DSCI353-353m-453: Class 04a p Multilevel Modeling

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09 February, 2023

Contents

4.1.2.1 Reading, Homeworks, Projects, SemProjects

- Readings:
  - For today: ISLR5
  - For Thursday: ISLR6 (R4DS9-16)
  - Next is: Deep Learning with R (2nd Ed.)
- Laboratory Exercises:
  - LE2 is Due next Tuesday Feb. 14th
- Office Hours: (Class Canvas Calendar for Zoom Link)
  - Wednesdays @ 4:00 PM to 5:00 PM

  - Saturdays @ 3:00 PM to 4:00 PM
  - **Office Hours are on Zoom, and recorded**
- Semester Projects
  - Office Hours for SemProjs: Mondays at 4pm on Zoom
  - DSCI 453 Students Biweekly Updates Due
    - Update # is Due ** **
  - DSCI 453 Students
    - Next Report Out # is Due ** **
  - All DSCI 353/353M/453, E1453/2453 Students:
    - Peer Grading of Report Out #1 is Due ** **
  - Exams
    - MidTerm: **Thursday March 9th**, in class or remote, 11:30 - 12:45 PM
    - Final: **Thursday May 4th**, 2023, 12:00PM - 3:00PM, Nord 356 or remote

4.1.2.2 Textbooks

4.1.2.2.1 Introduction to R and Data Science

For students new to R, Coding, Inferential Statistics

- Peng: R Programming for Data Science
- Peng: Exploratory Data Analysis with R
- OIS = Diez, Barr, Çetinkaya-Runde: Open Intro Stat v4

4.1.2.2.2 Textbooks for this class

- R4DS = Wickham, Grolemund: R for Data Science
- ISLR = James, Witten, Hastie, Tibshirani: Intro to Statistical Learning with R
- ESL = Trevor Hastie, Tibshirani, Friedman: Elements of Statistical Learning
- DLwR = Chollet, Allaire: Deep Learning with R

DL1 to DL6 are "Deep Learning" articles in 3-readings/2-articles/

4.1.2.3 Tidyverse Cheatsheets, Functions and Reading Your Code

Look at the Tidyverse Cheatsheet

- **Tidyverse For Beginers Cheatsheet**
  - In the Git/20s-dsci353-353m-453-prof/3-readings/3-CheatSheets/ folder
- **Data Wrangling with dplyr and tidyr Cheatsheet**]

Tidyverse Functions & Conventions

- The pipe operator `%>%`
- Use `dplyr::filter()` to subset data row-wise.
- Use `dplyr::arrange()` to sort the observations in a data frame
- Use `dplyr::mutate()` to update or create new columns of a data frame
- Use `dplyr::summarize()` to turn many observations into a single data point
- Use `dplyr::arrange()` to change the ordering of the rows of a data frame
- These can be combined using `dplyr::group_by()`

    - which lets you perform operations “by group”.
- The `%in%` matches conditions provided by a vector using the c() function

Reading Your Code: Whenever you see

- The assignment operator `<-`, think **“gets”**
- The pipe operator, `%>%`, think **“then”**

4.1.2.4 Syllabus

4.1.2.5 Intro to Frequentist (Multilevel) Generalised Linear Models (GLM) in R with `glm` and `lme4`

- This will provide a basic introduction to generalized linear models (GLM)
    - using the frequentist approach.

Specifically, we’ll focuses on the use of logistic regression

- in both binary-outcome and
    - count/porportion-outcome scenarios,
- and the respective approaches to model evaluation.

We’ll use the Thai Educational Data example

- from Chapter 6 of the book - Multilevel analysis: Techniques and applications [1]. - by Joop Hox, Mirjam Moerbeek, and Rens van de Schoot

Furthermore, we’ll demonstrate

- the multilevel extension of GLM models
    - with the `lme4` package in R.

Lastly, more distributions and link functions

- in the GLM framework are discussed.

4.1.2.5.1 More readings

- This is meant for beginners and we won’t
    - delve into technical details and complex models.

For a detailed introduction into frequentist multilevel models,

- see this LME4 Tutorial.

For an extensive overview of GLM models,

- see the Wikipedia GLM article here.

Also glmnet is a very nice “glm” package

- and has a good glmnet site

The Bayesian version of this tutorial

- can also be found here.

4.1.2.5.2 The packages being used (all installed on Markov and ODS Desktop)

- We’ll be using
    - `lme4` for multilevel modelling (this tutorial uses version 1.1-31);
    - `tidyverse` for data manipulation and plotting with `ggplot2`;

Day:Date	Foundation	Practicum	Readings(optional)	Due(optional)
w01a:Tu:1/17/23	Markov Cluster	R, Rstudio IDE, Git		(LE0)
w01b:Th:1/19/23	Stat. Learning, Approach	Bash, Git, Class Repo	ISLR1,2 (R4DS-1-3)	
w02a:Tu:1/24/23	Lin. Regr. Bias-Var.	SemProjs; Regr. Ovrvw	ISLR3,(R4DS-4-6)	**(LE0:Due)** LE1
w02b:Th:1/26/23	Train/Test, Bias vs. Vari.	Tidyverse Review	DL01 DL02 (R4DS-7,8)	
w02Pr:Fr:1/27/23	**ADD DROP**	**DEADLINE**		**453 Update 1**
w03a:Tu:1/31/23	Logistic Regr. Classif	Pred. Analytics, Regr.	DL03,ISLR4	
w03b:Th:2/2/23	LDA/QDA	ggPlot2, Code Expect.	DL04, DL05	**LE1:Due**, LE2
w03:Sa:2/4/23				**LE1:Due**
w04a:Tu:2/7/23	Resample Cross-Valid.	Multilevel Mod.	ISLR5	
w04b:Th:2/9/23	DL, ML Overview	ML with NNs	ISLR6 (R4DS9-16)	
w04Pr:Fr:2/10/23				**453 Update 2**
w05a:Tu:2/14/23	Bootstrap		DL2R1, DL06,07	**LE2:Due**, LE3
w05b:Th:2/16/23	Subset Selec., Shrink.	Mixed Effects	DLwR2	
w05Pr:Fr:2/17/23				**453 Rep. Out 1**
w06a:Tu:2/21/23	Mod. Selec.	Dim. Red.	ISLR7	
w06b:Th:2/23/23	Beyond Linear Modls	Feature Select., Caret	ISLR10, (R4DS22-25)	**LE3:Due**, LE4
w06Pr:Fr:2/24/23				**453 Update 3**
w07a:Tu:2/28/23	Dec. Trees, Rand. Forest.	Tidy Modeling	ISLR8, DL08,09	
w07b:Th:3/2/23	MidTerm Review, SVM	SVM, SVR, ROC	ISLR9 (R4DS26-30)	**Peer Review 1**
w08a:Tu:3/7/23	R-Keras/TensorFlow2		DLwR1	
w08b:Th:3/9/23	**MIDTERM EXAM**		DL10,11	**LE4:Due** LE5
w08Pr:Fr:3/10/23				**453 Update 4**
Tu:3/14/23	**SPRING**	**BREAK**	ISLR10	
Th:3/16/23	**SPRING**	**BREAK**	DL12,13	
w09a:Tu:3/21/23	Deep Learning	TF2 Keras Intro	Pocket Perceptron	ISLR10, DLR3
w09b:Th:3/23/23	Computer Vision, CNN	CNN w/TF2, Overfit	DLR4	
w09Pr:Fr:3/24/23				**453 Rep. Out 2**
w10a:Tu:3/28/23	Deep Learn Intro	NN Types	DLR5	
w10b:Th:3/30/23	DL CNN,RNN ImageNet	NN Types, CNN wTF2	Hinton ImageNet	
w10Pr:Fr:3/31/23				**453 Upd.5 & PrRev 2**
Sa:4/1/23				**LE5:Due** LE6
w11a:Tu:4/4/23	Fitting NNs	AUC,Prec,Recall Fruit		
w11b:Th:4/6/23	NLP, Graphs & ML		LeCun DL Rev. 2015	
w12a:Tu:4/11/23	Graphs & ML	NLP with sequences	DLR6	
w12b:Th:4/13/23	NLP w attention	Graph Repr Proc Wrk-flw		**LE6:Due** LE7
w13a:Tu:4/18/23	DL Frameworks	Explaining DL w Lime		
w13b:Th:4/20/23	Linux Distros XGBoost	Explain Preds	Deep Dream	
w13Pr:Fr:4/21/23				**453 Rep. Out 3 Due**
w14a:Tu:4/25/23	Tranformers			
w14b:Th:4/27/23	Final Exam Review	Torch NN & DeepLearn		**LE7:Due**
w14Pr:Fr:4/28/23				**Peer Rev 3 Due**
	FINAL EXAM	**Th. 5/4/23, 12-3pm**	Nord 356 & Zoom	
	453 Final PDF Report	**Fr. 4/29, 11:59pm**		

Figure 1: IT Fundamentals: Applied Data Science with R, Syllabus

  - `haven` for reading `sav` format data;
  - `jtools` for handling of model summaries;
  - `ggstance` for visualisation purposes;
  - `ROCR` for calculating area under the curve (AUC);
  - `performance` for calculating intra-class correlation (ICC).
  - `effects` for plotting parameter effects;
  - Basic knowledge of hypothesis testing and statistical inference;
  - Basic knowledge of correlation and regression;
  - Basic knowledge of coding in R;
  - Basic knowledge of plotting and data manipulation with tidyverse.

```
# install.packages(c("lme4", "tidyverse", "haven", "jtools", "ggstance", "ROCR"))
```

4.1.2.6 Introduction to GLM

- If you are already familiar with generalized linear models (GLM),
  - Its section4.6 of ISLR2
  - you can skip to the next section.

Otherwise, here is a short introduction to GLM

4.1.2.6.1 Recall that in a linear regression model,

- the object is to model the expected value of a continuous variable, Y,
- as a linear function of the predictor, $\epsilon = X\beta$.

The model structure is thus: $E(Y) = X\beta + \epsilon$,

- where ϵ refers to the residual error term.

The linear regression model assumes that Y

- is continuous and comes from a normal distribution,
- that ϵ is normally distributed and
- that the relationship between the linear predictor η and
- the expected outcome $E(Y)$ is strictly linear.

However, these assumptions are easily violated in many real world data examples,

- such as those with binary or proportional outcome variables and
- those with non-linear relationships between
  - the predictors and the outcome variable.

In these scenarios

- where linear regression models are clearly inappropriate,
- generalised linear models (GLM) are needed.

4.1.2.6.2 The GLM is the genearlised version of linear regression

- that allows for deviations from the assumptions underlying linear regression.

The GLM generalises linear regression

- by assuming the dependent variable Y
  - to be generated from any particular distribution in an exponential family
  - (a large class of probability distributions that includes
  - the normal, binomial, Poisson and gamma distributions, among others).

In this way, the distribution of Y

- does not necessarily have to be normal.

In addition, the GLM allows the linear predictor η

- to be connected to the expected value of the outcome variable, $E(Y)$,
- via a link function $g(.)$.

The outcome variable, Y, therefore, depends on η

- through $E(Y) = g^{-1}(\eta) = g^{-1}(X\beta)$.

In this way, the model does not assume

- a linear relationship between $E(Y)$ and η;
- instead, the model assumes a linear relationship
  - between $E(Y)$ and the transformed $g^{-1}(\eta)$.

This tutorial focuses on the probably most popular example of GLM: **logistic regression**.

Logistic regression has two variants,

- the well-known **binary logistic regression**
  - that is used to model binary outcomes (1 or 0; "yes" or "no"),
- and the less-known **binomial logistic regression**
  - suited to model count/proportion data.

Binary logistic regression assumes that Y

- comes from a Bernoulli distribution,
- where Y only takes a value of
  - 1 (target event) or 0 (non-target event).

Binary logistic regression connects $E(Y)$ and η

- via the logit link $\eta = logit(\pi) = log(\pi/(1-\pi))$,
  - where π refers to the probability of the target event ($Y = 1$).

Binomial logistic regression, in contrast,

- assumes a binomial distribution underlying Y,
  - where Y is interpreted as the number of target events,
  - can take on any non-negative integer value
  - and is binomially distributed with regards to
    * n number of trials and
    * π probability of the target event.

The link function is the same as that of binary logistic regression.

The next section details the example data (**Thai Educational Data**)

- followed by the demonstration of the use of
- both binary and binomial logistic regression.

4.1.2.7 Thai Educational Data

- The data used in this tutorial is the Thai Educational Data
  - that is also used as an example in Chapter 6
  - of Multilevel analysis: Techniques and applications.

The data can be downloaded from here.

And its also in the `2-class/data` folder

The data stems from a national survey of primary education in Thailand

- (Raudenbush & Bhumirat, 1992).

Each row in the data refers to a pupil.

- The outcome variable REPEAT is a dichotomous variable
  - indicating whether a pupil has repeated a grade during primary education.
- The SCHOOLID variable indicates the school of a pupil.
- The person-level predictors include: SEX (0 = female, 1 = male)
- and PPED (having had preschool education, 0 = no, 1 = yes).
- The school-level is MSESC,
  - representing school mean SES (socioeconomic status) scores.

The main research questions that this tutorial seeks to answer

- using the Thai Educational Data are:

Ignoring the clustering structure of the data,

- what are the effects of gender and preschool education
  - on whether a pupil repeats a grade?

Ignoring the clustering structure of the data,

- what is the effect of school mean SES
  - on the proportion of pupil repeating a grade?

Considering the clustering structure of the data,

- what are the effects of gender, preschool education and school mean SES
  - on whether a pupil repeats a grade?

These three questions are answered

- by using these following models, respectively:
  - binary logistic regression;
  - binomial logistic regression;
  - multilevel binary logistic regression.

4.1.2.8 Data Preparation

```
# if you don't have these packages installed yet, please use the install.packages("package_name") comma
library(lme4) # for multilevel models
```

4.1.2.8.1 Load necessary packages

```
## Loading required package: Matrix
```

```
library(tidyverse) # for data manipulation and plots
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.2 --
## v ggplot2 3.4.0      v purrr   1.0.0
## v tibble  3.1.8      v dplyr   1.0.10
## v tidyr   1.2.1      v stringr 1.5.0
## v readr   2.1.3      v forcats 0.5.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x tidyr::expand() masks Matrix::expand()
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x tidyr::pack()   masks Matrix::pack()
## x tidyr::unpack() masks Matrix::unpack()
```

```r
library(haven) #for reading sav data
library(sjstats) # used to be to calc icc. but now use performance::icc
library(effects) #for plotting parameter effects
```

```
## Loading required package: carData
## lattice theme set by effectsTheme()
## See ?effectsTheme for details.
```

```r
library(jtools) #for transforming model summaries
library(ROCR) #for calculating area under the curve (AUC) statistics
```

```r
# ThaiEdu_Raw <- read_sav("https://github.com/MultiLevelAnalysis/Datasets-third-edition-Multilevel-book/
ThaiEdu_Raw <- haven::read_sav("./data/thaieduc.sav")
head(ThaiEdu_Raw)
```

4.1.2.8.2 Import Data

```
## # A tibble: 6 x 5
##   SCHOOLID SEX       PPED      REPEAT    MSESC
##      <dbl> <dbl+lbl> <dbl+lbl> <dbl+lbl> <dbl>
## 1    10101 0 [girl]  1 [yes]   0 [no]       NA
## 2    10101 0 [girl]  1 [yes]   0 [no]       NA
## 3    10101 0 [girl]  1 [yes]   0 [no]       NA
## 4    10101 0 [girl]  1 [yes]   0 [no]       NA
## 5    10101 0 [girl]  1 [yes]   0 [no]       NA
## 6    10101 0 [girl]  1 [yes]   0 [no]       NA
```

```r
ThaiEdu_New <- ThaiEdu_Raw %>%
  mutate(
    SCHOOLID = factor(SCHOOLID),
    SEX = if_else(SEX == 0, "girl", "boy"),
    SEX = factor(SEX, levels = c("girl", "boy")),
    PPED = if_else(PPED == 0, "no", "yes"),
    PPED = factor(PPED, levels = c("no", "yes"))
  )

head(ThaiEdu_New)
```

4.1.2.8.3 Data Processing

```
## # A tibble: 6 x 5
##   SCHOOLID SEX   PPED  REPEAT    MSESC
##   <fct>    <fct> <fct> <dbl+lbl> <dbl>
## 1 10101    girl  yes   0 [no]       NA
## 2 10101    girl  yes   0 [no]       NA
## 3 10101    girl  yes   0 [no]       NA
## 4 10101    girl  yes   0 [no]       NA
## 5 10101    girl  yes   0 [no]       NA
## 6 10101    girl  yes   0 [no]       NA
```

```r
ThaiEdu_New %>%
  summarise_each(list( ~ sum(is.na(.)))) %>%
```

```
  gather()
```

4.1.2.8.4 Inspect Missing Data

```
## # A tibble: 5 x 2
##   key      value
##   <chr>    <int>
## 1 SCHOOLID     0
## 2 SEX          0
## 3 PPED         0
## 4 REPEAT       0
## 5 MSESC     1066
```

The data has 1066 observations missing for the MSESC variable.

The treatment of missing data is a complicated topic on its own.

For the sake of convenience,

- we simply list-wise delete the cases with missing data in this tutorial.

```
ThaiEdu_New <- ThaiEdu_New %>%
  filter(!is.na(MSESC))
```

4.1.2.9 Binomial Logistic Regression

```
ThaiEdu_New %>%
  group_by(SEX) %>%
  summarise(REPEAT = sum(REPEAT))
```

4.1.2.9.1 Explore Data: number of REPEAT by SEX and PPED

```
## # A tibble: 2 x 2
##   SEX   REPEAT
##   <fct>  <dbl>
## 1 girl     428
## 2 boy      639
```

```
ThaiEdu_New %>%
  group_by(PPED) %>%
  summarise(REPEAT = sum(REPEAT))
```

```
## # A tibble: 2 x 2
##   PPED  REPEAT
##   <fct>  <dbl>
## 1 no       673
## 2 yes      394
```

It seems that the number of pupils who repeated a grade

- differs quite a bit between the two genders,
- with more male pupils having to repeat a grade.

More pupils who did not have preschool education

- repeated a grade.

This observation suggests that SEX and PPED

- might be predictive of REPEAT.

4.1.2.9.2 Fit a Binary Logistic Regression Model

- R has the base package installed by default,
  - which includes the `glm` function that runs GLM.

The arguments for `glm` are similar to those for `lm`: formula and data.

However, `glm` requires an additional argument:

- `family`, which specifies the assumed distribution of the outcome variable;
  - within family we also need to specify the link function.
- The default of family is `gaussian(link = "identity")`,
  - which leads to a linear model
  - that is equivalent to a model specified by `lm`.
- In the case of binary logistic regression,
  - `glm` requires that we specify a binomial distribution
  - with the logit link, namely `family = binomial(link = "logit")`.

```
Model_Binary <- glm(
  formula = REPEAT ~ SEX + PPED,
  family = binomial(link = "logit"),
  data = ThaiEdu_New
)
summary(Model_Binary)
```

```
##
## Call:
## glm(formula = REPEAT ~ SEX + PPED, family = binomial(link = "logit"),
##     data = ThaiEdu_New)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.6844  -0.5630  -0.5170  -0.4218   2.2199
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.76195    0.05798 -30.387  < 2e-16 ***
## SEXboy       0.42983    0.06760   6.358 2.04e-10 ***
## PPEDyes     -0.61298    0.06833  -8.971  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 6140.8  on 7515  degrees of freedom
## Residual deviance: 6016.2  on 7513  degrees of freedom
## AIC: 6022.2
##
## Number of Fisher Scoring iterations: 4
```

4.1.2.9.3 Interpretation

- From the summary output above, we can see that
  - SEX positively and significantly
    - predicts a pupil's probability of repeating a grade,
  - while PPED negatively and significantly so.

Specifically, in comparison to being a girl,

- being a boy is more likely to repeat a grade.
- Having previous schooling is less likely to result in repeating a grade.

To interpret the value of the parameter estimates,

- we need to exponentiate the estimates.

The summ function from the `jtools` packages

- provides an easy to do so for any model fitted by `glm`. See below.

```
summ(Model_Binary, exp = T) # set "exp = T" to show esponentiated estimates; if you need standardised e
```

Observations	7516
Dependent variable	REPEAT
Type	Generalized linear model
Family	binomial
Link	logit

$\chi^2(2)$	124.55
Pseudo-R² (Cragg-Uhler)	0.03
Pseudo-R² (McFadden)	0.02
AIC	6022.21
BIC	6042.98

	exp(Est.)	2.5%	97.5%	z val.	p
(Intercept)	0.17	0.15	0.19	-30.39	0.00
SEXboy	1.54	1.35	1.75	6.36	0.00
PPEDyes	0.54	0.47	0.62	-8.97	0.00

Standard errors: MLE

Note that the interpretation of the parameter estimates

- is linked to the odds rather than probabilities.

The definition of odds is:

- P(event occurring)/P(event not occurring).

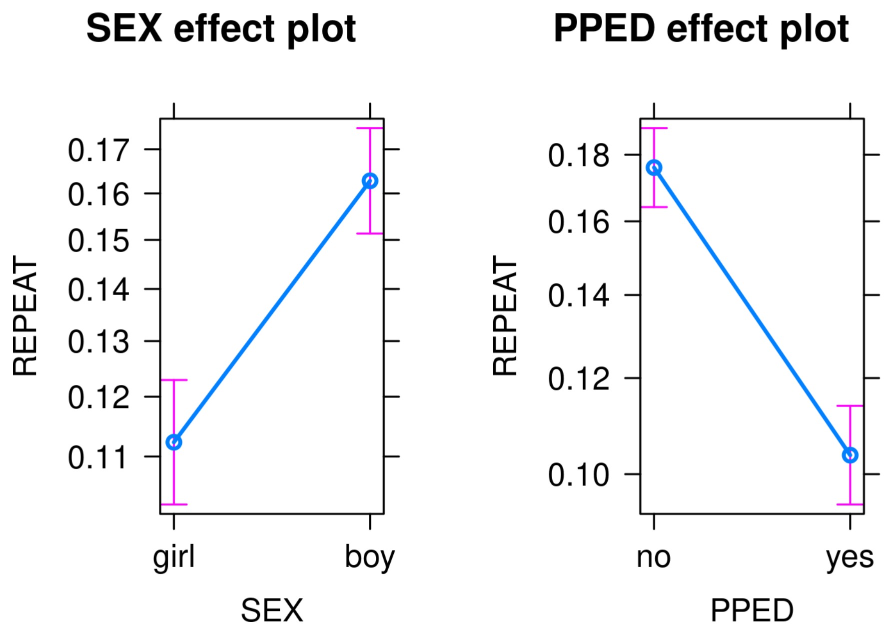
In this analysis, assuming everything else stays the same,

- being a boy increases the odds of repeating a grade by 54%,
  - in comparison to being a girl;
- having preschool education lowers the odds of repeating a grade
  - by $(1 - 0.54)\% = 46\%$,
  - in comparison to not having preschool education,
  - assuming everything else stays constant.

4.1.2.9.4 Visualisation of Parameter Effects

- To make the interpretation of the parameter effects even easier,
  - we can use the `allEffects` function from the effects package
  - to visualize the parameter effects. See below.

```
plot(allEffects(Model_Binary))
```



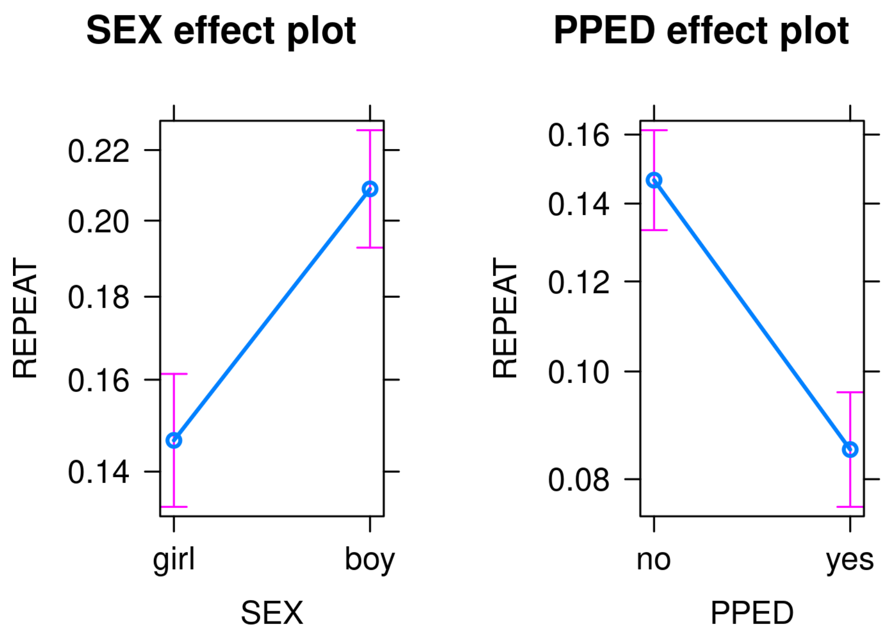
Note that in both plots, the y scale refers to

- the probability of repeating a grade rather than the odds.
- Probabilities are more interpretable than odds.
- The probability scores for each variable are calculated
  - by assuming that the other variables in the model are constant
  - and take on their average values.
- As we can see, assuming that a pupil has an average preschool education,
  - being a boy has a higher probability (~0.16) of repeating a grade
  - than being a girl ~0.11).
- Likewise, assuming that a pupil has an average gender,
  - having preschool education has a lower probability (~0.11)
  - of repeating a grade than not having preschool education (~0.18). -Note that in both plots the confidence intervals for the estimates
  - are also included to give us some idea
  - of the uncertainties of the estimates.

Note that the notion of average preschool education and gender

- may sound strange, given they are categorical variables (i.e. factors).
- If you are not comfortable with the idea of assuming an average factor,
  - you can specify your intended factor level as the reference point,
  - by using the `fixed.predictors = list(given.values = ...)` argument
  - in the `allEffects` function. See below:

```
plot(allEffects(Model_Binary, fixed.predictors =
                  list(given.values = c(
                    SEXboy = 0, PPEDyes = 0
                  ))))
```



Setting `SEXboy = 0` means that for the PPED effect plot,

- the reference level of the SEX variable is set to 0;
- `PPEDyes = 0` results in the 0 being the reference level
  - of the `PPED` variable in the SEX effect plot.

Therefore, as the two plots above show,

- assuming that a pupil has no preschool education,
  - being a boy has a higher probability (~0.20) of repeating a grade
  - than being a girl ~0.14);
- assuming that a pupil is female,
  - having preschool education has a lower probability (~0.09)
  - of repeating a grade than not having preschool education (~0.15).

4.1.2.9.5 Model Evaluation: Goodness of Fit

- There are different ways to evaluate the goodness of fit
  - of a logistic regression model.

Likelihood ratio test

- A logistic regression model has a better fit to the data if the model,
  - compared with a model with fewer predictors,
  - demonstrates an improvement in the fit.

This is performed using the likelihood ratio test,

- which compares the likelihood of the data under the full model
- against the likelihood of the data under a model with fewer predictors.

Removing predictor variables from a model

- will almost always make the model fit less well
  - (i.e. a model will have a lower log likelihood),
- but it is useful to test whether the observed difference in model fit
  - is statistically significant.

```r
#specify a model with only the `SEX` variable
Model_Binary_Test <- glm(
  formula = REPEAT ~ SEX,
  family = binomial(link = "logit"),
  data = ThaiEdu_New
)

#use the `anova()` function to run the likelihood ratio test
anova(Model_Binary_Test, Model_Binary, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: REPEAT ~ SEX
## Model 2: REPEAT ~ SEX + PPED
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1      7514     6099.1
## 2      7513     6016.2  1   82.941 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As we can see, the model with both SEX and PPED predictors

- provide a significantly better fit to the data
  - than does the model with only the SEX variable.
- Note that this method can also be used to determine
  - whether it is necessary to include one or a group of variables.

AIC

- The Akaike information criterion (AIC) is another measure for model selection.

Different from the likelihood ratio test,

- the calculation of AIC not only regards the goodness of fit of a model,
  - but also takes into account the simplicity of the model.
- In this way, AIC deals with the trade-off
  - between goodness of fit and complexity of the model,
  - and as a result, discourages overfitting.
- A smaller AIC is preferred.

```r
Model_Binary_Test$aic
```

```
## [1] 6103.148
```

```r
Model_Binary$aic
```

```
## [1] 6022.207
```

With a smaller AIC value, the model with both SEX and PPED predictors

- is preferred to the one with just the SEX predictor.

Correct Classification Rate

- The percentage of correct classification is another useful measure
  - to see how well the model fits the data.

```r
# use the `predict()` function to calculate the predicted probabilities of pupils in the original data
Pred <- predict(Model_Binary, type = "response")
Pred <- if_else(Pred > 0.5, 1, 0)
```

```r
ConfusionMatrix <-
  table(Pred, pull(ThaiEdu_New, REPEAT)) #`pull` results in a vector
#correct classification rate
sum(diag(ConfusionMatrix)) / sum(ConfusionMatrix)
```

```
## [1] 0.8580362
```

```r
ConfusionMatrix
```

```
##
## Pred    0    1
##    0 6449 1067
```

We can see that the model correctly classifies 85.8% of all the observations.

However, a closer look reveals that the model

- predicts all of the observations to belong to class "0",
  - meaning that all pupils are predicted not to repeat a grade.
- Given that the majority category of the REPEAT variable is 0 (No),
  - the model does not perform better in classification
  - than simply assigning all observations to the majority class 0 (No).

4.1.2.9.6 AUC (area under the curve).

- An alternative to using correct classification rate
  - is the Area under the Curve (AUC) measure.

The AUC measures discrimination, that is,

- the ability of the test to correctly classify
  - those with and without the target response.

In the current data, the target response is repeating a grade.

- We randomly pick one pupil from the "repeating a grade" group
  - and one from the "not repeating a grade" group.
- The pupil with the higher predicted probability
  - should be the one from the "repeating a grade" group.
- The AUC is the percentage of randomly drawn pairs for which this is true.
- This procedure sets AUC apart from the correct classification rate
  - because the AUC is not dependent
  - on the imbalance of the proportions of classes in the outcome variable.
- A value of 0.50 means that the model does not classify better than chance.
- A good model should have an AUC score much higher than 0.50
  - (preferably higher than 0.80).

```r
# Compute AUC for predicting Class with the model
Prob <- predict(Model_Binary, type = "response")
Pred <- prediction(Prob, as.vector(pull(ThaiEdu_New, REPEAT)))
AUC <- performance(Pred, measure = "auc")
AUC <- AUC@y.values[[1]]
AUC
```

```
## [1] 0.6013622
```

With an AUC score of 0.60,

- the model does not discriminate well.

4.1.2.10 Binomial Logistic Regression

- As mentioned in the beginning,
    - logistic regression can also be used to model count or proportion data.

Binary logistic regression assumes that the outcome variable

- comes from a Bernoulli distribution
    - (which is a special case of binomial distributions)
    - where the number of trial n is 1 and
    - thus the outcome variable can only be 1 or 0.
- In contrast, binomial logistic regression
    - assumes that the number of the target events
    - follows a binomial distribution with n trials and probability q.
- In this way, binomial logistic regression allows the outcome variable
    - to take any non-negative integer value
    - and thus is capable of handling count data.

The Thai Educational Data records information about individual pupils

- that are clustered within schools.

By aggregating the number of pupils who repeated a grade by school,

- we obtain a new data set where each row represents a school,
- with information about the proportion of pupils
    - repeating a grade in that school.
- The MSESC (mean SES score) is also on the school level;
- therefore, it can be used to predict proportion or count of pupils
    - who repeat a grade in a particular school. See below.

```
ThaiEdu_Prop <- ThaiEdu_New %>%
  group_by(SCHOOLID, MSESC) %>%
  summarise(REPEAT = sum(REPEAT),
            TOTAL = n()) %>%
  ungroup()
```

4.1.2.10.1 Transform Data

```
## `summarise()` has grouped output by 'SCHOOLID'. You can override using the
## `.groups` argument.
```

```
head(ThaiEdu_Prop)
```

```
## # A tibble: 6 x 4
##   SCHOOLID MSESC REPEAT TOTAL
##   <fct>    <dbl>  <dbl> <int>
## 1 10103     0.88      1    17
## 2 10104     0.2       0    29
## 3 10105    -0.07      5    18
## 4 10106     0.47      0     5
## 5 10108     0.76      3    19
## 6 10109     1.06      9    21
```

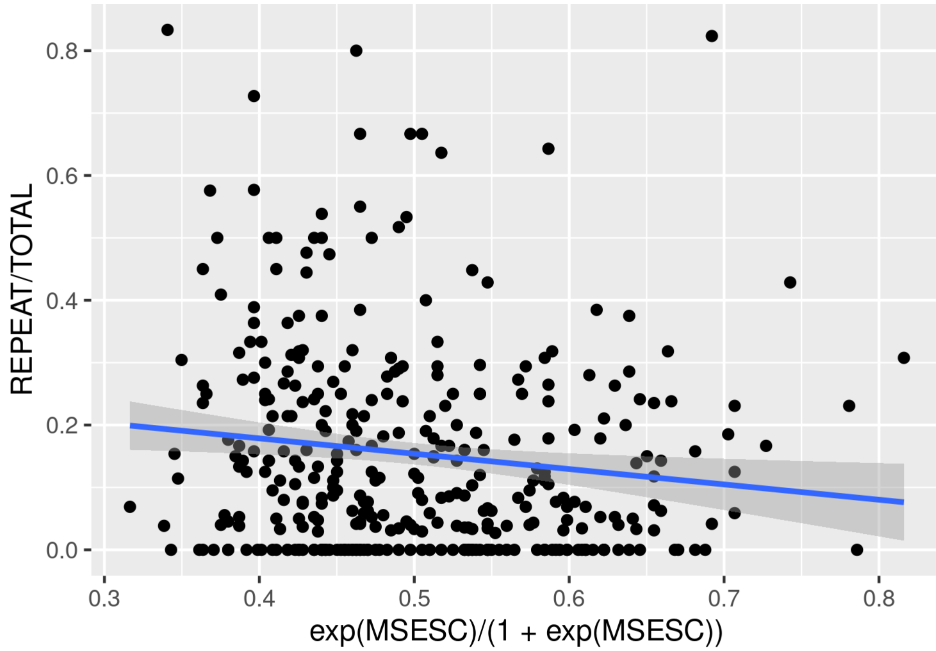
In this new data set,

- `REPEAT` refers to the number of pupils who repeated a grade;
- `TOTAL` refers to the total number of students in a particular school.

```
ThaiEdu_Prop %>%
  ggplot(aes(x = exp(MSESC) / (1 + exp(MSESC)), y = REPEAT / TOTAL)) +
  geom_point() +
  geom_smooth(method = "lm")
```

4.1.2.10.2 Explore Data

```
## `geom_smooth()` using formula = 'y ~ x'
```



We can see that the proportion of students who repeated a grade

- is negatively related to the inverse-logit of `MSESC`.

Note that we model the variable `MSESC` as its inverse-logit

- because in a binomial regression model,
- we assume a linear relationship between
  - the inverse-logit of the linear predictor
  - and the outcome (i.e. proportion of events),
- not linearity between the predictor itself and the outcome.

4.1.2.10.3 Fit a Binomial Logistic Regression Model

- To fit a binomial logistic regression model, we also use the `glm` function.

The only difference is in the specification

- of the outcome variable in the formula.

We need to specify both

- the number of target events (REPEAT)
- and the number of non-events (TOTAL-REPEAT)
- and wrap them in `cbind()`.

```
Model_Prop <- glm(
  formula = cbind(REPEAT, TOTAL - REPEAT) ~ MSESC,
```

```r
  family = binomial(logit),
  data = ThaiEdu_Prop
)

summary(Model_Prop)
```

```
##
## Call:
## glm(formula = cbind(REPEAT, TOTAL - REPEAT) ~ MSESC, family = binomial(logit),
##     data = ThaiEdu_Prop)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -3.3629  -1.8935  -0.5083   1.1674   6.9494
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.80434    0.03324 -54.280  < 2e-16 ***
## MSESC       -0.43644    0.09164  -4.763 1.91e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1480.7  on 355  degrees of freedom
## Residual deviance: 1457.3  on 354  degrees of freedom
## AIC: 2192
##
## Number of Fisher Scoring iterations: 5
```

4.1.2.10.4 Interpretation

- The parameter interpretation in a binomial regression model
  - is the same as that in a binary logistic regression model.

We know from the model summary above

- that the mean `SES` score of a school is negatively related to
- the odds of students repeating a grade in that school.

To enhance interpretability, we use the `summ()` function again

- to calculate the exponentiated coefficient estimate of `MSESC`.

Since `MSESC` is a continuous variable,

- we can standardize the exponentiated `MSESC` estimate
- (by multiplying the original estimate with the SD of the variable,
  - and then then exponentiating the resulting number).

```r
# Note that to use the summ() function for a binomial regression model, we need to make the outcome var
REPEAT <- pull(filter(ThaiEdu_Prop, !is.na(MSESC)), REPEAT)
TOTAL <- pull(filter(ThaiEdu_Prop, !is.na(MSESC)), TOTAL)
summ(Model_Prop, exp = T, scale = T)
```

We can see that with a `SD` increase in `MSESC`,

- the odds of students repeating a grade is lowered by $1 - 85\% = 15\%$.

Observations	356
Dependent variable	cbind(REPEAT, TOTAL - REPEAT)
Type	Generalized linear model
Family	binomial
Link	logit

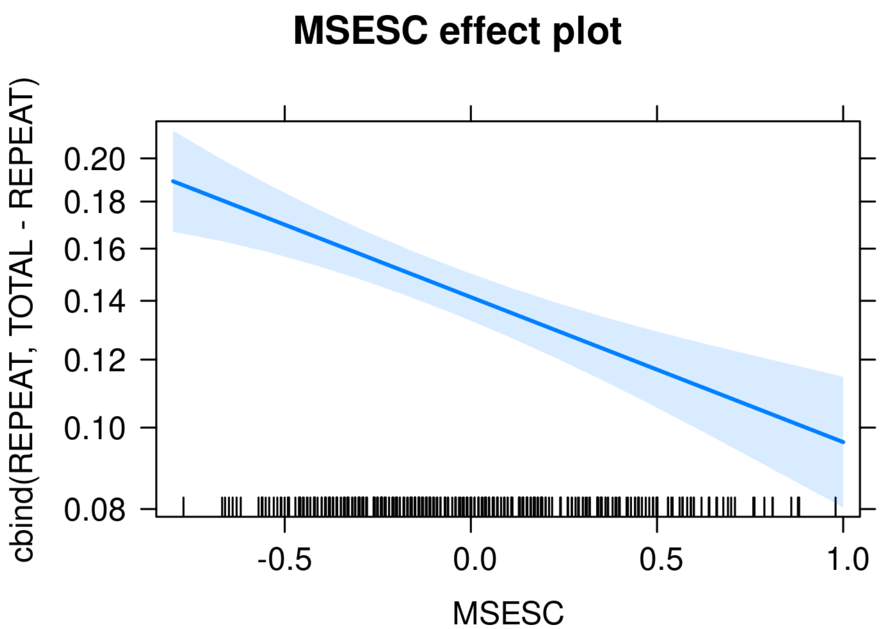
$\chi^2(1)$	23.36
Pseudo-R^2 (Cragg-Uhler)	0.06
Pseudo-R^2 (McFadden)	0.01
AIC	2191.96
BIC	2199.71

	exp(Est.)	2.5%	97.5%	z val.	p
(Intercept)	0.16	0.15	0.18	-54.26	0.00
MSESC	0.85	0.79	0.91	-4.76	0.00

Standard errors: MLE; Continuous predictors are mean-centered and scaled by 1 s.d.

We can visualize the effect of `MSESC`.

```
plot(allEffects(Model_Prop))
```



The plot above shows the expected influence of MSESC

- on the probability of a pupil repeating a grade.

Holding everything else constant, as MSESC increases,

- the probability of a pupil repeating a grade lowers
  - (from 0.19 to 0.10).
- The blue shaded areas indicate the 95% confidence intervals
  - of the predicted values at each value of MSESC.

4.1.2.11 Multilevel Binary Logistic Regression

- The binary logistic regression model introduced earlier
    - is limited to modelling the effects of pupil-level predictors;

The binomial logistic regression

- is limited to modelling the effects of school-level predictors.

To incorporate both pupil-level and school-level predictors,

- we can use multilevel models,
- specifically, multilevel binary logistic regression.

If you are unfamiliar with multilevel models,

- you can use Multilevel analysis: Techniques and Applications for reference
- and this tutorial for a good introduction to multilevel models
    - with the `lme4` package in R.

In addition to the motivation above,

- there are more reasons to use multilevel models.

For instance, as the data are clustered within schools,

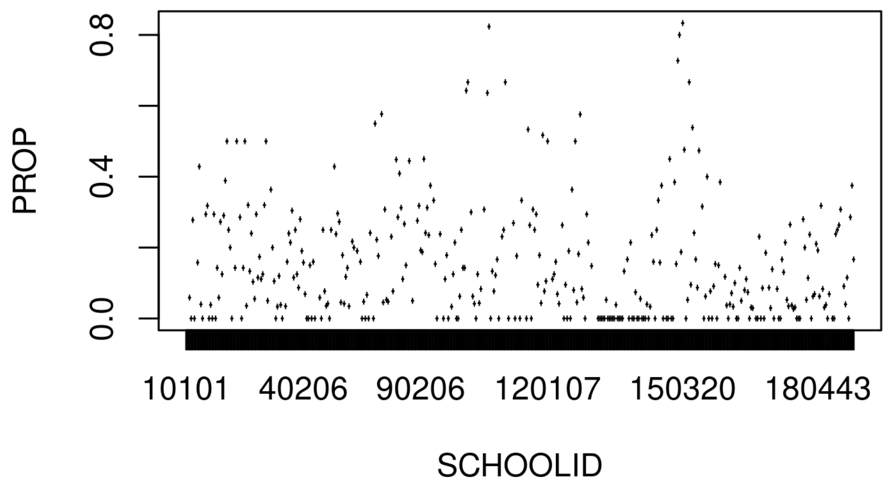
- it is likely that pupils from the same school
    - are more similar to each other than those from other schools.
- Because of this, in one school,
    - the probability of a pupil repeating a grade may be high,
    - while in another school, low.
- Furthermore, even the relationship between the outcome
    - (i.e. repeating a grade)
- and the predictor variables (e.g. gender, preschool education, SES)
    - may be different across schools.
- Also note that there are missing values in the MSESC variable.
- Using multilevel models can appropriately address these issues.

See the following plot as an example.

The plot shows the proportions of students repeating a grade across schools.

- We can see vast differences across schools.
- Therefore, we may need multilevel models.

```
ThaiEdu_New %>%
  group_by(SCHOOLID) %>%
  summarise(PROP = sum(REPEAT) / n()) %>%
  plot()
```



4.1.2.11.1 Center Variables

- Prior to fitting a multilevel model,
  - it is necessary to center the predictors
    - by using an appropriately chosen centering method
    - (i.e. grand-mean centering or within-cluster centering),
  - because the centering approach matters
    - for the interpretation of the model estimates.
  - Following the advice of Enders and Tofighi (2007),
    - we should use within-cluster centering
    - for the first-level predictors `SEX` and `PPED`,
  - and grand-mean centering
    - for the second-level predictor `MSESC`.

```
ThaiEdu_Center <- ThaiEdu_New %>%
  mutate(SEX = if_else(SEX == "girl", 0, 1),
         PPED = if_else(PPED == "yes", 1, 0)) %>%
  group_by(SCHOOLID) %>%
  mutate(SEX = SEX - mean(SEX),
         PPED = PPED - mean(PPED)) %>%
  ungroup() %>%
  mutate(MSESC = MSESC - mean(MSESC, na.rm = T))

head(ThaiEdu_Center)
```

```
## # A tibble: 6 x 5
##   SCHOOLID    SEX   PPED REPEAT    MSESC
##   <fct>     <dbl>  <dbl> <dbl+lbl> <dbl>
## 1 10103    -0.647 -0.882 0 [no]    0.870
## 2 10103    -0.647 -0.882 0 [no]    0.870
## 3 10103    -0.647  0.118 0 [no]    0.870
## 4 10103    -0.647  0.118 0 [no]    0.870
## 5 10103    -0.647  0.118 0 [no]    0.870
## 6 10103    -0.647  0.118 0 [no]    0.870
```

4.1.2.11.2 Intercept Only Model

- To specify a multilevel model,
  - we use the `glmer` function from the `lme4` package.

Note that the random effect term

- should be included in parentheses.

In addition, within the parentheses,

- the random slope term(s) and
- the cluster terms should be separated by `|` (the "pipe" symbol).

Note that we use an additional argument

- `control = glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun=2e5))`
- in the `glmer` function
  - to specify a higher number of maximum iterations than default (10000).
- This might be necessary because a multilevel model
  - may require a large number of iterations to converge.

We start by specifying an intercept-only model,

- in order to assess the impact of the clustering structure of the data.

```
Model_Multi_Intercept <- glmer(
  formula = REPEAT ~ 1 + (1 | SCHOOLID),
  family = binomial(link = "logit"),
  data = ThaiEdu_Center,
  control = glmerControl(optimizer = "bobyqa",
                         optCtrl = list(maxfun = 2e5))
)

summary(Model_Multi_Intercept)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: REPEAT ~ 1 + (1 | SCHOOLID)
##    Data: ThaiEdu_Center
## Control: glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 2e+05))
##
##      AIC      BIC   logLik deviance df.resid
##   5547.1   5560.9  -2771.5   5543.1     7514
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.6254 -0.4174 -0.2487 -0.1765  4.7824
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  SCHOOLID (Intercept) 1.646    1.283
## Number of obs: 7516, groups:  SCHOOLID, 356
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -2.22481    0.08391  -26.52   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Below we calculate

- the `ICC` (intra-class correlation)

- of the intercept-only model.

```
performance::icc(Model_Multi_Intercept)
```

```
## # Intraclass Correlation Coefficient
##
##     Adjusted ICC: 0.333
##   Unadjusted ICC: 0.333
```

An `ICC` of 0.33 means

- that 33% of the variation in the outcome variable
  - can be accounted for by the clustering structure of the data.
- This provides evidence that a multilevel model
  - may make a difference to the model estimates,
  - in comparison with a non-multilevel model.
- Therefore, the use of multilevel models is necessary and warrantied.

4.1.2.11.3 Full Model

- It is good practice to build a multilevel model step by step.

However, as this tutorial's focus is not on multilevel modelling,

- we go directly from the intercept-only model
  - to the full-model that we are ultimately interested in.

In the full model,

- we include not only fixed effect terms of `SEX`, `PPED` and `MSESC`
  - and a random intercept term,
- but also random slope terms for `SEX` and `PPED`.

Note that we specify `family = binomial(link = "logit")`,

- as this model is essentially a binary logistic regression model.

```
Model_Multi_Full <-
  glmer(
    REPEAT ~ SEX + PPED + MSESC + (1 + SEX + PPED | SCHOOLID),
    family = binomial(link = "logit"),
    data = ThaiEdu_Center,
    control = glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 2e5))
  )
```

```
## boundary (singular) fit: see help('isSingular')
```

```
?isSingular
```

boundary (singular) fit: see ?isSingular

```
summary(Model_Multi_Full)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: REPEAT ~ SEX + PPED + MSESC + (1 + SEX + PPED | SCHOOLID)
##    Data: ThaiEdu_Center
## Control: glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 2e+05))
##
##      AIC      BIC   logLik deviance df.resid
```

```
##   5468.0   5537.2  -2724.0   5448.0     7506
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.4509 -0.4003 -0.2451 -0.1732  5.6938
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  SCHOOLID (Intercept) 1.66585  1.2907
##           SEX         0.15439  0.3929    0.56
##           PPED        0.04748  0.2179   -0.61  0.32
## Number of obs: 7516, groups:  SCHOOLID, 356
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -2.2727     0.0886 -25.652  < 2e-16 ***
## SEX           0.4093     0.1105   3.703 0.000213 ***
## PPED         -0.5555     0.1534  -3.621 0.000293 ***
## MSESC        -0.5054     0.2173  -2.326 0.020020 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##       (Intr) SEX   PPED
## SEX   0.017
## PPED  0.024  0.064
## MSESC 0.048  0.064 0.077
## optimizer (bobyqa) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

The results (pertaining to the fixed effects) are similar

- to the results of the previous
  - binary logistic regression and binomial logistic regression models.
- On the pupil-level,
  - `SEX` has a significant and positive influence
  - on the odds of a pupil repeating a grade,
  - while `PPED` has a significant and negative influence.
- On the school-level, `MSESC` has a significant and negative effect
  - on the outcome variable.
- Let's also look at the variance of the random effect terms.

Again, we can use the `summ()` function

- to retrieve the exponentiated coefficient estimates for easier interpretation.

```
summ(Model_Multi_Full, exp = T)
```

Observations	7516
Dependent variable	REPEAT
Type	Mixed effects generalized linear model
Family	binomial
Link	logit

We can also use the `allEffects` function

AIC	5467.99
BIC	5537.23
Pseudo-R^2 (fixed effects)	0.02
Pseudo-R^2 (total)	0.36

Fixed Effects				
	exp(Est.)	S.E.	z val.	p
(Intercept)	0.10	0.09	-25.65	0.00
SEX	1.51	0.11	3.70	0.00
PPED	0.57	0.15	-3.62	0.00
MSESC	0.60	0.22	-2.33	0.02

Random Effects		
Group	Parameter	Std. Dev.
SCHOOLID	(Intercept)	1.29
SCHOOLID	SEX	0.39
SCHOOLID	PPED	0.22

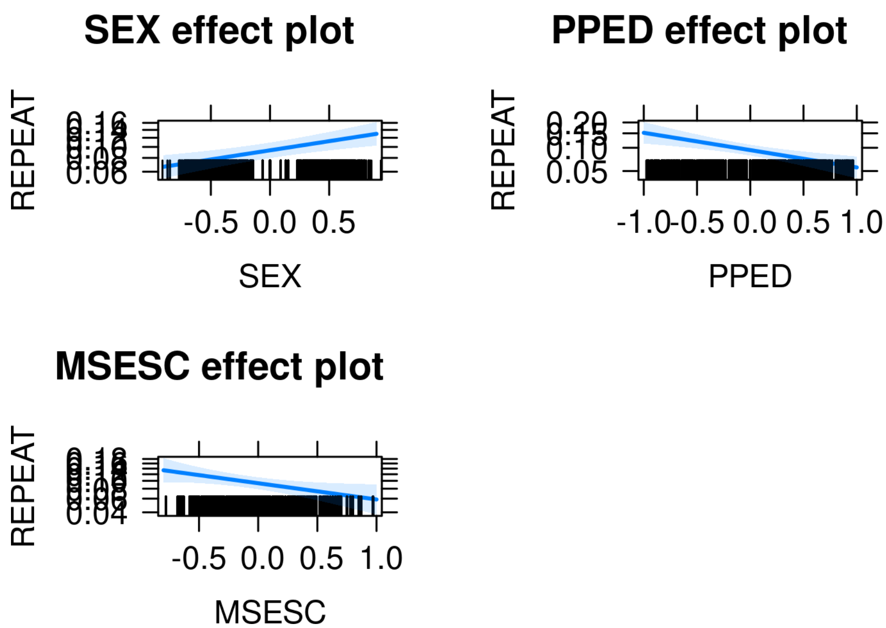
Grouping Variables		
Group	# groups	ICC
SCHOOLID	356	0.34

- to visualize the effects of the parameter estimates.

Note that because the first-level categorical variables

- (`SEX` and `PPED`) are centered,
- they are treated as continuous variables in the model
- and as well in the following effect plots.

```
plot(allEffects(Model_Multi_Full))
```



In addition to the fixed-effect terms,

- let's also look at the random effect terms.

From the `ICC` value before,

- we know that it's necessary to include a random intercept.

However, the necessity of including

- random slopes for `SEX` and `PPED` is less clear.

To find this out, we can use

- the likelihood ratio test
  - and `AIC`
- to judge whether the inclusion of the random slope(s)
  - improves model fit.

```
# let's fit a less-than-full model that leaves out the random slope term of `SEX`
Model_Multi_Full_No_SEX <-
  glmer(
    REPEAT ~ SEX + PPED + MSESC + (1 + PPED | SCHOOLID),
    family = binomial(link = "logit"),
    data = ThaiEdu_Center,
    control = glmerControl(optimizer = "bobyqa",
                           optCtrl = list(maxfun = 2e5))
  )
```

```
## boundary (singular) fit: see help('isSingular')
```

boundary (singular) fit: see ?isSingular

```
# let's fit a less-than-full model that leaves out the random slope term of `PPED`
Model_Multi_Full_No_PPED <-
  glmer(
    REPEAT ~ SEX + PPED + MSESC + (1 + SEX | SCHOOLID),
    family = binomial(link = "logit"),
    data = ThaiEdu_Center,
```

```r
    control = glmerControl(optimizer = "bobyqa", optCtrl =
                             list(maxfun = 2e5))
  )

# let's fit a less-than-full model that leaves out the random slope terms of both `SEX` and `PPED`
Model_Multi_Full_No_Random_Slope <-
  glmer(
    REPEAT ~ SEX + PPED + MSESC +
      (1 | SCHOOLID),
    family = binomial(link = "logit"),
    data = ThaiEdu_Center,
    control = glmerControl(optimizer = "bobyqa",
                           optCtrl = list(maxfun = 2e5))
  )
```

Likelihood ratio test:

```r
# compare the full model with that model that excludes `SEX`
anova(Model_Multi_Full_No_SEX, Model_Multi_Full, test = "Chisq")
```

```
## Data: ThaiEdu_Center
## Models:
## Model_Multi_Full_No_SEX: REPEAT ~ SEX + PPED + MSESC + (1 + PPED | SCHOOLID)
## Model_Multi_Full: REPEAT ~ SEX + PPED + MSESC + (1 + SEX + PPED | SCHOOLID)
##                         npar    AIC    BIC  logLik deviance  Chisq Df
## Model_Multi_Full_No_SEX    7 5466.6 5515.1 -2726.3   5452.6
## Model_Multi_Full          10 5468.0 5537.2 -2724.0   5448.0 4.6054  3
##                         Pr(>Chisq)
## Model_Multi_Full_No_SEX
## Model_Multi_Full            0.2031
```

```r
# compare the full model with that model that excludes `PPED`
anova(Model_Multi_Full_No_PPED, Model_Multi_Full, test = "Chisq")
```

```
## Data: ThaiEdu_Center
## Models:
## Model_Multi_Full_No_PPED: REPEAT ~ SEX + PPED + MSESC + (1 + SEX | SCHOOLID)
## Model_Multi_Full: REPEAT ~ SEX + PPED + MSESC + (1 + SEX + PPED | SCHOOLID)
##                          npar    AIC    BIC  logLik deviance  Chisq Df
## Model_Multi_Full_No_PPED    7 5462.9 5511.4 -2724.4   5448.9
## Model_Multi_Full           10 5468.0 5537.2 -2724.0   5448.0 0.9052  3
##                          Pr(>Chisq)
## Model_Multi_Full_No_PPED
## Model_Multi_Full             0.8242
```

```r
anova(Model_Multi_Full_No_Random_Slope, Model_Multi_Full, test = "Chisq")
```

```
## Data: ThaiEdu_Center
## Models:
## Model_Multi_Full_No_Random_Slope: REPEAT ~ SEX + PPED + MSESC + (1 | SCHOOLID)
## Model_Multi_Full: REPEAT ~ SEX + PPED + MSESC + (1 + SEX + PPED | SCHOOLID)
##                                  npar    AIC    BIC  logLik deviance  Chisq Df
## Model_Multi_Full_No_Random_Slope    5 5463.2 5497.8 -2726.6   5453.2
## Model_Multi_Full                   10 5468.0 5537.2 -2724.0   5448.0 5.2249  5
##                                  Pr(>Chisq)
## Model_Multi_Full_No_Random_Slope
```

```
## Model_Multi_Full                        0.3891
```

From the all insignificant likelihood ratio test results

- (Pr(>Chisq) > 0.05),
- we can conclude that there is no significant improvement in model fit
  - by adding any random slope terms.

AIC:

```r
AIC(logLik(Model_Multi_Full)) #full model
```

```
## [1] 5467.985
```

```r
AIC(logLik(Model_Multi_Full_No_SEX)) #model without SEX
```

```
## [1] 5466.591
```

```r
AIC(logLik(Model_Multi_Full_No_PPED)) #model without PPED
```

```
## [1] 5462.89
```

```r
AIC(logLik(Model_Multi_Full_No_Random_Slope)) #model without random slopes
```

```
## [1] 5463.21
```

From the AIC results, we see that

- including random slope terms
  - either does not substantially improve AIC (indicated by lower AIC value)
  - or leads to worse AIC (i.e. higher).
- Therefore, we also conclude
  - there is no need to include the random effect term(s).

4.1.2.12 Other Family (Distribution) and Link Functions

- So far, we have introduced binary and binomial logistic regression,
  - both of which come from the binomial family with the logit link.

However, there are many more distribution families and link functions

- that we can use in `glm` analysis.

For instance, to model binary outcomes,

- we can also use the `probit` link
  - or the complementary log-log (`cloglog`)
- instead of the logit link.

To model count data,

- we can also use Poisson regression,
- which assumes that the outcome variable
  - comes from a Poisson distribution
  - and uses the logarithm as the link function.

For an overview of possible `glm` models,

- see the Wikipedia page for GLM.

4.1.2.13 Links

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