FIN-MVBA	sMVBA⋆-BLS
101	4200	8414	2764	↓34%	↓67%
201	16172	16800	1941	↓88%	↓88%

Implementation and Evaluation. We implemented our HMVBA in Python 3 and deployed it on AWS EC2 `t2.medium` instances evenly distributed across 13 AWS regions[1]. For a fair comparison, we further developed Python implementations of FIN-MVBA [32] and the actual state of the art "classical" MVBA protocol sMVBA⋆, which is obtained by trivially instantiating the Dumbo-MVBA⋆ framework in [3] with sMVBA in [22]. We tested the three MVBA protocols with various input sizes L and network sizes N. The results demonstrate that (1) The current hash-based MVBA FIN-MVBA is indeed sacrificing performance, as it is consistently worse than sMVBA⋆. (2) our new HMVBA consistently outperforms the other two MVBAs when the input size is fixed and the scale increases starting from moderate N. As Table II shows, our HMVBA brings significant improvement. Moreover, our MVBA establishes a clearly better throughput-latency trade-off at a reasonable scale. See Sect.VII for details.

[1]Our code: https://anonymous.4open.science/r/hash-mvba-7C57/.

B. Challenges and Our Techniques

The goal of MVBA is to decide on a "valid" input, a natural idea is to have all nodes agree on an input from a random node such that, with a constant probability, the value is valid. We found that existing MVBA schemes follow this common approach, which we call "Lock-Elect-Vote" (LEV).

The existing LEV approach. FIN-MVBA [32] employs a reliable broadcast protocol RBC [40] (which ensures agreement among all honest nodes even if the sender is malicious; see section IV) to "lock" input messages. Then, it invokes a leader election protocol, which is usually realized by a common coin, to decide whose input is going to be the prospective output. Finally, an ABA (asynchronous binary agreement), actually a variant called reproposable ABA [39], is applied to "vote" on the status of the elected leader's RBC instance. When ABA outputs 1, it means at least one honest node inputs 1, which implies that the honest node has received the corresponding value from the elected RBC instance. RBC's property will then ensure all honest nodes will eventually receive that same value. FIN-MVBA gives a hash-based instantiation, as components have efficient hash-based instantiations [35], [40]. However, since using RBC to disseminate an ℓ-bit value to n nodes incurs $\mathcal{O}(\ell n+\lambda n^2)$ bits of communication, invoking n parallel RBC instances leads to the communication complexity of $\mathcal{O}(\ell n^2+\lambda n^3)$. With threshold signatures, we can employ a "cheaper" broadcast primitive, provable broadcast PB [2], [4] (with communication cost of $\mathcal{O}(\ell n+\lambda n)$) to replace RBC. In doing so, we get rid of the cubic term, since n parallel PB instances only cost us $\mathcal{O}(\ell n^2+\lambda n^2)$. Unfortunately, threshold signatures typically require algebraic structures that are not available in conventional hash functions or other lightweight tools, let alone the reliance on a trusted setup. [2]

Why hash-based quadratic MVBA **is hard under an adaptive adversary?** In the LEV, for locking n messages we use n instances of broadcast with a strong consistency guarantee. As mentioned, using n instances of RBC or PB results in different challenges: the former requires $\mathcal{O}(n^3)$ communication cost, while the latter demands efficient and suitable algebraic structures. An alternative method is to use a concatenation of $n-f$ hash-based signature as the proof, which however blows up the communication cost to $\mathcal{O}(n^3)$ again. MVBA with quadratic communication without using expensive cryptographic tools appears to be beyond the LEV paradigm. We find it easy to construct an MVBA protocol with quadratic communication complexity and optimal resilience, if we only focus on *static adversaries*. At a high-level, we can select a few nodes in the beginning, such that at least one of them is honest with an overwhelming probability; Then, we let all selected nodes broadcast their inputs (via RBC), and let all nodes agree on the broadcasted values (via ABA) and finally decide on a valid input. We call this construction "Elect-Lock-Vote"(ELV). While the adaptive security failure of LEV is largely due to the failure in the "locking" step. Specifically, as we are using very few ($< f$) parallel broadcast to lock messages, an adaptive adversary can target all the senders to stop the protocol. We are facing a dilemma: on one hand, avoiding cubic communication (while not using threshold signatures) prevents us from locking $\mathcal{O}(n)$ inputs. On the other hand, if there are only $o(n)$ inputs to be locked, an adaptive adversary may simply corrupt all the selected senders and make the protocol stuck.

Our approach towards adaptive security: "Disseminate-Elect-Agree". To break out of this dilemma, we start with a "dissemination" step (which does not use RBC thus avoiding the high communication). Subsequently, we use a coin to conduct the "election", if the selected value is disseminated by a malicious node, then there is no consistency guarantee. We need a more powerful "Vote" technique to pair with the efficient "dissemination" to compensate for the absence of RBC for locking. To illustrate, let us assume the following "ideal" dissemination to design the remaining techniques, then we discuss how to realize the dissemination part efficiently.

- **Ideal dissemination.** All nodes are engaged in this phase to disseminate their inputs to the entire network. At the end of this phase, every honest node should have the inputs provided by all other honest nodes.

Now after the ideal dissemination and election, if the elected node (as sender in dissemination) is honest, every honest node is holding a same value; otherwise, they may have different values (or some may not have one). To conquer the challenge for agreement on the final output, a binary agreement (ABA) to vote as in previous constructions seems insufficient. Instead, we observe that a classical yet overlooked BFT primitive, Multi-Valued Byzantine Agreement with Weak Validity (MBA) [43], [44], is closer to our need as the more powerful "Vote" step. *Weak validity* means if all honest nodes have the same input, then they will output that value; no guarantee otherwise. Now, if the selected input is from an honest node, such that every other honest node already has it, then they must decide on this value; if the selected one is malicious, since now MBA has no guarantee, we should at least let the honest nodes be aware of the failure such that they can restart from the coin step. So before invoking an MBA protocol to vote, each honest node will multi-cast its current "notification" (either a received fragment, or nothing, just using $\perp$). More care is needed for the details (see Sec.V). We rephrase this new paradigm as "Disseminate-Elect-Agree".

Realizing dissemination with quadratic communication. We now turn to the construction of dissemination. Note that the ideal dissemination can be trivially realized in a synchronous network; Every node simply multicasts its input such that every other honest node can have it at the end of the round. The communication cost of such dissemination will be merely $\mathcal{O}(\ell n^2)$. However, subtleties arise due to asynchrony: some honest nodes may have not finished the multicast step when the election starts. We will have to relax the requirement of

[2] Remark that from the feasibility point of view, as inspired by recent works [30], [31], one may construct such a threshold signature by making non-blackbox use of a hash-based signature [41] and a hash-based succinct argument system [42], which, however, is much heavier than BLS [28].

ideal dissemination and only ask for a constant fraction of honest inputs to be disseminated to all honest nodes.

Even for the relaxed dissemination, there are subtleties around. Let us consider a straightforward approach as a baseline, where each node performs the following tasks: (1) multicast its input; (2) when receiving an input value from node $\mathcal{P}_i$ for the first time, respond OK to $\mathcal{P}_i$; (3) whenever receiving OK from $n-f$ distinct nodes, multicast DONE; (4) whenever receiving DONE from $n-f$ distinct nodes, move to the next phase. In doing so, we can guarantee there are at least $n-2f$ honest nodes (we call them good senders hereafter) who managed to deliver their input messages to at least $n-2f$ honest nodes. However, even when a good sender is elected, there can still be f honest nodes (we call them unfortunate receivers) that have not received the corresponding message. What is worse, an adaptive adversary may corrupt the elected good sender, retract the message that has not been delivered, and send different messages to these unfortunate receivers.

We rectify this situation by letting all nodes exchange information about the value received from the elected node, such that honest nodes shall use the value endorsed by the majority of other nodes as input for MBA. When $n \geq 5f+1$, there could be $3f+1$ honest nodes having received the message from an elected good sender. In the step for exchanging information, their voice will form a majority in every honest node's view.

Pushing to optimal communication using erasure code. All the above discussions assume that every node multicasts its entire input, which results in a communication cost of $\mathcal{O}(\ell n^2)$. However, as in MVBA each node outputs only one value, so it is unnecessary for every node to keep track of all input values from other nodes. We therefore adopt the dispersal-then-recast methodology introduced in [3]. In the MVBA design of [3], instead of having each node directly send its input to everyone, they consider that each node first disperses fragments of its input to all nodes. These fragments have a smaller size compared to the full input value. What's more, all other honest nodes can reconstruct the input of the elected node using a sufficient number of fragments. We bend the methodology into our design of dissemination, using a hash-based Merkle tree to help nodes identify the correct fragments, resulting in $\mathcal{O}(\ell n + \lambda n^2 \log n)$ bit complexity.

Applying non-intrusion secure MBA and further optimizations. Our HMVBA makes novel use of MBA to achieve agreement on messages. However, directly inputting entire messages into MBA can still be communication-intensive, potentially undermining prior efforts. Nevertheless, our dissemination phase already ensures a certain level of data availability: if an honest node uses a message as the input of MBA, all other honest nodes also can obtain this message during recast. Hence, rather than using MBA to agree on the entire message, we leverage it to agree on a short hash **digest**. In conventional MBA, the output of MBA might be a digest provided by the adversary, resulting in the unavailability of the original value for some honest nodes and causing an agreement issue. Fortunately, the *non-intrusion security* property in MBA [45] addresses this concern. This property ensures that if the output of MBA is not $\perp$, it must be the input of an honest node, ensuring data availability for the agreed digest.

We instantiated the MBA (actually IT-secure), featuring $\mathcal{O}(1)$ rounds, $\mathcal{O}(n^2)$ messages, and $\mathcal{O}(\ell n^2)$-bit communication cost. However, for practical efficiency, the MBA in [45] requires 6 additional rounds of multicast beyond its ABA components, which we aim to optimize. By leveraging the fact that our MVBA operates with $n \geq 5f+1$, we design a more efficient IT-secure MBA with only 2 extra rounds of multicast alongside ABA in the same setting, while maintaining the same asymptotic performance as [45].

C. Discussion

Using our HMVBA as an optimistic Fast Path. Our HMVBA could directly improve the efficiency for practical scenarios where $1/5$ resilience is a reasonable assumption. More broadly, for asynchronous consensus, (that is continuously run as a state machine replication), our HMVBA can be used as an *optimistic* fast path. There exist extensive investigations of optimistic consensus protocols in both asynchronous and synchronous settings [10], [46]–[48], that a fast optimistic protocol is usually run; while if condition gets worse, a slower but "securer" protocol can still ensure security. The rationale for such a hybrid paradigm is that, condition may usually be "optimistically" good, e.g., leader (if there is one) is honest, all parties are honest, or a larger portion of parties are honest.

One key factor of the fast path for this hybrid paradigm is to ensure the safety property, i.e., agreement, even in the bad case. Once we have this guarantee, we can just run a faster optimistic protocol e.g., our HMVBA. If optimistic condition holds (e.g., up to $1/5$ corruption), it already ensures all the properties of the consensus protocols. Even if worst case condition happens, i.e., under $1/3$ corruption, only liveness will be influenced, which is easy to detect. For example, we may follow the ideas of [47], [48] that two chains can be concurrently run: the first running the fast path, i.e., our HMVBA; while the second running a slow-but-with-$1/3$-resilience MVBA protocol (such as [49]). Usually the fast path outputs much faster; but if the parties notice that the slow path is already catching up or even outputting faster, they can all simply output according to the slow path (and jointly switch back to fast path if it outputs faster again). We leave details of such optimistic protocols for future work.

We make two remarks here: our HMVBA can be efficiently adapted to ensure *agreement* even if $1/3$ of nodes are corrupted. Specifically, we can replace our specially designed MBA protocol with a standard optimal-resilient MBA protocol [45], allowing the network to agree on the same accumulator value, which ensures that output values are identical across all honest nodes. Also, existing optimistic paths have zero fault tolerance (e.g., in [47], [48]), meaning the final latency might be determined by that of the slow path if there are some malicious nodes. In contrast, our construction offers a more *robust* fast path, making it more likely for the system to benefit from the smaller latency provided by our HMVBA protocol.

II. Related Work

(Multi-valued) Asynchronous Byzantine Agreement. The most basic form of asynchronous Byzantine consensus is asynchronous binary agreement (ABA), where each node has a binary value of 0 or 1 as input and will agree on a binary value. The validity of ABA is defined in an *unanimous* manner, *i.e.*, if all honest nodes input the same binary value, they will agree on this value. As there are only two candidate input values, the validity of ABA implies the so-called *strong validity*, *i.e.*, the output is always the input of some honest node, which is a very useful property for applications. Mostefaoui et al. 's seminal work [35] presented an ABA protocol with $\mathcal{O}(1)$ rounds and $\mathcal{O}(n^2)$ communication complexity, not relying on any cryptographic tools beyond the coin. There are follow-up works [50] for improving the concrete performance of [35].

Multi-valued byzantine agreement (MBA) is a natural extension to ABA for handling multi-bit inputs. There is a straightforward reduction from MBA to ABA, by applying multiple ABA instances to agree on each bit. However, for an ℓ-bit input, the expected running time and the message complexity of ℓ parallel ABA instances will be blown up to $\mathcal{O}(\log \ell)$ and $\mathcal{O}(\ell n^2)$, respectively. Mostefaoui and Raynal [45] presented an optimized MBA with $\mathcal{O}(1)$ rounds, $\mathcal{O}(n^2)$ messages, and $\mathcal{O}(\ell n^2)$ communication complexity. For large-size inputs, say $\ell \gg \lambda$, Nayak et al. [51] presented a general framework that reduced the ℓn^2 term to ℓn in communication complexity using pairing-based cryptography tools. Alternatively, Li and Chen [44] presented an MBA with communication complexity of $\mathcal{O}(\ell n + n^2 \log n)$, achieving perfect security without using any cryptographic tools. Note that the MBA discussed above guarantees that when all honest nodes have the same input value, they will agree on that value. But for other cases, there is no guarantee on what value they will agree on; the output could be a default value $\perp$. Some works, including [45] considered a slightly stronger validity called *non-intrusion validity*, which means if the output $v \neq \perp$, then v must be an input of an honest node. The non-intrusion property has been leveraged and explored in consequent works [52], [53]. Compared with [45], our MBA in Section VI improves the concrete communication and rounds cost while assuming $n \geq 5f + 1$ which aligns with our MVBA.

Multi-valued Validated Asynchronous Byzantine Agreement. The weak validity of MBA is insufficient for many natural cases. A dream version of validity would be that the nodes always agree on the input of an honest node, which, often called *strong validity*, is known to be out of reach for large input sizes [54]. To address this issue, Cachin et al. [1] introduced *external validity*, named MVBA, which guarantees that the nodes can always agree on a "valid" value satisfying a predefined predicate function, as long as all honest nodes input valid values. Note that MVBA and MBA (with weak validity) are generally incomparable, as MVBA's output may be controlled by the adversary in any input case. However, this issue can be mitigated by carefully designing a predicate that the output should satisfy.

Since the first MVBA was introduced in [1], many subsequent works [2], [3], [22] have been proposed to either improve communication complexity or reduce the number of concrete rounds. However, all these MVBA schemes rely on threshold signatures, whose current constructions are based on algebraic assumptions that are vulnerable to quantum attackers. Additionally, threshold signatures require a trusted setup, which can be problematic in various settings. Due to the drawbacks of using heavy cryptographic tools like threshold signatures, several recent works [29], [32], [55] shifted their focus on studying MVBA in the information-theoretical setting (also known as signature free setting) or the hash-based setting, where the hash function is the only cryptographic tool and used in a blackbox manner. These works actually give a framework that could be instantiated in both settings, while their hash-based instantiations enjoy better performance. Particularly, Das et al. [29], [56] presented MVBA has $\mathcal{O}(\log n)$ rounds, $\mathcal{O}(n^3)$ messages, and the communication complexity of $\mathcal{O}(\ell n^2 + \lambda n^3)$. Duan et al. [32] improved Das et al.'s result to $\mathcal{O}(1)$ rounds. Nonetheless, there exists a significant asymptotic efficiency gap between current hash-based MVBA constructions and classical constructions [1]–[3].

Subsequent Works. There are two follow-up works on hash-based MVBA that aim to improve resilience. In particular, Feng et al. [49] presented a hash-based MVBA protocol with optimal resilience and $O(\kappa n \ell + \kappa \lambda n^2 \log n)$ communication complexity, where κ is a statistical security parameter. Komatovic et al. [57] introduced a construction that offers sub-optimal yet improved resilience, specifically $1/3 - \epsilon$, with a communication complexity of $O(\gamma(\epsilon) n \ell + \gamma(\epsilon) \lambda n^2 \log n)$, where $\gamma(\epsilon) = O\left(\frac{1}{\epsilon^2}\right)$. Both works followed our framework of "Disseminate-Elect-Agree," using the same dissemination phase as ours but designing a stronger agreement phase to achieve better resilience. We stress that [57] and [49] are theoretical in nature, as their communication complexities are higher than ours, and they did not implement their protocols. In contrast, our protocol has been tested and shown to be significantly more efficient than state-of-the-art MVBA protocols.

III. Model and Goal

A. System model

The system involves a set of n known nodes labeled as $\{\mathcal{P}_1, \dots, \mathcal{P}_n\}$ which are connected through pairwise authenticated channels. Throughout this paper, we primarily consider an adaptive and computationally bounded adversary ($\mathcal{A}$), capable of corrupting up to $f < n/5$ nodes at any point during the protocol execution. Nodes not corrupted by the adaptive adversary at a certain stage of the protocol are termed "so-far-uncorrupted." However, once the adaptive adversary corrupts a node $\mathcal{P}_i$, that node $\mathcal{P}_i$ becomes fully controlled by the adaptive adversary and can act maliciously. A node is considered honest if it has never been corrupted by the adaptive adversary. Specifically, if a "so-far-uncorrupted" node $\mathcal{P}_i$ sent a message to $\mathcal{P}_j$ and then got corrupted by $\mathcal{A}$ before it was delivered, we allow the adversary to retract this message, preventing its delivery. Additionally, we concentrate on asynchronous networks, where no assumptions are made about the timing of message transmissions. Moreover, the adversary can intentionally delay messages but must eventually deliver all messages sent among honest nodes. We do not require a PKI setup and do not use digital signatures in any form, aligning with the unauthenticated setting. The only cryptographic tool employed in our scheme is a collision-resistant hash function.

B. Goal: hash-based asynchronous MVBA

Our goal is to design an efficient multi-valued validated Byzantine agreement (MVBA) protocol [1]–[3]. The formal definition of MVBA is as follows.

Definition 1: In an MVBA protocol involving an external global predicate function denoted as $\mathsf{Predicate} : \{0,1\}^{\ell} \rightarrow \{1,0\}$, each honest node has its input value that conforms to the predefined global function Predicate. The objective is to generate a unanimous output, ensuring that the resulting output also adheres to the predetermined global function Predicate. Formally, the protocol strives to attain the following properties, with all but negligible probability:

- **Termination.** If each honest node $\mathcal{P}_i$ takes an input value v_i such that $\mathsf{Predicate}(v_i) = 1$, then the protocol ensures that every honest node outputs a value v.
- **External-Validity.** If an honest node $\mathcal{P}_i$ outputs a value v, it guarantees that $\mathsf{Predicate}(v) = 1$.
- **Agreement.** If one honest node $\mathcal{P}_i$ outputs v and another honest node $\mathcal{P}_j$ outputs v', it guarantees that v is equal to v', i.e., $v = v'$.

"Quality" was introduced by Abraham et al. [2]. It indicates that the probability of the output value is determined by the adversary. If this probability is less than 1, it can prevent the adversary from entirely determining the output.

- **Quality.** If an honest node outputs an value v, then it is ensured that the probability of v being input by the adversary is bounded by a constant $\mathtt{p} \in (0, 1)$ [58].

IV. Notations and Preliminaries

In this section, we introduce the definitions of several fundamental building blocks that are employed in our paper.

Notations: The notation $\Pi[\mathsf{ID}]$ is employed to denote an instance of the protocol Π with the identifier ID. Additionally, the notation $y \leftarrow \Pi[\mathsf{ID}](x)$ signifies the action of invoking $\Pi[\mathsf{ID}]$ with input x and obtaining y as the output. We sometimes use $[k]$ to represent the integers from 1 to k. Throughout this paper, λ denotes the cryptographic security parameter, representing the size of the (threshold) signature and the length of the hash value.

Collision-resistant Hash Function. A cryptographic collision-resistant hash function ensures that a computationally limited adversary cannot find two distinct inputs that produce the same hash value, except for a negligible probability.

Erasure code scheme. The (k, n)-erasure code scheme [59] consists of two deterministic algorithms, referred to as Enc and Dec. The Enc algorithm takes a vector $\mathbf{v} = (v_1, \cdots, v_k)$ consisting of k data fragments and maps it into a vector $\mathbf{m} = (m_1, \cdots, m_n)$ containing n code fragments. Crucially, the Dec algorithm allows for the reconstruction of $\mathbf{v}$ using any set of k elements from the code vector $\mathbf{m}$. Throughout the paper, we consider a $(f+1, n)$-erasure code scheme, and we employ the terms fragment and codeword interchangeably when the context is unambiguous.

Vector commitment (VC). A VC scheme is specified as a tuple of algorithms denoted as $(\mathsf{VCom}, \mathsf{Open}, \mathsf{VerifyOpen})$. The VCom algorithm produces a commitment vc for an input vector $\mathbf{m}$. When provided with both (m_i, i) and vc, the Open algorithm generates a succinct proof π_i. This proof is designed to verify that the element m_i corresponds to the i-th committed element in the vector $\mathbf{m}$. The position proof can be verified by the VerifyOpen algorithm.

Remark: In this VC scheme, the hiding property is not required, and the VCom algorithm is deterministic. To implement the VC protocol, we consider using a Merkle tree based on hash functions, as described in [60]. In this instantiation, the size of commitment vc is $\mathcal{O}(\lambda)$ bits, and the openness proof π is $\mathcal{O}(\lambda \log n)$ bits in size.

Common coin $\&$ **Election**. In our protocol design, we follow the approach of prior works [15], [35], [50] by assuming the existence of common coins, a concept introduced by Rabin in [61]. This common coin provides an unpredictable and unbiased random value that is shared among all nodes in the network. We model this common coin as an oracle, denoted as $random()$, which all nodes can query using a string `event`. To prevent the adversary from preemptively knowing the coin values of honest nodes, we impose the condition that $random()$ responds to a query with `event` only when at least $f+1$ nodes have previously queried $random()$ with the same `event`. This requirement ensures that at least one non-faulty node has requested the coin.

The common coin is employed in the random leader election protocol, denoted as $\mathsf{Election}[id]$ [2], which is designed to output a uniformly sampled index $\ell \in [n]$.

Reliable broadcast (RBC)**.** In RBC [40], there exists a sender whose goal is to broadcast a value to all nodes. More formally, an RBC protocol satisfies the following properties: *Totality*: if an honest node outputs v, then all honest nodes output v; *Agreement*: if any two honest nodes output v and v' respectively, then $v = v'$; *Validity*: if the sender is honest and inputs v, then all honest nodes output v.

Asynchronous binary agreement (ABA)**.** In an ABA protocol [35], honest nodes provide a single bit as input and output a common bit value $b \in \{0, 1\}$ that is the input of at least one honest node. Formally, an ABA protocol aims to fulfill the following properties, with all but negligible probability: *Termination*: if all honest nodes input a bit, either 0 or 1, then every honest node outputs a bit $b \in \{0, 1\}$; *Agreement*: if any two honest nodes output b and b' receptively, then $b = b'$; *Validity*: if any honest node outputs a bit $b \in \{0, 1\}$, then at least one honest node had b as its input.

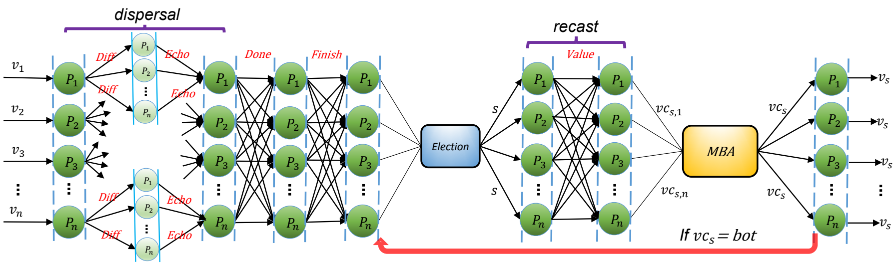


Fig. 1: The execution flow of our HMVBA

Multi-valued Byzantine agreement (MBA)**.** In MBA, honest nodes provide input values $v_i \in \{0,1\}^\ell \cup \{\perp\}$, where the values are not limited to 0/1. Formally, MBA satisfies the following properties except with negligible probability: *Termination*: if all honest nodes input a value v_i, then every honest node output a value v; *Agreement*: if any two honest nodes output v and v' receptively, then $v = v'$; *Weak Validity*: if all honest nodes input the same value v, then all honest nodes output v. Besides the above conventional properties of an MBA, it also satisfies the following *non-intrusion validity* [45], [62]: if one honest node outputs v and $v \neq \perp$, then v is the input of some honest node.

V. Adaptive Asynchronous Multi-valued validated Byzantine agreement

In this section, we introduce our hash-based MVBA protocol, designed to withstand adaptive adversaries and denoted as HMVBA. The HMVBA protocol achieves an optimal time complexity of $\mathcal{O}(1)$ and an optimal message complexity of $\mathcal{O}(n^2)$. Additionally, it achieves optimal communication complexity, specifically $\mathcal{O}(n\ell)$, when the size of the input values ℓ is at least $\lambda n \log n$.

A. Overview of the HMVBA protocol

As we discussed in Introduction, our HMVBA follows "Disseminate-Elect-Agree" paradigm. Now we turn to explain how HMVBA realizes each part of the framework. The workflow of our HMVBA is delineated in Figure 1.

To attain the optimal communication complexity of $\mathcal{O}(n\ell)$, we let each node disseminate their input via an erasure-code-based *Dispersal phase*, rather than through simply multicasting. The Merkle tree is utilized to help the network identify the correct codewords. Specifically, as illustrated in Figure 1, when an honest node receives a valid value v, it uses the deterministic Enc algorithm to generate the codewords $\{m_1, m_2, \ldots, m_n\}$ and computes the vector commitment vc using the VCom algorithm. The honest node then sends codeword m_i along with some auxiliary information to $\mathcal{P}_i$. This approach ensures a communication complexity of $\mathcal{O}(n\ell)$.

Election can be realized by the underlying common coin. After the sender $\mathcal{P}_s$ is chosen by Election, the network starts to recast the input value of $\mathcal{P}_s$ and enters the "Agree" part, trying to agree on the input of $\mathcal{P}_s$. In our protocol, each invocation of Election ensures that all honest nodes can recast the same valid value with a constant probability. This guarantee is crucial for both the performance and the complexities of the protocol.

B. Details of the HMVBA protocol

Our HMVBA protocol is detailed in Algorithm 1. Below is a detailed description of the process of the HMVBA protocol:

1. Dispersal phase (lines 1-16). Nodes disperse their input value v through DIFF messages; each DIFF message includes a vector commitment, a codeword, and a position proof. When a node receives a valid DIFF message from a sender for the first time, it responses an ECHO message to the sender. Once a node receives $n - f$ ECHO messages from distinct nodes, it informs all nodes that its dispersal is completed via DONE messages. When a node receives $n - f$ DONE messages from distinct nodes, it will send a FINISH message to all nodes. If an honest node receives $f + 1$ FINISH messages from distinct nodes, it will also send a FINISH message to all nodes if it has not done so already.

2. Election **& Recast phase** (lines 17-33). Once a node receives $n-f$ FINISH messages from distinct nodes, it initiates a common coin protocol Election to randomly select a leader node $\mathcal{P}_s$. Nodes exchange the DIFF messages received from the elected node and attempt to reconstruct a value. Specifically, upon receiving the result $\mathcal{P}_s$ from Election, each node $\mathcal{P}_i$ checks if it has previously received a valid DIFF message from the sender $\mathcal{P}_s$. If $\mathcal{P}_i$ has received it, $\mathcal{P}_i$ multicasts $(\text{VALUE}, k, \mathcal{P}_s, \mathsf{vc}, m_i, \pi_i)$. If $\mathcal{P}_i$ has not received a valid message, it multicasts $(\text{VALUE}, k, \mathcal{P}_s, \perp, \perp, \perp)$. Honest nodes wait for $n - f$ VALUE messages from distinct nodes. If there are at least $n - 3f \geq 2f + 1$ messages carrying the same vc, each node randomly selects $f + 1$ messages from these $2f + 1$ messages and tries to decode the $f + 1$ codewords to generate an output value M_i. If the value M_i satisfies the condition $\mathsf{Predicate}(M_i) = 1$, it will set VCom_i equal to the vector commitment VCom of the value M_i. Otherwise, it will set VCom_i equal to $\perp$.

3. MBA phase (lines 34-43). All honest nodes invoke MBA with the vector commitment vc_i as input. Suppose MBA returns a value vc'. If $\mathsf{vc}' \neq \perp$ and vc' is the input of MBA, then output the M_i generated in the recast phase. If $\mathsf{vc}' \neq \perp$ and vc' is not the input of MBA, then wait for $f + 1$ valid VALUE messages from distinct nodes such that $|store[\mathsf{vc}']| = f + 1$, and then decode it to output M_i. If $\mathsf{vc}' = \perp$, repeat the Election process until an externally valid value is obtained.

C. Security and Complexity analysis

Security analysis. We now establish the security of Algorithm 1 in the following theorem, and provide a proof sketch.

Algorithm 1 HMVBA protocol with external Predicate, for each node $\mathcal{P}_i$: $n \geq 5f+1$

let $S[i] \leftarrow \bot$ for $i \in [n]$, $store \leftarrow \{\ \}$, $flag \leftarrow 0$,$abandons \leftarrow 0$

▷ Dispersal

1 **upon** receiving an input value v s.t. $\mathsf{Predicate}(v) = 1$ **do**
2 $\boldsymbol{m} := \{m_1, \cdots, m_n\} \leftarrow \mathsf{Enc}(n, f, v)$, where v is parsed as a $f+1$ vector
3 $\mathsf{vc} \leftarrow \mathsf{VCom}(\boldsymbol{m})$;
4 **for** each $j \in [n]$ **do**
5 $\pi_j \leftarrow \mathsf{Open}(\mathsf{vc}, m_j, j)$
6 **send** $(\textsc{Diff}, \mathsf{vc}, m_j, \pi_j)$ to $\mathcal{P}_j$
7 **upon** receiving $(\textsc{Diff}, \mathsf{vc}, m_i, \pi_i)$ from $\mathcal{P}_j$ for the first time **do**
8 **if** $abandons = 0$ and $\mathsf{VerifyOpen}(\mathsf{vc}, m_i, i, \pi_i) = 1$ **then**
9 $S[j] \leftarrow (j, \mathsf{vc}, m_i, \pi_i)$ ▷ store fragment
10 **send** $(\textsc{Echo}, 1)$ to $\mathcal{P}_j$
11 **upon** receiving $(\textsc{Echo}, 1)$ from $n-f$ nodes **do**
12 **multicast** $(\textsc{Done}, 1)$
13 **upon** receiving $(\textsc{Done}, 1)$ from $n-f$ nodes **do**
14 **multicast** $(\textsc{Finish}, 1)$
15 **upon** receiving $(\textsc{Finish}, 1)$ from $f+1$ nodes **do**
16 **multicast** $(\textsc{Finish}, 1)$ if it has not yet been sent
17 **upon** receiving $(\textsc{Finish}, 1)$ from $n-f$ nodes **do**
18 $abandons \leftarrow 1$ ▷ abandon all Dispersal
19 **for** each $k \in \{1, 2, 3, \dots\}$ **do**
20 $s \leftarrow \mathsf{Election}[k]$ ▷ threshold $f+1$
21 **if** $S[s] := (j, \mathsf{vc}, m_i, \pi_i)$ **then**
22 **multicast** $(\textsc{Value}, k, s, \mathsf{vc}, m_i, \pi_i)$
23 **else**
24 **multicast** $(\textsc{Value}, k, s, \bot, \bot, \bot)$ ▷ $S[s] = \bot$
25 **upon** receiving $(\textsc{Value},k,s,\mathsf{vc},m_j,\pi_j)$ from $\mathcal{P}_j$ for the first time **do**
26 **if** $m_j \neq \bot$ and $\mathsf{VerifyOpen}(\mathsf{vc}, m_j, j, \pi_j) = 1$ **then**
27 $store[\mathsf{vc}] \leftarrow store[\mathsf{vc}] \cup (j, m_j)$
28 **upon** $|store[\mathsf{vc}]| = n - 3f$ **do**
▷ pick $f+1$ elements in $store[\mathsf{vc}]$ when decoding
29 $M_i \leftarrow \mathsf{Dec}(store[\mathsf{vc}])$
30 **if** $\mathsf{Predicate}(M_i) = 1$ and $\mathsf{VCom}(\mathsf{Enc}(n, f, M_i)) = \mathsf{vc}$ **then**
31 $\mathsf{vc}_i \leftarrow \mathsf{vc}$, $flag \leftarrow 1$
32 **upon** receiving $\textsc{Value}$ from $n-f$ nodes and $flag = 0$ **do**
33 $\mathsf{vc}_i \leftarrow \bot$; $flag \leftarrow 1$
34 **upon** $flag = 1$ **do**
35 $\mathsf{vc}' \leftarrow \mathsf{MBA}[k](\mathsf{vc}_i)$ ▷ see Algorithm 2
36 **if** $\mathsf{vc}' \neq \bot$ **then**
37 **if** $\mathsf{vc}' = \mathsf{vc}_i$ **then**
38 output M_i
39 **else**
40 **wait** until $|store[\mathsf{vc}']| = f+1$
41 output $M_i \leftarrow \mathsf{Dec}(store[\mathsf{vc}'])$
42 **else**
43 $M_i \leftarrow \bot$; $flag \leftarrow 0$; $store \leftarrow \{\}$

Theorem 1: Assuming the hash function is collision-resistant, and the MBA protocol is adaptively secure, our HMVBA in Algorithm 1, aided by a common coin, is a secure MVBA against any adaptive and computationally bounded adversary corrupting up to f among $n \geq 5f+1$ nodes.

Proof 5.1 (sketched): We first establish the some useful facts about our dissemination phase, and then prove the security properties of our HMVBA based on these facts.

First, we prove that if a "so-far-uncorrupted" node $\mathcal{P}_s$ successfully disperses its input (i.e., multicasts $(\textsc{Done}, 1)$), then all honest nodes can recover its original input value M, even if the node later becomes corrupted by an adaptive adversary. When $\mathcal{P}_s$ multicasts $\textsc{Done}$, at least $n-f$ nodes have received fragments of message M from $\mathcal{P}_s$, with at least $n-2f$ of them being honest nodes. Meanwhile, there are up to f honest nodes may not have received any fragments. Subsequently, an adaptive adversary can later corrupt $\mathcal{P}_s$ and send incorrect fragments (not corresponding to the fragments in M) to these f nodes that have not received the correct fragments. In doing so, the adversary can provide up to $2f$ incorrect fragments in the recasting phase. To successfully reconstruct the original message M, it is necessary to receive at least $4f+1$ correct fragments, which is why we need to require $n \geq 5f+1$.

Second, we show that when an honest node initiates the leader election, a sufficient number of "so-far-uncorrupted" nodes have already completed their dispersal, and every honest node can enter the leader election phase. Specifically, an honest node $\mathcal{P}_i$ invokes Election only when it receives $n-f$ $\textsc{Finish}$ messages, which indicates that at least $f+1$ honest nodes have sent the $\textsc{Finish}$ messages. According to line 15, every honest node will multicast the $\textsc{Finish}$ message, so that every honest node can receive enough $\textsc{Finish}$ messages to enter the election phase. On the other hand, it is easy to see that at least one honest node has received $n-f$ $\textsc{Done}$ messages, and at least $n-2f$ honest nodes have finished their dispersal.

Now, we turn to prove the security properties.

Agreement: Note that the *agreement* of MBA ensures that all honest nodes must agree on the same vector commitment value. Due to the collision-resistance of the underlying hash function and thus the binding property of VC, if two nodes output v and v' whose commitments are the same, then $v = v'$.

Termination: As we argued above, all honest nodes can enter the leader election phase. Then, following the termination of MBA, all honest nodes can agree on a commitment value. As we have analyzed above, at least $n-2f$ honest nodes have finished the dispersal phase, and if one of them is elected as the leader, all honest nodes can obtain its input value. In this case, due to the *validity* of MBA, all nodes will agree on this value. On the other hand, if MBA returns $\bot$, the network can repeat the leader election until a "good" leader is elected. The remaining case is that not every honest node has obtained valid value, but MBA returns a commitment value which is not $\bot$. In this case, ensured by the *non-intrusion* security of MBA, at least one honest node has received at least $n-3f$ valid fragments which can be decoded to a valid value w.r.t. the agreed commitment. Therefore, every honest node can receive at least $f+1$ valid fragments for obtaining the value.

External-validity: It is granted as an honest node will only output a value satisfying the external predicate.

Quality: As the network is trying to agree on an input of a randomly selected node. When an honest node who has finished the dispersal phase is selected, then the network can agree on the input of this node. Therefore, there is a constant probability that the output is an honest node's input.

Efficiency analysis. As depicted in Algorithm 1, the cost breakdown of the HMVBA protocol can be summarized into three distinct phases: In the dispersal phase, which incurs $\mathcal{O}(n^2)$ messages and $\mathcal{O}(n\ell + \lambda n^2 \log n)$ bit complexity, where each node sends a total of $\mathcal{O}(n)$ messages, including $\textsc{Diff}$, ECHO, DONE, and FINISH messages. In the Election & Recast phase, beyond a single common coin invocation, the recast phase requires one all-to-all multicast of VALUE messages, incurring $\mathcal{O}(n^2)$ messages exchange and $\mathcal{O}(n\ell + \lambda n^2 \log n)$ bits of communication. In the MBA phase, there is only one MBA instance, Assuming the use of the MBA protocol [35], where the time complexity of MBA is $\mathcal{O}(1)$, message complexity is $\mathcal{O}(n^2)$, and communication complexity is $\mathcal{O}(n^2\ell)$. Moreover, the Election & Recast phase and the MBA phase are expected to be repeated two times. To summarize, the HMVBA in Algorithm 1 has constant running time, $\mathcal{O}(n^2)$ message complexity, and $\mathcal{O}(n\ell + \lambda n^2 \log n)$ communication complexity, where ℓ is the input size.

VI. ASYNCHRONOUS MULTI-VALUED BYZANTINE AGREEMENT WITH WEAK VAILIDITY

In this section, we propose a novel MVBA that significantly outperforms the one in [45]. Specifically, our MBA reduces the all-to-all broadcast step by at least four compared to the MBA in [45], while maintaining the same asymptotic complexity. Our MBA construction assumes an adaptively secure ABA protocol, which can be instantiated using the IT ABA protocol [35] combined with a common coin, offering $\mathcal{O}(n^2)$ message complexity, $\mathcal{O}(n^2\lambda)$ communication complexity, and $\mathcal{O}(1)$ time complexity.

A. *Overview of the* MBA *protocol*

Our construction is primarily based on the following principle: If all honest nodes input the same value v, then all honest nodes will output v. To achieve this, we first perform a "filter" procedure to retain only the "good cases," where a good case is at least the majority of nodes have the same input. After this filtering process, all honest nodes invoke ABA to determine whether to output a non-$\perp$ value, based on the output of ABA. If ABA outputs 1, it signifies the occurrence of a good case. To achieve agreement, our protocol ensures that all honest nodes output the same value in the good case. To maintain weak validity, we also guarantee that the output value matches the input of the majority of nodes.

It is crucial to ensure that when all honest nodes have the same input, they can collectively identify the occurrence of a good case. This enables all honest nodes to input 1 when invoking ABA. Additionally, we must ensure that even if not all honest nodes input the same values, the protocol will not get stuck. Therefore, in all scenarios, our protocol guarantees that all honest nodes will always have a value as input for ABA, ensuring termination.

B. *Details of the* MBA *protocol*

In this section, we present a comprehensive description of the construction of our MBA protocol.

1. Filter phase (lines 1-7). When a node $\mathcal{P}_i$ receives an input value v, it multicasts (VALUE, v) to all nodes. All nodes wait for VALUE messages from $n-f$ distinct nodes. Once the node $\mathcal{P}_i$ receives (VALUE, v') from $n-2f$ distinct nodes, where $v' \neq 0$, it multicasts (ECHO, v') message to all nodes.

Algorithm 2 MBA protocol, for each node $\mathcal{P}_i$: $n \geq 5f+1$

let $flag \leftarrow 0$
1 **upon** receiving an input value v **do**
2 **multicast** (VALUE, v)
3 **upon** receiving $(\text{VALUE}, *)$ from $n-f$ nodes **do**
4 **if** (VALUE, v') was received from $n-2f$ nodes **then**
5 **multicast** (ECHO, v')
6 **else**
7 **multicast** $(\text{ECHO}, 0)$
8 **upon** receiving $(\text{ECHO}, *)$ from $n-f$ nodes **do**
9 **if** (ECHO, v') was received from $n-2f$ nodes and $v' \neq 0$ **then**
10 $flag \leftarrow 1$
11 **else**
12 $flag \leftarrow 0$
13 **wait** $b \leftarrow \text{ABA}(flag)$
14 **if** $b = 0$ **then**
15 **output** $\perp$
16 **if** $b = 1$ **then**
17 **wait** until receiving (ECHO, v') from f+1 nodes and $v' \neq 0$
18 **output** v'

This (ECHO, v') message serves as the signal that at least $n-2f$ honest nodes have the same input. Otherwise, it multicasts $(\text{ECHO}, 0)$ message to all nodes.

2. ABA phase (lines 8-13). For any node $\mathcal{P}_i$, upon receiving ECHO messages from $n-f$ distinct nodes, if at least $n-2f$ ECHO messages carry the same non-zero value v', then it will input 1 to ABA; otherwise, it takes 0 as its input.

3. Output phase (lines 14-18). In this phase, all nodes output a value based on the output of ABA. If the output is 0, then all nodes output $\perp$. If the output is 1, then all nodes output a non-$\perp$ value. Specifically, if ABA outputs 1, then all honest nodes wait until receiving (ECHO, v') from $f+1$ distinct nodes, where $v' \neq 0$. Then, all nodes output value v'.

C. *Security and Complexity analysis*

Security analysis. We establish the security of MBA in the following theorem and provide a sketch of the proof.

Theorem 2: Assuming the underlying ABA protocol is adaptively secure, our MBA in Algorithm 2 is a secure MBA against any adaptive and computationally bounded adversary corrupting up to f among $n \geq 5f+1$ nodes.

Proof 6.1 (sketched): Agreement: When an honest node $\mathcal{P}_i$ outputs $v' \neq \perp$, ABA must return 1. Due to the validity of ABA, there must be at least one honest node that provided 1 to ABA, so it has received at least (ECHO, v') from at least $n-2f$ nodes. It follows that at least $n-3f = 2f+1$ honest nodes sent (ECHO, v'), so every honest nodes can receive (ECHO, v') from at least $f+1$ nodes. Then, we show there are no distinct v' and v'', such that both of them are carried by at least $f+1$ ECHO messages. Note an honest node multicasts (ECHO, v') only when it has received message (VALUE, v') from $n-2f$ distinct nodes. Since $n \geq 5f+1$, at least $n-3f \geq 2f+1$ honest nodes multicast (VALUE, v'). As a result, if two different honest nodes $\mathcal{P}_i$ and $\mathcal{P}_j$ multicast (ECHO, v') and (ECHO, v''), respectively, and if $v' \neq 0$ and $v'' \neq 0$, then $v' = v''$. Therefore, if an honest node outputs v', all other honest nodes output v'.

Termination: Due to the termination property of ABA, all honest nodes eventually receive a binary value from ABA. When

ABA returns 0, all nodes can terminate with $\perp$. Otherwise, if ABA returns 1, all honest nodes will decide on the same value as we argued above.

Weak validity: When all honest nodes have the same input v, then every honest node can receive enough VALUE and ECHO messages carrying v. Therefore, all honest nodes provide 1 as their inputs to ABA. Due to the validity of ABA, all honest nodes receive 1 from ABA and then decide on the value v.

Non-intrusion: If an honest $\mathcal{P}_i$ returns v', then there exists an honest $\mathcal{P}_j$ having received (ECHO, v') from at least $n-2f$ nodes. It follows that at least $n-3f$ honest nodes received (VALUE, v') from at least $n-2f$ nodes. So v' is the input of at least $n-3f$ honest nodes, while $n-3f > 1$ as $n \geq 5f+1$.

Efficiency of MBA. The cost breakdown of Algorithm 2 consists of three phases. In the filter phase, each node sends n messages, resulting in $\mathcal{O}(n^2)$ messages and $\mathcal{O}(n^2\ell)$ bit complexity due to the message size ℓ. In the ABA phase, beyond the invocation of the common coin, the protocol exchanges $\mathcal{O}(n^2)$ messages with a bit complexity of $\mathcal{O}(n^2)$. Finally, the output phase incurs no communication cost, as it requires no message exchange. Hence, the MBA implemented in Algorithm 2 has constant running time, $\mathcal{O}(n^2)$ message complexity, and $\mathcal{O}(n^2\ell)$ communication complexity, where ℓ is the input size.

VII. Implementation and Evaluations

We implemented and evaluated the performance of HMVBA within a WAN environment. Throughout our study, we systematically compared HMVBA with several common MVBA protocols, including sMVBA* [22] and hash-based FIN-MVBA [32]. In our evaluations, we examined two variants of sMVBA*: sMVBA*-BLS and sMVBA*-ECDSA. In sMVBA*-BLS, we employed sMVBA [22] as the underlying MVBA to instantiate the Dumbo-MVBA* [3], utilizing the BLS threshold signature. Conversely, in sMVBA*-ECDSA, we substituted the BLS threshold signature with the concatenation of $n-f$ ECDSA signatures. Additionally, we emphasize that our experiments did not incorporate any network layer optimizations.

Test environment. The experiments are conducted among AWS EC2 `t2.medium` instances evenly distributed in 13 AWS regions: N. Virginia, Ohio, N. California, Oregon, Canada Central, Mumbai, Tokyo, Seoul, Osaka, Singapore, Sydney, Ireland, and São Paulo. Each `t2.medium` instance has 2 Intel Xeon processors of speed up to 3.4GHz Turbo CPU clock and 4 GB memory. It also provides a baseline bandwidth of 256 Mbps and a peak bandwidth of 1024 Mbps.

The one-shot agreement is evaluated with different input sizes (L) and network sizes (N). Each input L consists of B batches of transactions. In our experiments, a single transaction is represented as a string of 250 bytes, which approximates the size of a typical Bitcoin transaction with one input and two outputs. Hence, we express L as $250 \times B$, where the batch size B ranges from 0 (mini-payload, an empty string) to 7×10^3 in our experiments. Moreover, we conducted tests with six different network sizes, specifically $N = 6$, 16, 31, 61, 101, and 201. The parameter f, denoting the number of corrupted nodes, is set as the optimal threshold. It is consistently set to the maximum integer that satisfies the condition $N \geq 5f+1$, where N represents the network size.

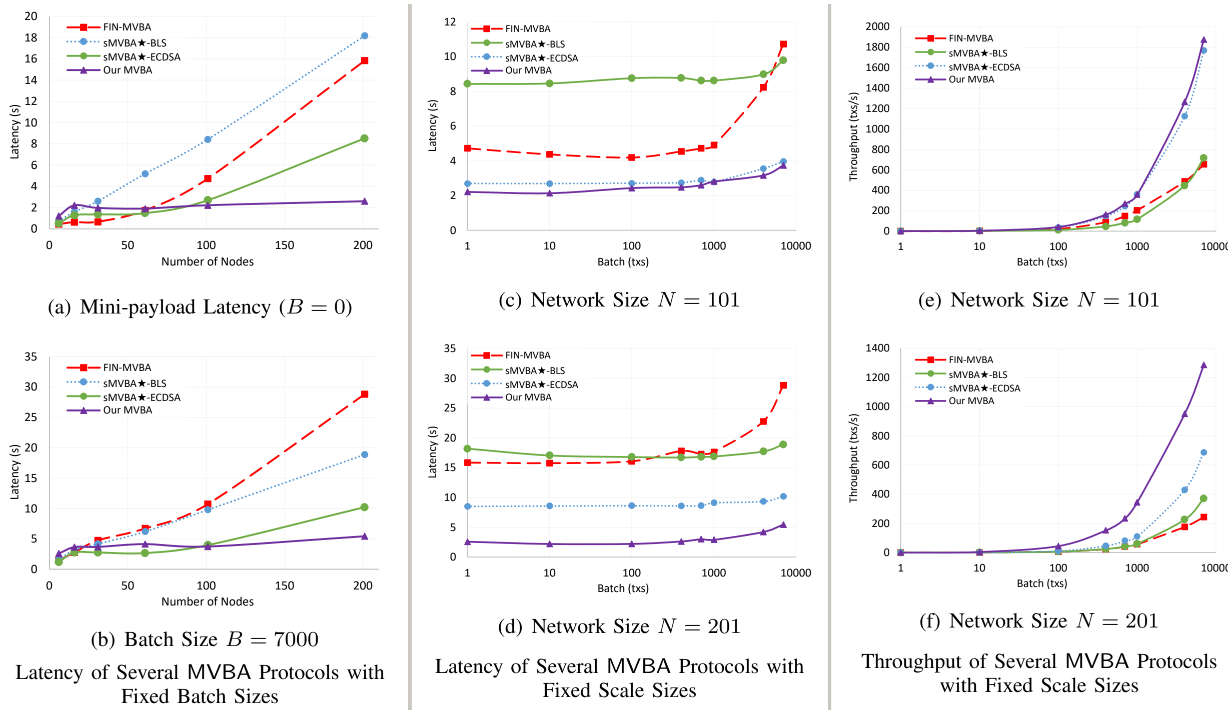
In our test, all N nodes take an input and participate in the instantiation simultaneously. The latency for each node is defined as the time difference between receiving an input and outputting all transactions. To measure the latency among all nodes, we utilize the 20% trimmed mean. Initially, we obtained ten latency measurements by conducting repeated assessments ten times for each specific test configuration under a fixed network size and a fixed input size. Subsequently, the average latency is determined by applying the 20% trimmed mean to the collected latency values.

Implementation details. All asynchronous protocols are written as multi-process Python 3 programs, and are developed upon the open source code of *Dumbo_NG* [25] [3]. The pair-to-pair communication channels between every two nodes are set up using unauthenticated TCP sockets. The Python program initiates three processes on each node, comprising a protocol process responsible for protocol execution, a client process managing data transmission, and a server process managing data reception. Concurrent tasks within each process are managed by *gevent* [4] Python library.

We adapt the source code of sMVBA* [3] in *Dumbo_NG* [25] [5] to fit our testing framework. All components in FIN-MVBA [32] are implemented from scratch, including its WRBC and RABA constructions. For our own HMVBA, we adopted the ABA component from *ADKG* [6] in [29]. Everything else is newly implemented, except for the basic cryptographic components present in *Dumbo_NG*, such as erasure code, Merkle tree, and ECDSA signature. Common coins and leader election protocols are needed by all tested MVBA protocols. They are implemented by hashing the session ID, which is shared across the entire network. Thus, the implementation of these sub-protocols does not introduce any fairness issues.

Latency Improvements with Fixed Input Sizes. Our HMVBA achieves a substantial reduction in basic latency (e.g., batch size $B = 1$) compared to FIN-MVBA by 53% and 84% when $N = 101$ and 201, respectively. Similarly, it also demonstrates latency reductions of 18% and 70% compared to sMVBA*-ECDSA under the same conditions of $N = 101$ and 201. As for sMVBA*-BLS, the reductions in latency are even greater. A mini-payload input, an empty string realized by $B = 0$, is also tested as the input of MVBAs against multiple group sizes. Fig. 2(a) shows that HMVBA outperforms FIN-MVBA and the two variants of sMVBA* for a scale $N \geq 101$. These outcomes align with our asymptotic comparisons, as our MVBA effectively reduces the $\mathcal{O}(\lambda n^3)$ term in the communication cost of FIN-MVBA.

[3] https://github.com/yylluu/Dumbo_NG
[4] http://www.gevent.org/
[5] https://github.com/yylluu/Dumbo_NG/tree/main/dumbomvbastar
[6] https://github.com/sourav1547/adkg/blob/adkg/adkg/broadcast/binaryagreement.py



(a) Mini-payload Latency ($B = 0$)

(b) Batch Size $B = 7000$

Latency of Several MVBA Protocols with Fixed Batch Sizes

(c) Network Size $N = 101$

(d) Network Size $N = 201$

Latency of Several MVBA Protocols with Fixed Scale Sizes

(e) Network Size $N = 101$

(f) Network Size $N = 201$

Throughput of Several MVBA Protocols with Fixed Scale Sizes

Fig. 2: Performance of Several MVBA Protocols

For larger input sizes (e.g., $B = 7000$), our HMVBA continues to outperform FIN-MVBA and achieves latency reductions starting from an even smaller scale (e.g., $N = 31$). This reflects our asymptotic improvement by reducing the $\mathcal{O}(\ell n^2)$ term in the communication cost of FIN-MVBA to $\mathcal{O}(\ell n)$. In contrast, since sMVBA* already benefits from the $\mathcal{O}(\ell n)$ term, our HMVBA does not lower the latency of sMVBA* on a smaller scale. However, HMVBA still can reduce latency under the same conditions that enable it to outperform sMVBA* in basic latency comparisons. This can be indicative of the impact of the gap in computational complexity, as observed when $N = 101$ and 201. Refer to Fig. 2(b).

Latency & Throughput Improvements with Fixed Network Size. To illustrate our advantages in handling various input sizes in a fixed-size network, we demonstrates results of all four MVBA protocols with a fixed two network sizes of $N = 101$ and 201, and various input batch sizes ranging from $B = 1$ to 7000. As depicted in Fig. 2(c) & 2(d), our HMVBA consistently reduces latency across all tested input sizes. Furthermore, with fixed network sizes of $N = 101$ and 201, our HMVBA demonstrates greater throughput across all tested input sizes, cf Fig. 2(e) & 2(f). This aligns with our advantages in less total latency illustrated in Fig. 2(c) & 2(d).

Better Latency-throughput Trade-off. In Fig. 3, the throughput-latency trade-offs in the four MVBA protocols are demonstrated at three reasonable network scales: $N = 61$, 101 and 201. These curves do not exhibit an L-shape, suggesting that the tested MVBA protocols have not reached the network-bound, and the peak throughput is yet to be observed. Despite not exhausting their bandwidths, our HMVBA has already demonstrated a significant advantage over FIN-MVBA and sMVBA*-BLS in all three scales, and sMVBA*-ECDSA in the two larger scales. Specifically, when $N = 61$, our HMVBA achieves a maximum throughput of 1690 transactions per second under the current testing condition, whereas FIN-MVBA has never reached this throughput under the same testing conditions. On the other hand, although our HMVBA outpaces sMVBA*-BLS, it is yet to outpace sMVBA*-ECDSA at this scale. When $N = 101$, our HMVBA exhibits slightly better latency compared to sMVBA*-ECDSA at the same throughput. Moreover, its latency is approximately half of that observed in FIN-MVBA. Furthermore, as the network size N increases to 201, the latency gap between our HMVBA and sMVBA*-ECDSA significantly widens. In this case, sMVBA*-ECDSA experiences a latency that is twice that of our HMVBA, while sMVBA*-BLS sees an even larger latency. Additionally, FIN-MVBA incurs an greater latency, surpassing four times the latency of our HMVBA to achieve the same throughput.

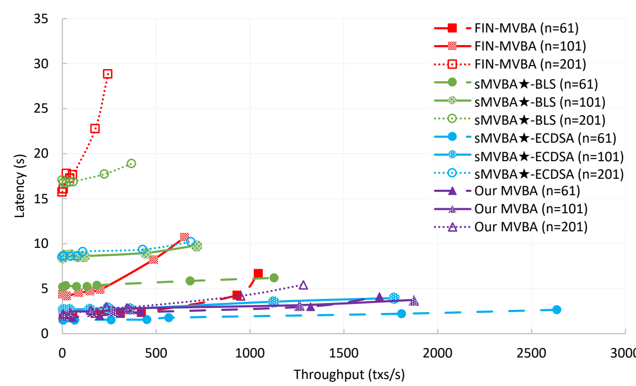


Fig. 3: Latency vs. Throughput of Several MVBA Protocols

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