Classification Methods

Chapter 4 – Part I

Logistic Regression

Outline

- **Cases:**
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  - Credit Card Default Data
- **Why Not Linear Regression?**
- **Simple Logistic Regression**
  - Logistic Function
  - Interpreting the coefficients
  - Making Predictions
  - Adding Qualitative Predictors
- **Multiple Logistic Regression**

Case 1: Brand Preference for Orange Juice

We would like to predict what customers prefer to buy:

- **Citrus Hill or Minute Maid orange juice?**

The Y (Purchase) variable is categorical: 0 or 1

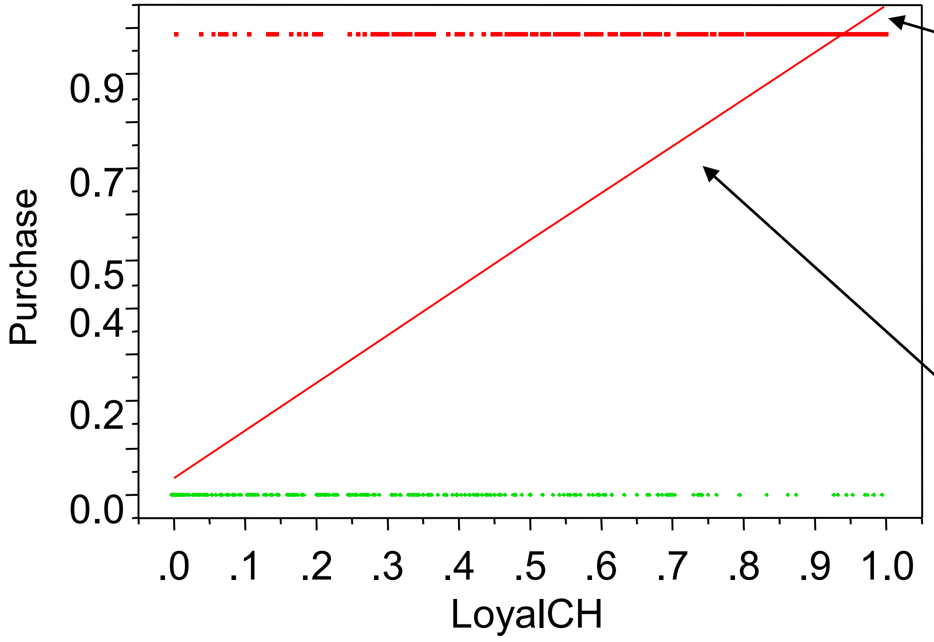
The X (LoyalCH) variable is a numerical value (between 0 and 1)

- **which specifies the how much the customers are loyal to the Citrus Hill (CH) orange juice**

Can we use Linear Regression when Y is categorical?

Why not Linear Regression?

- **When Y only takes on values of 0 and 1,**
  - Why standard linear regression in inappropriate?



How do we interpret values greater than 1?

How do we interpret values of Y between 0 and 1?

Problems

The regression line $\beta_0+\beta_1X$ can take on any value

- between negative and positive infinity

In the orange juice classification problem,

- Y can only take on two possible values: 0 or 1.

Therefore the regression line almost always predicts

- the wrong value for Y in classification problems

Solution: Use Logistic Function

Instead of trying to predict Y,

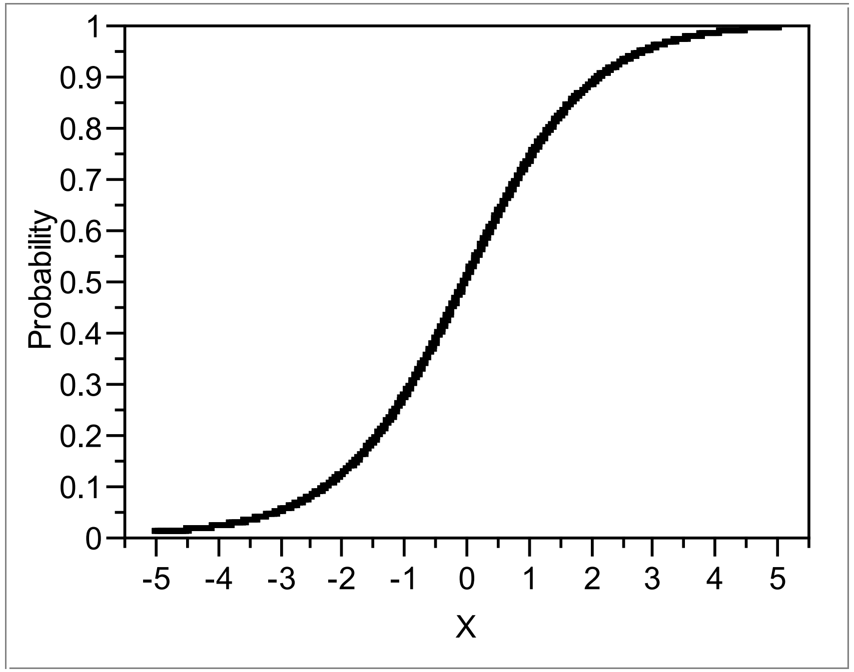
- **let's try to predict P(Y = 1),**
- **i.e., the probability a customer buys Citrus Hill (CH) juice.**

Thus, we can model P(Y = 1) using a function that gives outputs between 0 and 1.

We can use the logistic function

Logistic Regression!

$$\boxed{p = P(Y=1) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}}$$



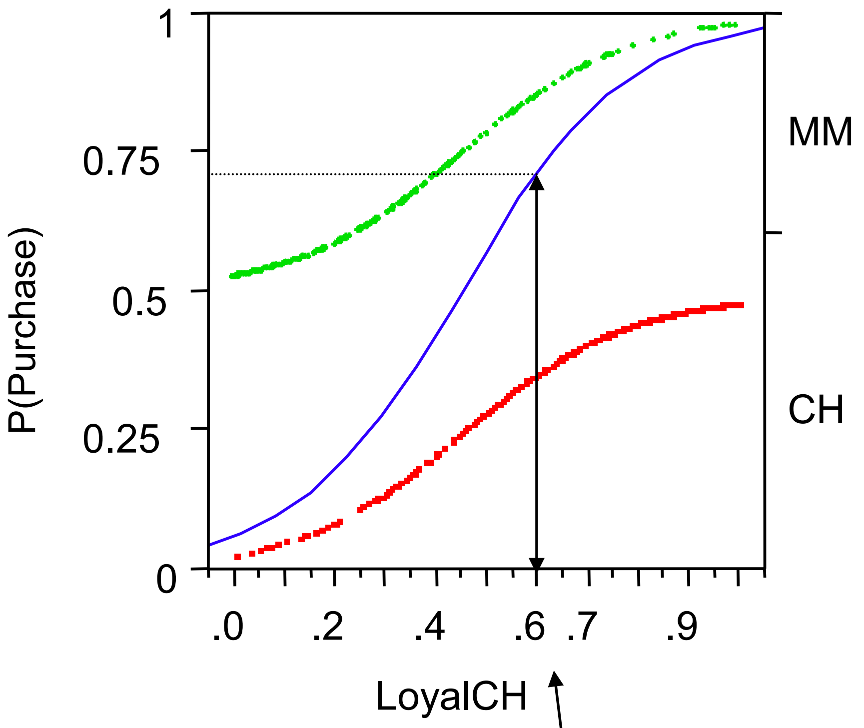
Logistic Regression

Logistic regression is very similar to linear regression

We come up with b_0 and b_1 to estimate β_0 and β_1.

We have similar problems and questions

- **as in linear regression**
  - e.g. Is β_1 equal to 0?
  - How sure are we about our guesses for β_0 and β_1?



If LoyalCH is about .6 then Pr(CH) ≈ .7.

Case 2: Credit Card Default Data

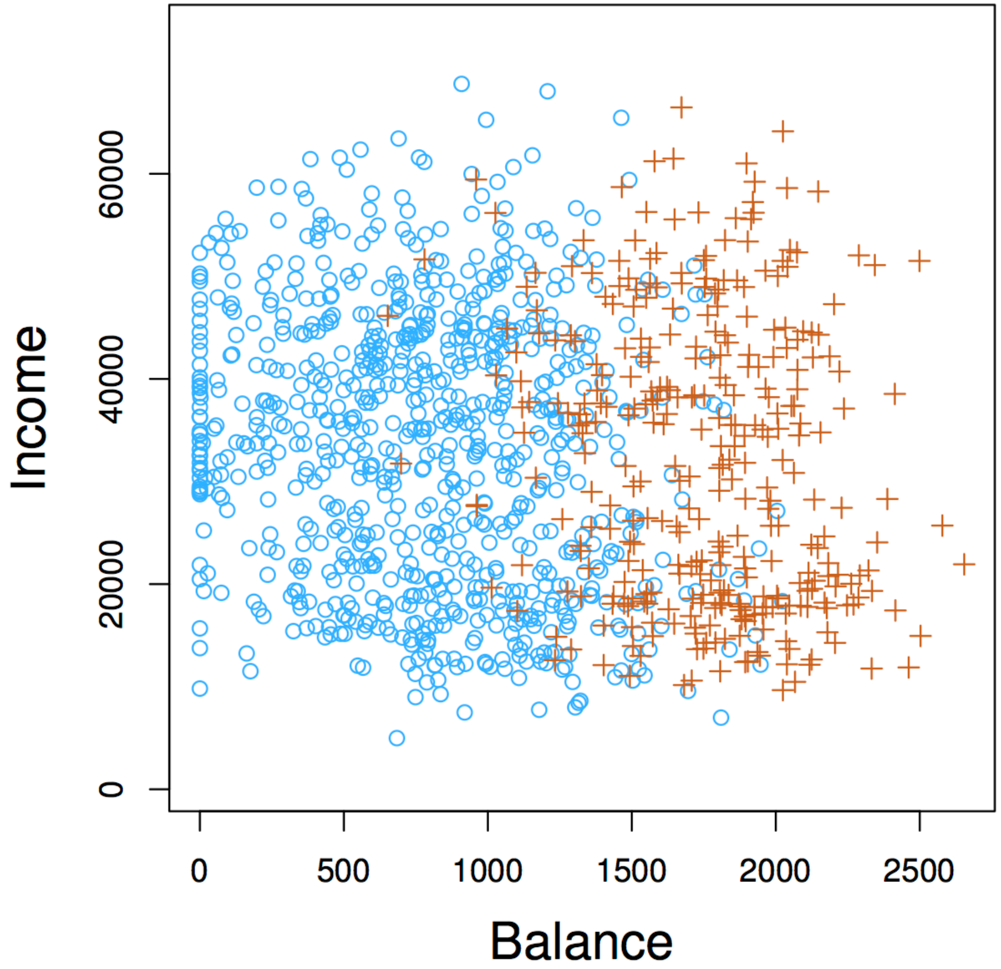
- **We would like to be able to predict**
  - customers that are likely to default

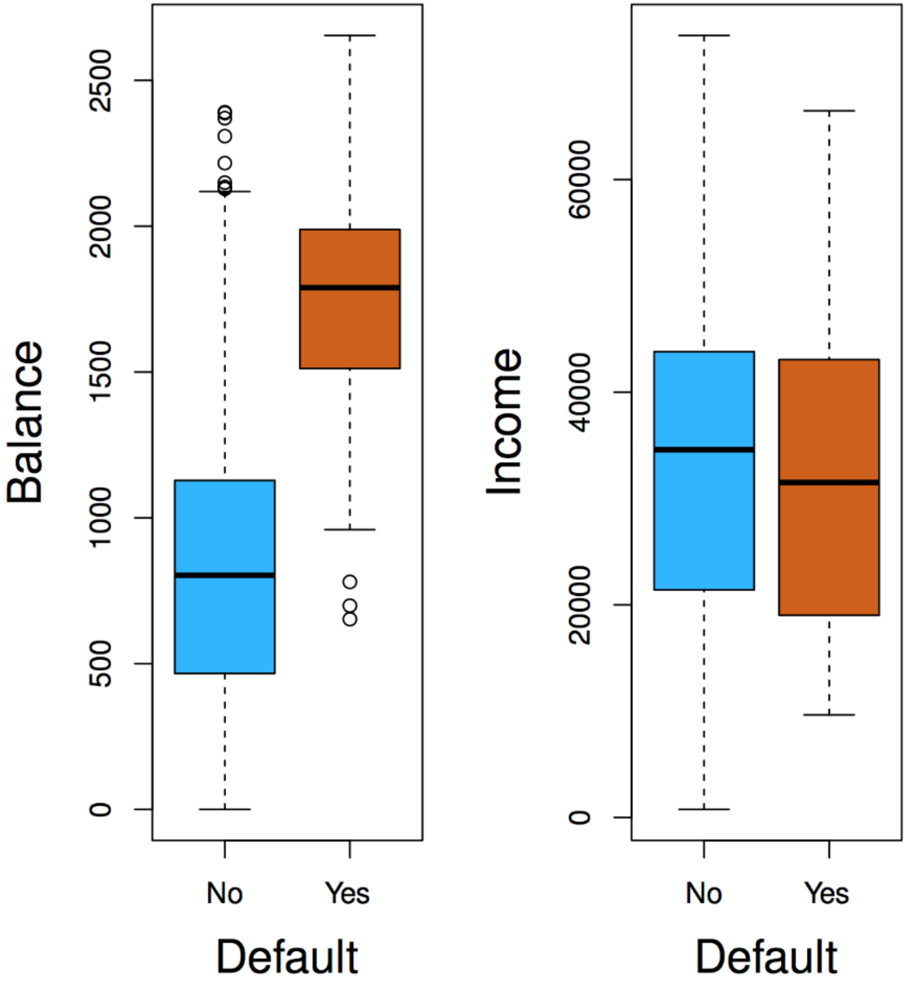
- **Possible X variables are:**
  - Annual Income
  - Monthly credit card balance

- **The Y variable (Default) is <u>categorical</u>:**
  - Yes or No

- **How do we check the relationship between Y and X?**

The Default Dataset

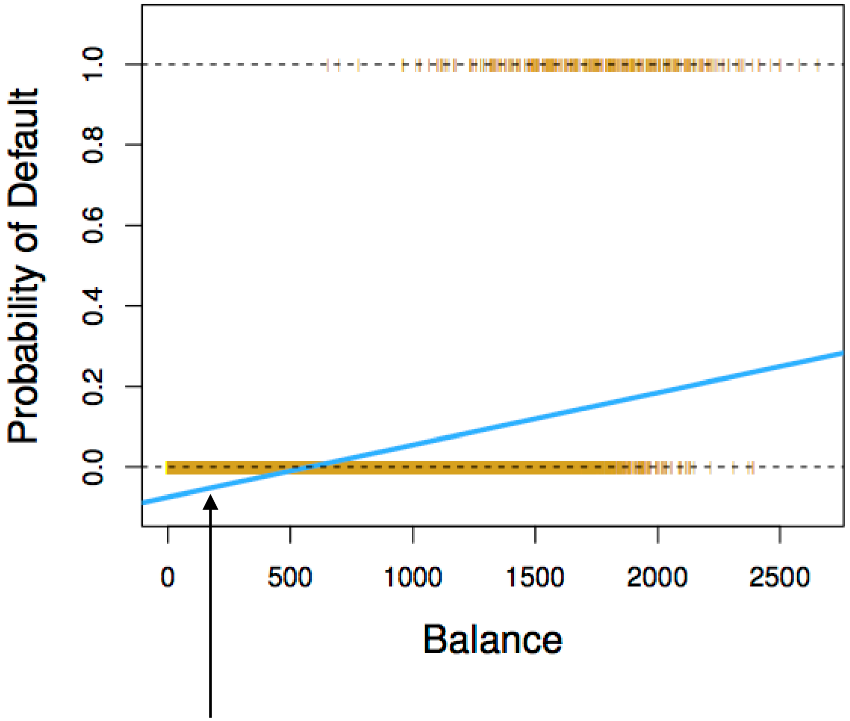




Why not Linear Regression?

If we fit a linear regression to the Default data,

- **then for very low balances**
  - we predict a negative probability,
- **and for high balances**
  - we predict a probability above 1!



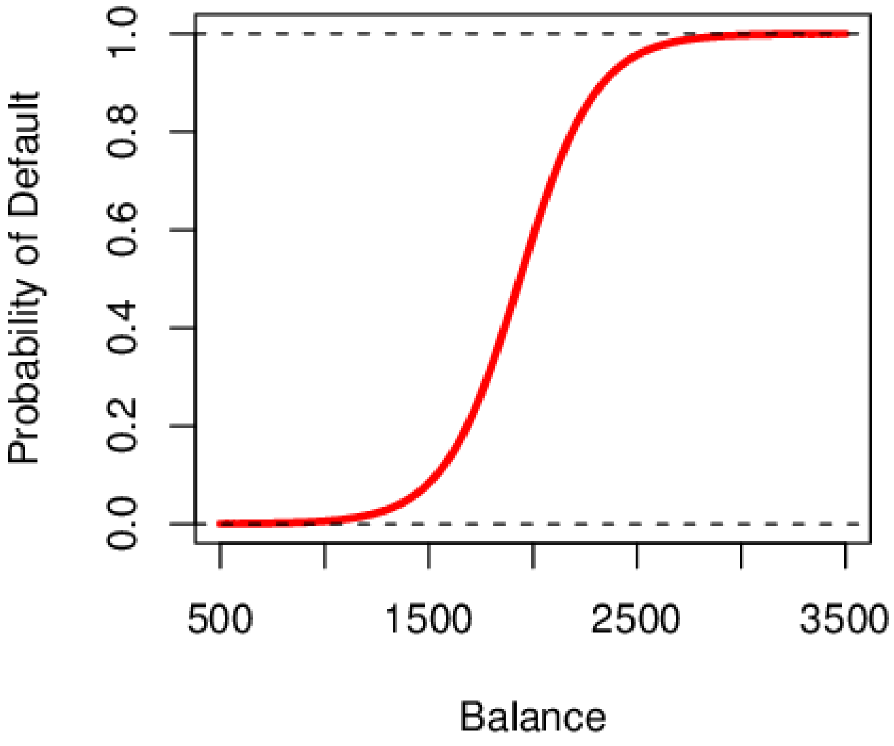
When Balance < 500,

Pr(default) is negative!

Logistic Function on Default Data

Now the probability of default

- **is close to, but not less than zero**
  - for low balances.
- **And close to but not above 1**
  - for high balances



Interpreting β_1

Interpreting what β_1 means is not very easy with logistic regression,

- simply because we are predicting P(Y) and not Y.

If $\beta_1 = 0$, this means that

- there is no relationship between Y and X.

If $\beta_1 > 0$, this means that

- when X gets larger so does the probability that Y = 1.

If $\beta_1 < 0$, this means that

- when X gets larger, the probability that Y = 1 gets smaller.

But how much bigger or smaller

- depends on where we are on the slope

Are the coefficients significant?

We still want to perform a hypothesis test

- to see whether we can be sure that β_0 and β_1
- Are significantly different from zero.

	Coefficient	Std. Error	Z-statistic	P-value
Intercept	-10.6513	0.3612	-29.5	< 0.0001
balance	0.0055	0.0002	24.9	< 0.0001

We use a Z test instead of a T test,

- but of course that doesn't change the way we interpret the p-value

Here the p-value for balance is very small, and b_1 is positive,

- so we are sure that if the balance increases,
- then the probability of default will increase as well.

Making Prediction

Suppose an individual has an average balance of $1000.

- **What is their probability of default?**

The predicted probability of default for an individual with a balance of $1000

- **is less than 1%.**

$$\hat{p}(X) = \frac{e^{\hat{\beta}_0+\hat{\beta}_1 X}}{1+e^{\hat{\beta}_0+\hat{\beta}_1 X}} = \frac{e^{-10.6513+0.0055\times 1000}}{1+e^{-10.6513+0.0055\times 1000}} = 0.00576$$

For a balance of $2000, the probability is much higher,

- **and equals to 0.586 (58.6%).**

Qualitative Predictors in Logistic Regression

We can predict if an individual defaults

- **by checking if she is a student or not.**

Thus we can use a qualitative variable “Student”

- **coded as (Student = 1, Non-student =0).**
- **b_1 is positive:**

This indicates students tend to have

higher default probabilities than non-students

	Coefficient	Std. Error	Z-statistic	P-value
Intercept	-3.5041	0.0707	-49.55	< 0.0001
student[Yes]	0.4049	0.1150	3.52	0.0004

$$\widehat{\Pr}(\texttt{default=Yes}|\texttt{student=Yes}) = \frac{e^{-3.5041+0.4049\times 1}}{1+e^{-3.5041+0.4049\times 1}} = 0.0431,$$

$$\widehat{\Pr}(\texttt{default=Yes}|\texttt{student=No}) = \frac{e^{-3.5041+0.4049\times 0}}{1+e^{-3.5041+0.4049\times 0}} = 0.0292.$$

Multiple Logistic Regression

We can fit multiple logistic

- **just like regular regression**

$$p(X) = \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}.$$

Multiple Logistic Regression- Default Data

Predict Default using:

- Balance (quantitative)
- Income (quantitative)
- Student (qualitative)

	Coefficient	Std. Error	Z-statistic	P-value
Intercept	-10.8690	0.4923	-22.08	< 0.0001
balance	0.0057	0.0002	24.74	< 0.0001
income	0.0030	0.0082	0.37	0.7115
student[Yes]	-0.6468	0.2362	-2.74	0.0062

Predictions

A student

- **with a credit card balance of $1,500**
- **and an income of $40,000**
- **has an estimated probability of default**

$$\hat{p}(X) = \frac{e^{-10.869+0.00574\times 1500+0.003\times 40-0.6468\times 1}}{1+e^{-10.869+0.00574\times 1500+0.003\times 40-0.6468\times 1}} = 0.058.$$

An Apparent Contradiction!

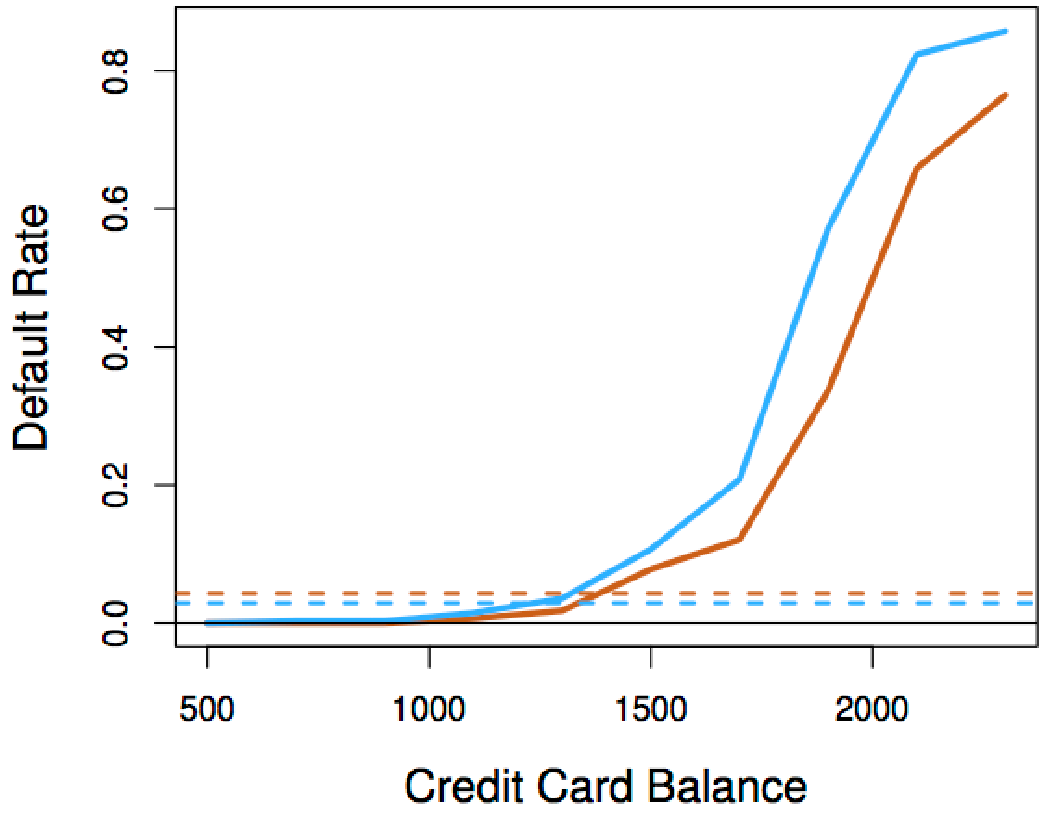
	Coefficient	Std. Error	Z-statistic	P-value
Intercept	-3.5041	0.0707	-49.55	< 0.0001
student[Yes]	0.4049	0.1150	3.52	0.0004

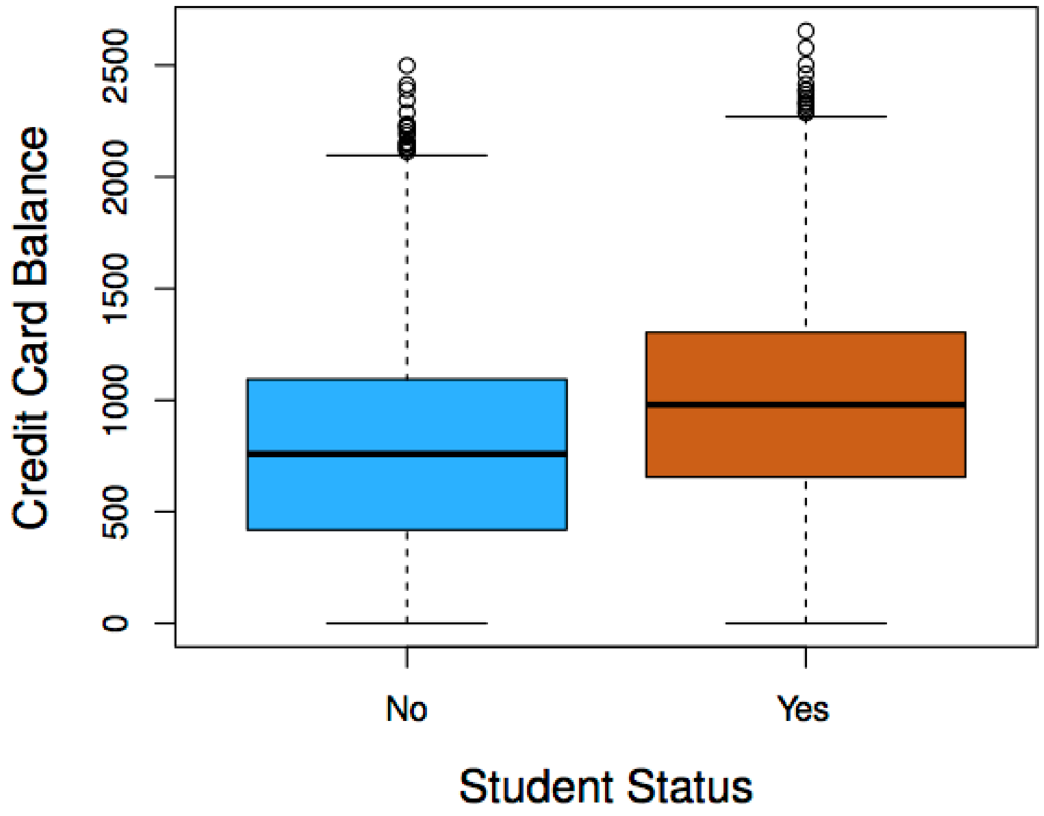
Positive

	Coefficient	Std. Error	Z-statistic	P-value
Intercept	-10.8690	0.4923	-22.08	< 0.0001
balance	0.0057	0.0002	24.74	< 0.0001
income	0.0030	0.0082	0.37	0.7115
student[Yes]	-0.6468	0.2362	-2.74	0.0062

Negative

Students (Orange) vs. Non-students (Blue)





To whom should credit be offered?

A student is risker than non students

- **if no information about the credit card balance is available**

However, that student is less risky than a non student

- **with the same credit card balance!**