What is Statistical Learning?

Chapter 02 – Part I

Outline

What Is Statistical Learning?

- Why estimate *f*?
- How do we estimate *f*?
- The trade-off between prediction accuracy and model interpretability
- Supervised vs. unsupervised learning
- Regression vs. classification problems

What is Statistical Learning?

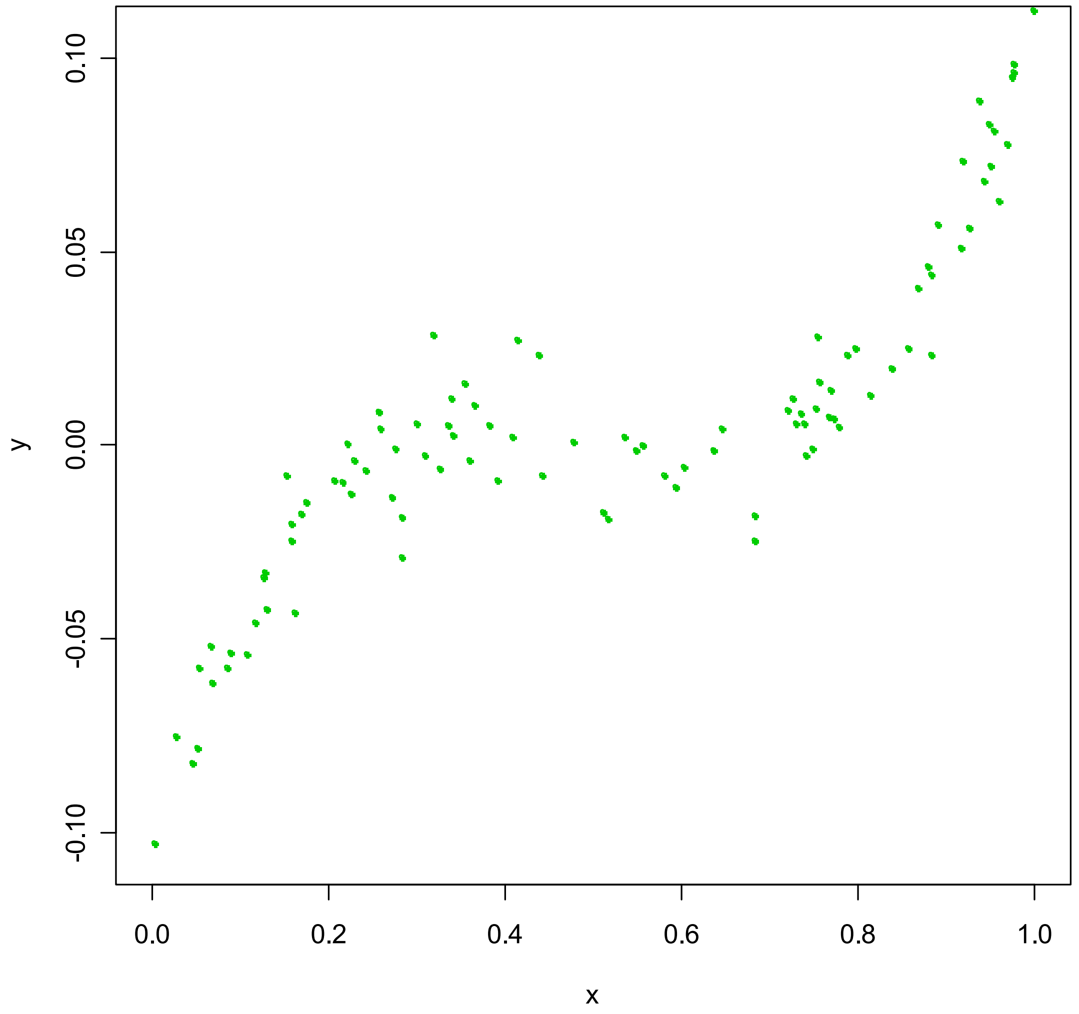
Suppose we observe Y_i and $X_i = (X_{i1}, \ldots, X_{ip})$ for $i = 1, \ldots, n$

We believe that there is a relationship between *Y* and at least one of the *X'*s.
We can model the relationship as

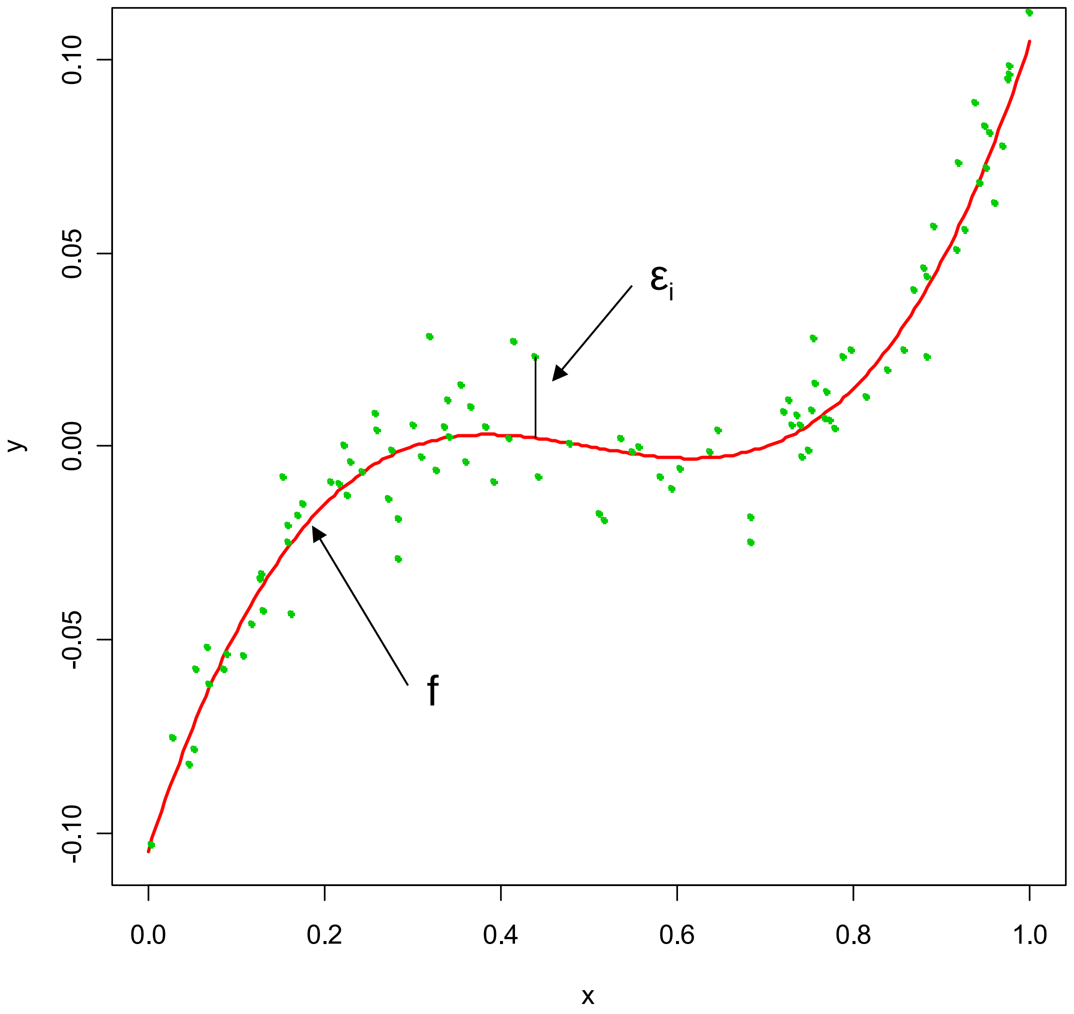
$$Y_i = f(\mathbf{X}_i) + \varepsilon_i$$

Where *f* is an unknown function and *ε* is a random error with mean zero.

A Simple Example



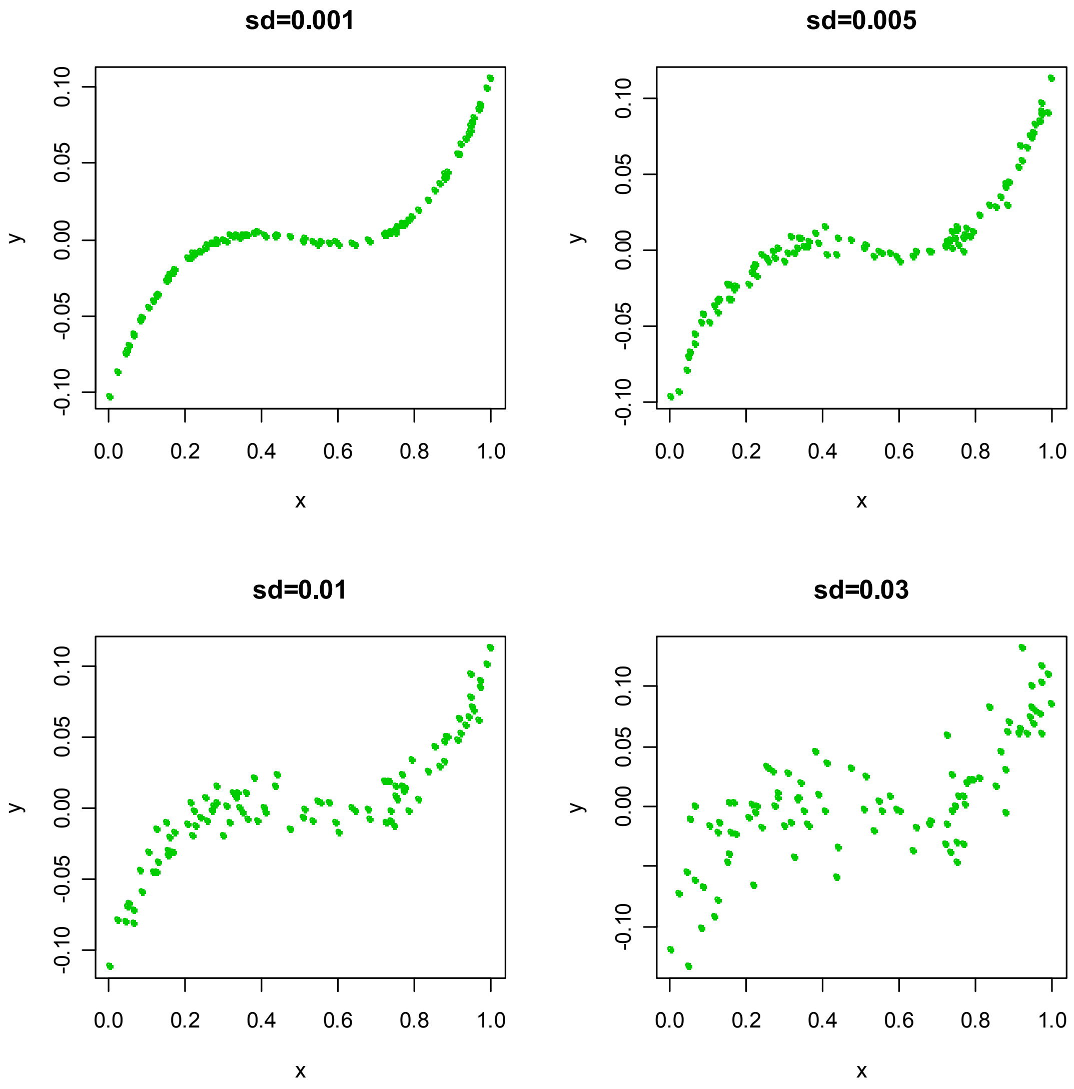
A Simple Example



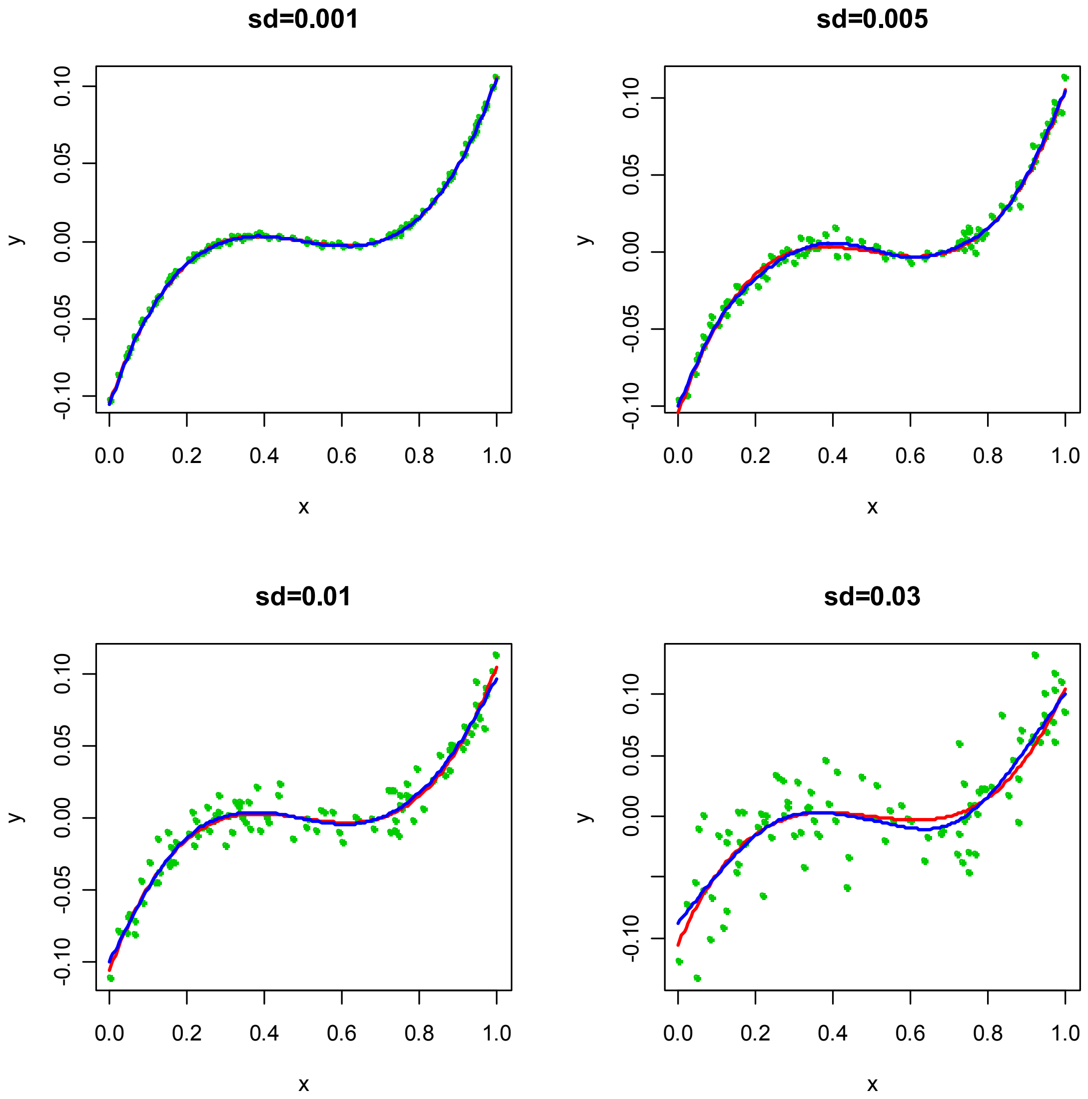
Different Standard Deviations

The difficulty of estimating *f*

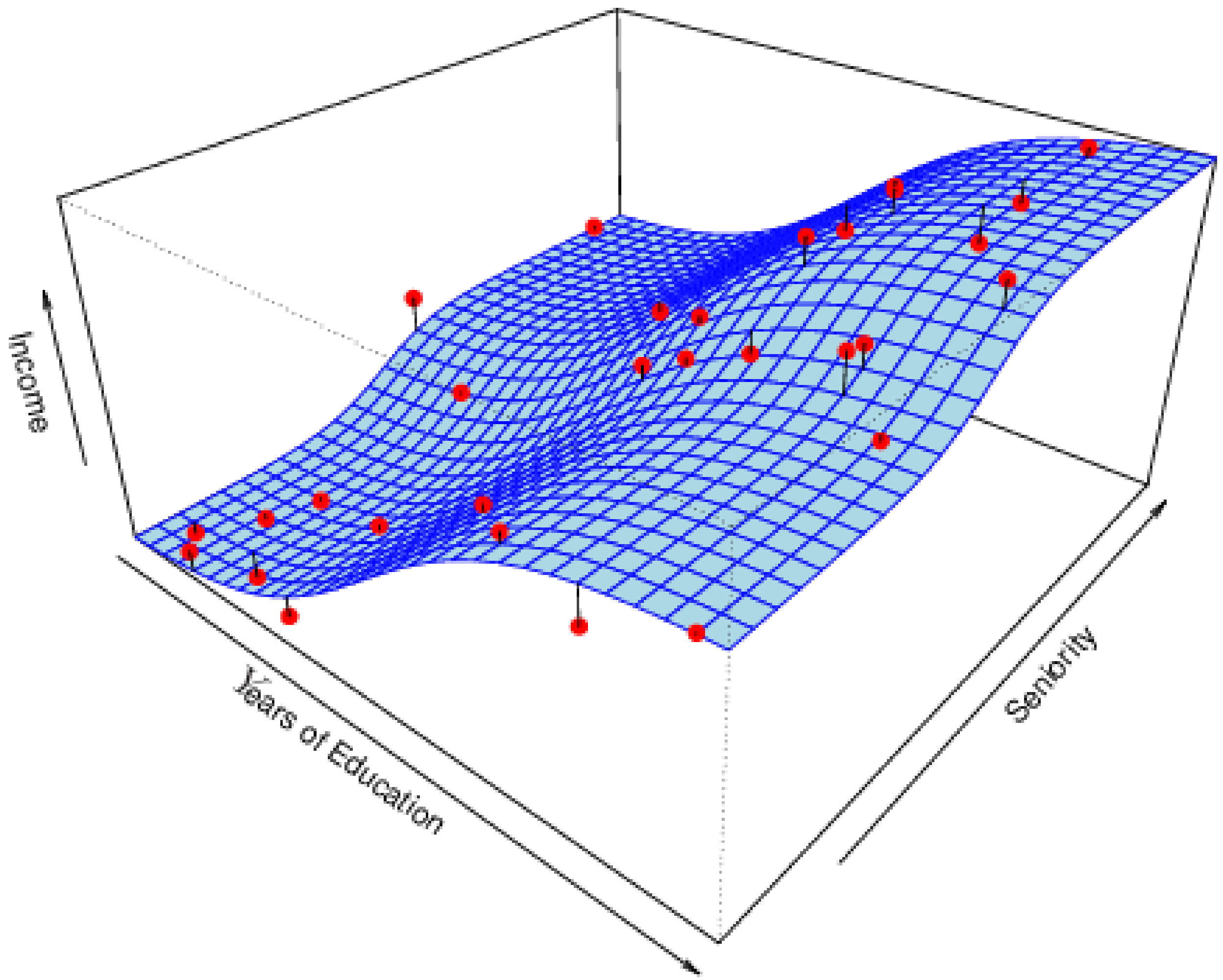
- **will depend on**
- **the standard deviation of the ε's.**



Different Estimates For *f*



Income vs. Education Seniority



Why Do We Estimate *f*?

Statistical Learning, and this course,

- **are all about how to estimate *f*.**

The term statistical learning

- **refers to using the data to "learn" *f*.**

Why do we care about estimating *f*?

There are 2 reasons for estimating *f*,

- **Prediction and**
- **Inference.**

1. Prediction

If we can produce a good estimate for *f*

- **(and the variance of ε is not too large)**

We can make accurate predictions for the response, *Y*,

- **based on a new value of *X*.**

Example: Direct Mailing Prediction

Interested in predicting how much money an individual will donate

- based on observations from 90,000 people
- on which we have recorded over 400 different characteristics.

Don’t care too much about each individual characteristic.

Just want to know:

- For a given individual should I send out a mailing?

2. Inference

Alternatively, we may also be interested in the type of relationship

- **between *Y* and the *X*'s.**

For example,

- Which particular predictors actually affect the response?
- Is the relationship positive or negative?
- Is the relationship a simple linear one or is it more complicated etc.?

Example: Housing Inference

Wish to predict median house price

- based on 14 variables.

Probably want to understand which factors

- have the biggest effect on the response
- and how big the effect is.

For example how much impact does a river view have

- on the house value etc.

How Do We Estimate *f*?

We will assume we have observed a set of training data

$$\{(\mathbf{X}_1, Y_1), (\mathbf{X}_2, Y_2), \ldots, (\mathbf{X}_n, Y_n)\}$$

We must then use the training data

- **and a statistical method to estimate *f*.**

Statistical Learning Methods:

- Parametric Methods
- Non-parametric Methods

Parametric Methods

It reduces the problem of estimating *f*

- **down to one of estimating a set of parameters.**

They involve a two-step model based approach

STEP 1:

- Make some assumption about the functional form of f, i.e. come up with a model.

The most common example is a linear model i.e.

$$f(\mathbf{X}_i) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip}$$

However, in this course we will examine

- **far more complicated, and flexible, models for *f*.**

In a sense the more flexible the model

- **the more realistic it is.**

Parametric Methods (cont.)

STEP 2:

Use the training data to fit the model

- **i.e. estimate *f***
- **or equivalently the unknown parameters such as $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$.**

The most common approach for estimating the parameters in a linear model

- **is ordinary least squares (OLS)**

However, this is only one way.

- **We will see in the course that there are often superior approaches.**

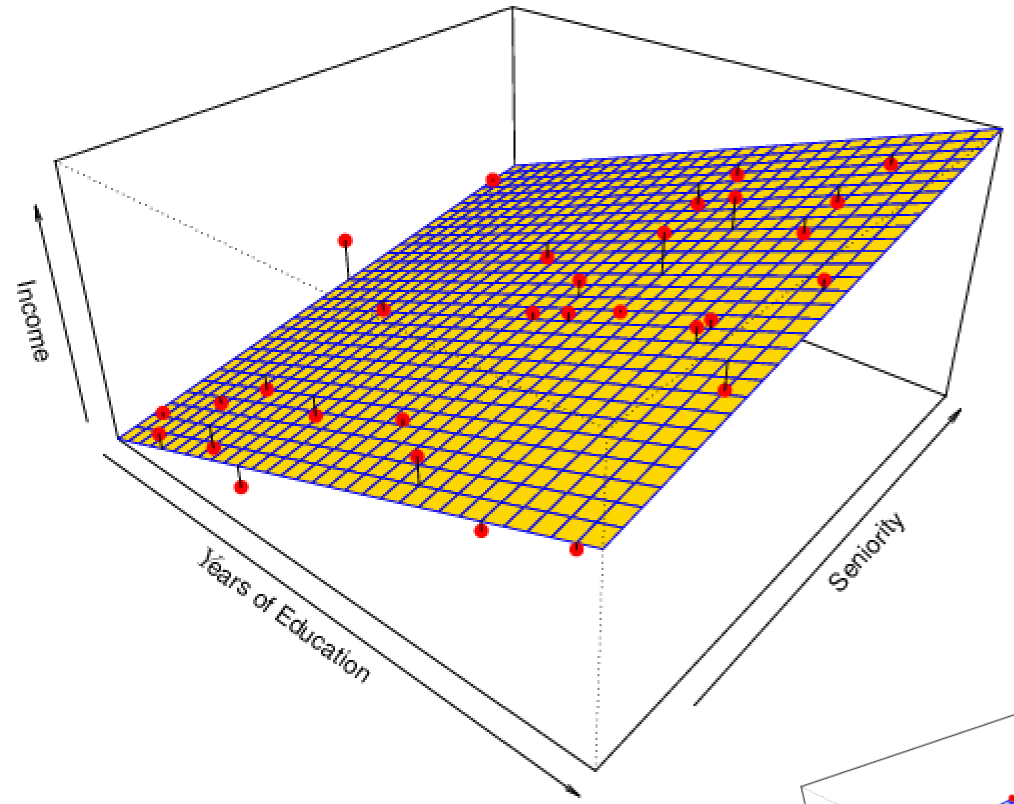
Example: A Linear Regression Estimate

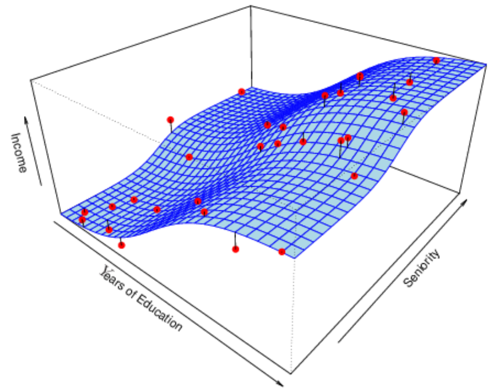
Even if the standard deviation is low

We will still get a bad answer

- **if we use the wrong model.**

$$f = \beta_0 + \beta_1 \times Education + \beta_2 \times Seniority$$





Non-parametric Methods

They do not make explicit assumptions

- **about the functional form of *f*.**

Advantages:

- **They accurately fit a wider range of possible shapes of *f*.**

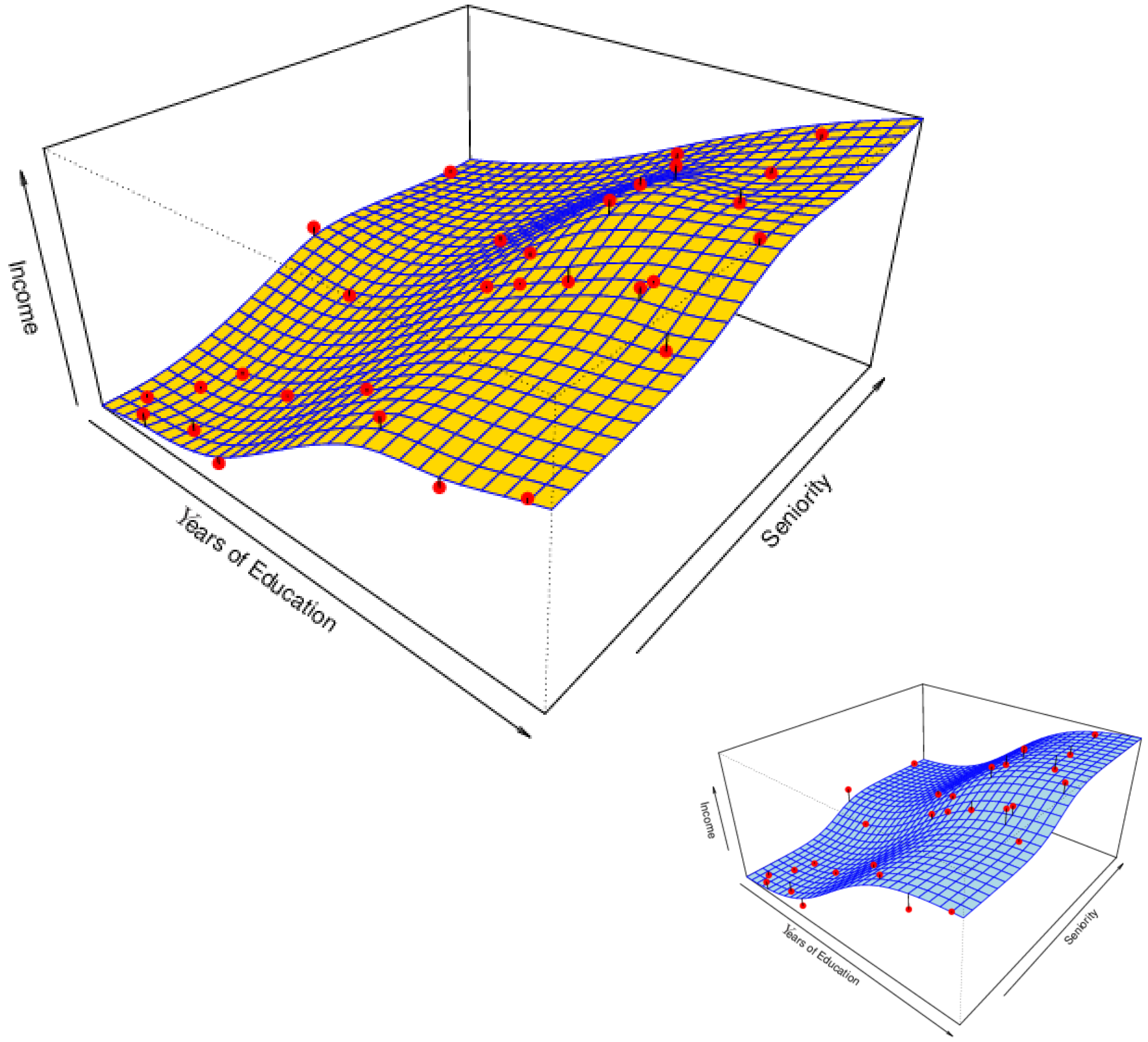
Disadvantages:

- **A very large number of observations is required**
- **to obtain an accurate estimate of *f***

Example: A Thin-Plate Spline Estimate

Non-linear regression methods

- **are more flexible and**
- **can potentially provide more accurate estimates.**



Tradeoff Between Prediction Accuracy & Model Interpretability

Why not just use a more flexible method

- **if it is more realistic?**
-

There are two reasons

Reason 1:

A simple method such as linear regression

- **produces a model which is much easier to interpret**
  - (the Inference part is better).
- **For example, in a linear model,**
  - β_j is the average increase in Y for a one unit increase in X_j
  - holding all other variables constant.

Tradeoff Between Prediction Accuracy and Model Interpretability?

Reason 2:

- Even if you are only interested in prediction,
  so the first reason is not relevant,
- it is often possible to get more accurate predictions
  with a simple, instead of a complicated, model.

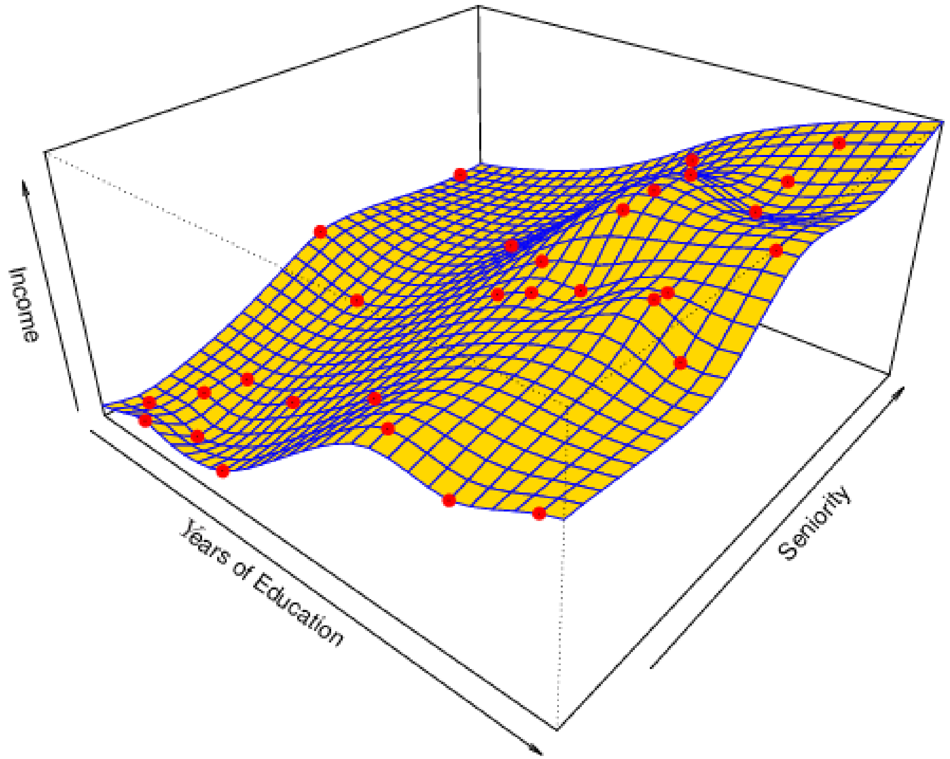
This seems counter intuitive

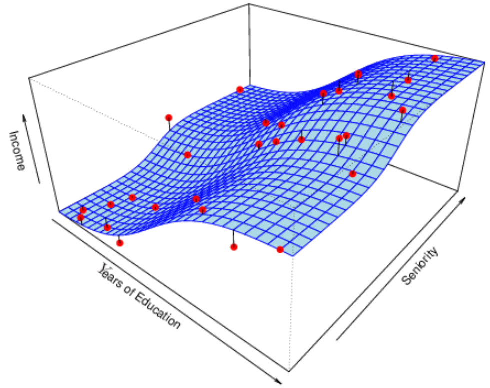
- **but has to do with the fact that**
- **it is harder to fit a more flexible model**

A Poor Estimate

Non-linear regression methods

- **can also be too flexible**
- **and produce poor estimates for *f*.**





Supervised vs. Unsupervised Learning

We can divide all learning problems into

- **Supervised and**
- **Unsupervised situations**

Supervised Learning:

- Supervised Learning is where both
  the predictors, X_i, and
  the response, Y_i, are observed.

This is the situation you deal with in Linear Regression

- Most of this course will also deal with supervised learning.

Unsupervised Learning:

In this situation only the X_i's are observed.

- We need to use the X_i's to guess what Y would have been and build a model from there.

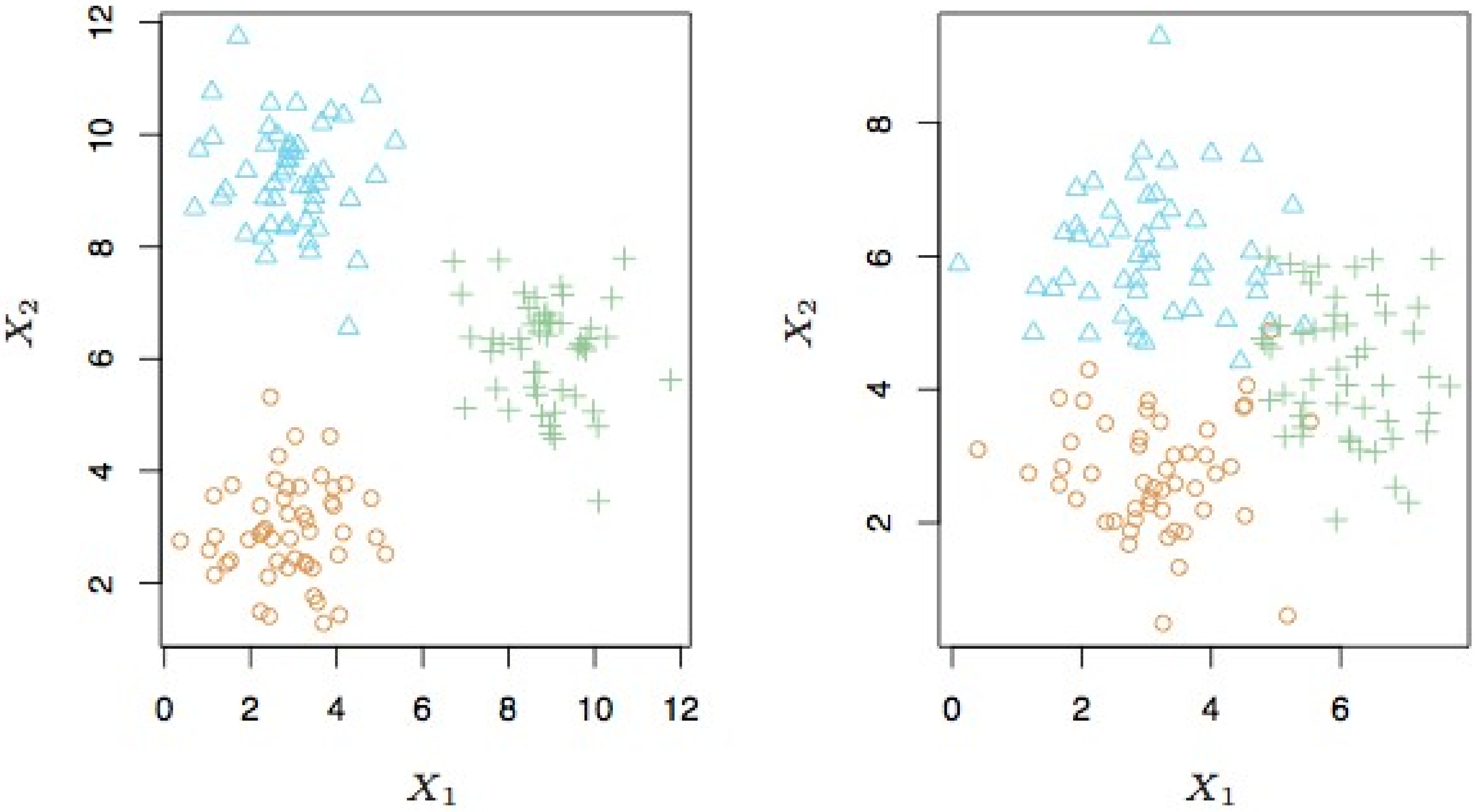
A common example is market segmentation

where we try to divide potential customers into groups based on their characteristics.

- **A common approach is clustering.**

- We will also consider unsupervised learning

A Simple Clustering Example



Regression vs. Classification

Supervised learning problems

- **can be further divided into regression and classification problems.**

Regression covers situations where *Y* is a continuous/numerical variable. e.g.

- Predicting the value of the Dow in 6 months.
- Predicting the value of a given house based on various inputs.

Classification covers situations where *Y* is a categorical variable e.g.

- Will the Dow be up (U) or down (D) in 6 months?
- Is this email a SPAM or not?

Different Approaches

We will deal with both types of problems in this course.

Some methods work well on both types of problem

- **e.g. Neural Networks**

Other methods work best

- **on Regression, e.g. Linear Regression,**
- **or on Classification, e.g. k-Nearest Neighbors.**