DSCI351-351m-451: Class 11a-p Sample Size, Effect Size and Stat Power

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10.2.3.1 Class Readings, Assignments, Syllabus Topics

10.2.3.1.1 Reading, Lab Exercises, SemProjects

- Readings:
  - For today: OIS 8
  - For next class: ISLR 1, 2.1, 2.2
- Laboratory Exercises:

    - LE 5: is due Thursday November 10th
    - LE 6: Will be posted Thursday November 10th
- Office Hours: (Class Canvas Calendar for Zoom Link)
    - Wednesday @ 4:00 PM to 5:00 PM, Will Oltjen
    - Saturday @ 3:00 PM to 4:00 PM, Kristen Hernandez
    - **Office Hours are on Zoom, and recorded**
- Semester Projects
    - DSCI 451 Students Biweekly Update #6 Due Friday Nov. 18th
    - DSCI 451 Students
        * Next **Report Out 3 is Due Frodau Mpvember 25th**
    - All DSCI 351/351M/451 Students:
        * **Peer Grading of Report Out #2 is Due November 8th**
    - Exams
        * Final: Monday December 19, 2022, 12:00PM - 3:00PM, Nord 356 or remote

10.2.3.2 Power Analysis, Sample Size, Effect Size

Power analysis allows you to

- determine the sample size required
    - to detect an effect of a given "effect size"
    - with a given degree of confidence.

Conversely, it allows you to determine

- the probability of detecting an effect
    - of a given effect size
- with a given level of confidence,
    - under sample size constraints.

If the probability is unacceptably low,

- you'd be wise to alter or abandon the experiment.

Lets see how to conduct power analyses

- for a variety of statistical tests,
    - including tests of proportions,
    - t-tests,
    - chi-square tests,
    - balanced one-way ANOVA,
    - tests of correlations,
    - and linear models.

Because power analysis applies to hypothesis testing situations,

- we'll start with a review of null hypothesis significance testing (NHST).

Then we'll review conducting power analyses within R,

- focusing primarily on the `pwr` package.

Finally, we'll consider other approaches to power analysis available with R.

10.2.3.2.1 Review of NHST

In statistical hypothesis testing,

- you specify a hypothesis about a population parameter
    - (your null hypothesis, or H0).

You then draw a sample from this population

- and calculate a statistic that's used
  - to make inferences about the population parameter.
- Assuming that the null hypothesis is true,
  - you calculate the probability of obtaining
  - the observed sample statistic or one more extreme.
- If the probability is sufficiently small,
  - you reject the null hypothesis in favor of its opposite
  - (referred to as the alternative or research hypothesis, H 1).

An example will clarify the process.

- Say you're interested in evaluating
  - the impact of cell phone use on driver reaction time.
- Your null hypothesis is Ho: $\mu_1 - \mu_2 = 0$,
  - where μ_1 is the mean response time for drivers using a cell phone
  - and μ_2 is the mean response time for drivers that are cell phone free
- (here, $\mu_1 - \mu_2$ is the population parameter of interest).

If you reject this null hypothesis,

- you're left with the alternate or research hypothesis,
  - namely H1: $\mu_1 - \mu_2 \neq 0$.
- This is equivalent to $\mu_1 \neq \mu_2$,
  - that the mean reaction times for the two conditions are not equal.

A sample of individuals

- is selected
  - and randomly assigned to one of two conditions.
- In the first condition,
  - participants react to a series of driving challenges in a simulator
  - while talking on a cell phone.
- In the second condition, participants complete the same series of challenges
  - but without a cell phone.
- Overall reaction time is assessed for each individual.

Based on the sample data, you can calculate the statistic

- $(\bar{X}_1 - \bar{X}_2/(s/\sqrt{n}))$
  - where $(\bar{X}_1$ and $\bar{X}_2$
  - are the sample reaction time means in the two conditions,
- s is the pooled sample standard deviation,
- and n is the number of participants in each condition.

If the null hypothesis is true

- and you can assume that reaction times are normally distributed,
- this sample statistic will follow
  - a t distribution
  - with $2n$–2 degrees of freedom.

Using this fact, you can calculate

- the probability of obtaining a sample statistic this large or larger.

If the probability (p) is smaller than some predetermined cutoff

- (say $p < .05$),
- you reject the null hypothesis
  - in favor of the alternate hypothesis.

This predetermined cutoff (0.05)

- is called the significance level of the test.

Note that you use sample data

- to make an inference about the population it's drawn from.

Your null hypothesis is

- that the mean reaction time of all drivers talking on cell phones
  - isn't different from
- the mean reaction time of all drivers who aren't talking on cell phones,
  - not just those drivers in your sample.

The four possible outcomes from your decision are as follows:

- 1. If the null hypothesis is false
  - and the statistical test leads you to reject it,
  - you've made a correct decision.
  - You've correctly determined that reaction time
  - is affected by cell phone use.
- 2. If the null hypothesis is true and you don't reject it,
  - again you've made a correct decision.
  - Reaction time isn't affected by cell phone use.
- 3. If the null hypothesis is true but you reject it,
  - you've committed a Type I error.
  - You've concluded that cell phone use affects reaction time
  - when it doesn't.
- 4. If the null hypothesis is false and you fail to reject it,
  - you've committed a Type II error.
  - Cell phone use affects reaction time,
  - but you've failed to discern this.

10.2.3.2.2 Controversy surrounding null hypothesis significance testing

Null hypothesis significance testing

- isn't without controversy
  - and detractors have raised numerous concerns about the approach,
  - particularly as practiced in the field of psychology.
- They point to
  - a widespread misunderstanding of p values,
  - reliance on statistical significance over practical significance,
  - the fact that the null hypothesis is never exactly true
  - and will always be rejected for sufficient sample sizes,
  - and a number of logical inconsistencies in NHST practices.

An interesting discussion is in 3-readings/2-articles

- [1]Bruce Thompson, Statistical Significance and Effect Size Reporting: Portrait of a Possible Future, Research in the Schools. 5 (1998) 33–38.
- [2]A. Gelman, E. Loken, The statistical crisis in science: data-dependent analysis–a" garden of forking paths"–explains why many statistically significant comparisons don't hold up, American Scientist. 102 (2014) 460–466. https://doi.org/10.1511/2014.111.460.

- [3]A. Gelman, The Problems With P-Values are not Just With P-Values, The American Statistician. (2016) 2.
- [4]B.B. McShane, D. Gal, A. Gelman, C. Robert, J.L. Tackett, Abandon Statistical Significance, The American Statistician. 73 (2019) 235–245. https://doi.org/10.1080/00031305.2018.1527253.

Researchers typically pays special attention to four quantities

- **Sample size** refers to the number of observations
  - in each condition/group of the experimental design or study protocol
- The **significance level** (also referred to as alpha)
  - defined as the probability of making a Type I error.
  - The significance level can also be thought of as
  - the probability of finding an effect that is not there.
- **Statistical Power** defined as
  - one minus the probability of making a Type II error.
  - Power can be thought of as
  - the probability of finding an effect that is there.
- **Effect size** is the magnitude of the effect
  - under the alternate or research hypothesis.
  - The formula for effect size depends on
  - the statistical methodology employed in the hypothesis testing.

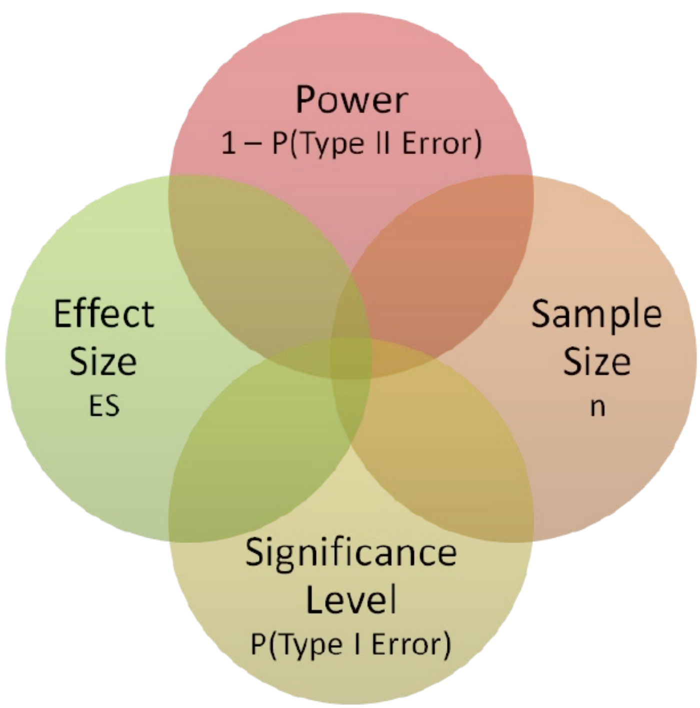


Figure 10.1. Four primary quantities considered in a study design power analysis. Given any three, you can calculate the fourth.

Figure 1: Fig. 10.1

Although the sample size and significance level

- are under the direct control of the researcher,
  - power and effect size are affected more indirectly.

- For example, as you relax the significance level
  - (in other words, make it easier to reject the null hypothesis),
  - power increases.
- Similarly, increasing the sample size increases power.

Your research goal is typically

- to maximize the power of your statistical tests
  - while maintaining an acceptable significance level
  - and employing as small a sample size as possible.
- That is, you want to maximize the chances of finding a real effect
  - and minimize the chances of finding an effect that isn't really there,
  - while keeping study costs within reason.

The four quantities

- **sample size, significance level, power, and effect size**
  - have an intimate relationship.
- Given any three,
  - you can determine the fourth.

You'll use this fact to carry out various power analyses

- as we move forward,
- we'll look at ways of implementing power analyses
  - using the R package pwr.
- And we'll briefly look at some highly specialized power functions
  - that are used in biology and genetics.

10.2.3.3 Implementing power analysis with the pwr package

The pwr package implements power analysis as outlined by Cohen (1988).

Some of the more important functions are listed in table 10.1.

For each function,

- the user can specify three of the four quantities
  - (sample size, significance level, power, effect size),
- and the fourth will be calculated.

Of the four quantities, effect size is often the most difficult to specify.

Calculating effect size typically requires

- some experience with the measures involved
  - and knowledge of past research.
- But what can you do if you have no clue
  - what effect size to expect in a given study?
- You'll look at this difficult question in the next section.

Here we'll apply `pwr` package functions

- to common statistical tests.

10.2.3.3.1 t-tests

When the statistical test to be used is a t-test,

- the pwr.t.test() function
  - provides a number of useful power analysis options.

Table 10.1 pwr package functions

Function	Power calculations for ...
pwr.2p.test	Two proportions (equal n)
pwr.2p2n.test	Two proportions (unequal n)
pwr.anova.test	Balanced one way ANOVA
pwr.chisq.test	Chi-square test
pwr.f2.test	General linear model
pwr.p.test	Proportion (one sample)
pwr.r.test	Correlation
pwr.t.test	t-tests (one sample, two samples, paired)
pwr.t2n.test	t-test (two samples with unequal n)

Figure 2: table 10.1

```
pwr.t.test(n=, d=, sig.level=, power=, alternative=)
```

where

- `n` is the sample size.
- `d` is the effect size defined as the standardized mean difference.

$$d = \frac{|\mu_1 - \mu_2|}{\sigma} \qquad \begin{aligned} \mu_1 &= \text{mean of group 1} \\ \mu_2 &= \text{mean of group 2} \\ \sigma^2 &= \text{common error variance} \end{aligned}$$

- `sig.level` is the significance level (0.05 is the default).
- `power` is the power level.
- `type` is two-sample t-test (`"two.sample"`), a one-sample t-test (`"one.sample"`), or a dependent sample t-test (`"paired"`) . A two-sample test is the default.
- `alternative` indicates whether the statistical test is two-sided (`"two.sided"`) or one-sided (`"less"` or `"greater"`). A two-sided test is the default.

Figure 3: t-test

The format is

Let's work through an example.

- Continuing the experiment from section 10.1
  - involving cell phone use and driving reaction time,
- assume that you'll be using a two-tailed independent sample t-test
  - to compare the mean reaction time
  - for participants in the cell phone condition
- with the mean reaction time for participants driving unencumbered.

Let's assume that you know from past experience

- that reaction time has a standard deviation of 1.25 seconds.
- Also suppose that a 1-second difference in reaction time
  - is considered an important difference.
- You'd therefore like to conduct a study
  - in which you're able to detect
  - an effect size of d = 1/1.25 = 0.8 or larger.
- Additionally, you want to be 90% sure
  - to detect such a difference if it exists,
- and 95% sure that you won't declare a difference
  - to be significant when it's actually due to random variability.
- How many participants will you need in your study?

Entering this information in the `pwr.t.test()` function, you have the following:

```
# t-tests
library(pwr)
pwr.t.test(
  d = .8,
  sig.level = .05,
  power = .9,
  type = "two.sample",
  alternative = "two.sided"
)
```

```
##
##      Two-sample t test power calculation
##
##               n = 33.82555
##               d = 0.8
##       sig.level = 0.05
##           power = 0.9
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

The results suggest that

- you need 34 participants in each group
  - (for a total of 68 participants)
- in order to detect an effect size of 0.8
  - with 90% certainty
  - and no more than a 5% chance of erroneously concluding
- that a difference exists
  - when, in fact, it doesn't.

Let's alter the question. Assume that in comparing the two conditions

- you want to be able to detect
  - a 0.5 standard deviation difference in population means.
- You want to limit the chances of falsely declaring
  - the population means to be different to 1 out of 100.
- Additionally, you can only afford
  - to include 40 participants in the study.
- What's the probability that you'll be able to detect
  - a difference between the population means that's this large,
  - given the constraints outlined?

Assuming that an equal number of participants

- will be placed in each condition, you have

```
pwr.t.test(
  n = 20,
  d = .5,
  sig.level = .01,
  type = "two.sample",
  alternative = "two.sided"
)
```

```
##
##      Two-sample t test power calculation
##
##               n = 20
##               d = 0.5
##       sig.level = 0.01
##           power = 0.1439551
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

10.2.3.3.2 ANOVA

You can also use the `pwr` package for ANOVA problems

```
pwr.anova.test(
  k = 5,
  f = .25,
  sig.level = .05,
  power = .8
)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##               k = 5
##               n = 39.1534
##               f = 0.25
##       sig.level = 0.05
##           power = 0.8
##
## NOTE: n is number in each group
```

10.2.3.3.3 Correlations

The `pwr.r.test()` function

- provides a power analysis for tests of correlation coefficients.

The format is as follows

- `pwr.r.test(n=, r=, sig.level=, power=, alternative=)`
- where
  - `n` is the number of observations,
  - `r` is the effect size (as measured by a linear correlation coefficient),
  - `sig.level` is the significance level,
  - `power` is the power level,
- and `alternative`
  - specifies a two-sided (`two.sided`)
  - or a one-sided (`less` or `greater`)
  - significance test.

For example, let's assume that you're studying

- the relationship between depression and loneliness.

Your null and research hypotheses are

- H0:   0.25
- versus H1: > 0.25
  - where is the population correlation
  - between these two psychological variables.

You've set your significance level to 0.05,

- and you want to be 90% confident
  - that you'll reject H0 if it's false.
- How many observations will you need?

This code provides the answer:

```
# correlations
pwr.r.test(
  r = .25,
  sig.level = .05,
  power = .90,
  alternative = "greater"
)
```

```
##
##      approximate correlation power calculation (arctangh transformation)
##
##               n = 133.2803
##               r = 0.25
##       sig.level = 0.05
##           power = 0.9
##     alternative = greater
```

Thus, you need to assess depression and loneliness

- in 134 participants
- in order to be 90% confident
  - that you'll reject the null hypothesis
  - if it's false.

10.2.3.3.4 Linear Models

For linear models (such as multiple regression),

- the `pwr.f2.test()` function can be used
  - to carry out a power analysis.
- The format is
  - `pwr.f2.test(u=, v=, f2=, sig.level=, power=)`
- where `u` and `v` are
  - the numerator and denominator degrees of freedom
- and `f2` is the effect size.

$$f^2 = \frac{R^2}{1 - R^2} \quad \text{where} \quad R^2 = \text{population multiple correlation}$$

$$f^2 = \frac{R_{AB}^2 - R_A^2}{1 - R_{AB}^2} \quad \text{where} \quad R_A^2 = \text{variance accounted for in the population by variable set A}$$

$$\mathrm{R}_{AB}^2 = \text{variance accounted for in the population by variable set A and B together}$$

The first formula for `f2`

- is appropriate when you're evaluating
  - the impact of a set of predictors on an outcome.
- The second formula is appropriate
  - when you're evaluating the impact of one set of predictors
  - above and beyond a second set of predictors (or covariates).

Let's say you're interested in whether a boss's leadership style

- impacts workers' satisfaction
  - above and beyond the salary and perks associated with the job.
- Leadership style is assessed by four variables,
  - and salary and perks are associated with three variables.
- Past experience suggests that salary and perks
  - account for roughly 30% of the variance in worker satisfaction.

From a practical standpoint,

- it would be interesting if leadership style
  - accounted for at least 5% above this figure.
- Assuming a significance level of 0.05,
  - how many subjects would be needed
  - to identify such a contribution with 90% confidence?

In this example

- `sig.level=0.05`
- `power=0.90`
- `u=3`
  - (total number of predictors minus the number of predictors in set B),
- and the effect size is
  - `f2` = (.35 – .30)/(1 – .35) = 0.0769.

Entering this into the function yields the following:

```
# linear models
pwr.f2.test(
  u = 3,
  f2 = 0.0769,
  sig.level = 0.05,
  power = 0.90
)
```

```
##
##      Multiple regression power calculation
##
##               u = 3
##               v = 184.2426
##              f2 = 0.0769
##       sig.level = 0.05
##           power = 0.9
```

10.2.3.3.5 Tests of Propotions

The `pwr.2p.test()` function

- can be used to perform a power analysis
  - when comparing two proportions.

The format is `pwr.2p.test(h=, n=, sig.level=, power=)`

- where `h` is the effect size
- and `n` is the common sample size in each group.

The effect size h is defined as

$$h = 2\arcsin\left(\sqrt{p_1}\right) - 2\arcsin\left(\sqrt{p_2}\right)$$

Figure 4: h

and can be calculated with the function `ES.h(p1, p2)`.

For unequal `ns`,

- the desired function is
- `pwr.2p2n.test(h =, n1 =, n2 =, sig.level=, power=)`

The `alternative= option`

- can be used to specify
  - a two-tailed (`two.sided`)
  - or one-tailed (`less` or `greater`) test.
- A two-tailed test is the default.

Let's say that you suspect

- that a popular medication relieves symptoms in 60% of users.
- A new (and more expensive) medication will be marketed
  - if it improves symptoms in 65% of users.
- How many participants will you need to include in a study
  - comparing these two medications

    - if you want to detect a difference this large?

Assume that you want to be 90% confident in a conclusion

- that the new drug is better
    - and 95% confident that you won't reach this conclusion erroneously.
- You'll use a one-tailed test
    - because you're only interested in assessing
    - whether the new drug is better than the standard.

```
# tests of proportions
pwr.2p.test(
  h = ES.h(.65, .6),
  sig.level = .05,
  power = .9,
  alternative = "greater"
)
```

```
##
##      Difference of proportion power calculation for binomial distribution (arcsine transformation)
##
##               h = 0.1033347
##               n = 1604.007
##       sig.level = 0.05
##           power = 0.9
##     alternative = greater
##
## NOTE: same sample sizes
```

Based on these results,

- you'll need to conduct a study with 1,605 individuals
    - receiving the new drug
- and 1,605 receiving the existing drug
- in order to meet the criteria.

10.2.3.3.6 χ^2 tests

χ^2 tests are often used

- to assess the relationship between two categorical variables.

The null hypothesis is typically

- that the variables are independent
    - versus a research hypothesis that they aren't.
- The `pwr.chisq.test()` function can be used
    - to evaluate the power, effect size, or requisite sample size
    - when employing a χ^2 test.

The format is

- `pwr.chisq.test(w =, N = , df = , sig.level =, power = )`
- where `w` is the effect size,
- `N` is the total sample size,
- and `df` is the degrees of freedom.

Here, effect size w is defined as

The summation goes from `1 to m`,

- where `m` is the number of cells in the contingency table.

$$w = \sqrt{\sum_{i=1}^{m} \frac{\left(p0_i - p1_i\right)^2}{p0_i}} \quad \text{where} \quad \begin{array}{l} p0_i = \text{cell probability in the ith cell under } H_0 \\ p1_i = \text{cell probability in the ith cell under } H_1 \end{array}$$

Figure 5: effect size for chi-squared test

The function `ES.w2(P)` can be used

- to calculate the effect size
  - corresponding to the alternative hypothesis
  - in a two-way contingency table.
- Here, `P` is a hypothesized two-way probability table.

As a simple example, let's assume that you're looking at

- the relationship between ethnicity and promotion.
- You anticipate that
  - 70% of your sample will be Caucasian,
  - 10% will be African American,
  - and 20% will be Hispanic.
- Further, you believe that
  - 60% of Caucasians tend to be promoted,
  - compared with 30% for African Americans
  - and 50% for Hispanics.

Your research hypothesis is

- that the probability of promotion
- follows the values in this table.

Table 10.2 Proportion of individuals expected to be promoted

Ethnicity	Promoted	Not Promoted
Caucasian	0.42	0.28
African American	0.03	0.07
Hispanic	0.10	0.10

Figure 6: promoted

For example, you expect that

- 42% of the population will be promoted Caucasians
  - ($.42 = .70 \times .60$)
- and 7% of the population will be non-promoted African Americans
  - ($.07 = .10 \times .70$).

Let's assume a significance level of 0.05

- and that the desired power level is 0.90.

The degrees of freedom in a two-way contingency table

- are (r – 1) × (c – 1),
  - where r is the number of rows
  - and c is the number of columns.

You can calculate the hypothesized effect size as follows

```
# chi-square tests
prob <- matrix(c(.42, .28, .03, .07, .10, .10),
               byrow = TRUE,
               nrow = 3)
ES.w2(prob)
```

```
## [1] 0.1853198
```

Using this information, you can calculate the necessary sample size by

```
pwr.chisq.test(
  w = .1853,
  df = 2,
  sig.level = .05,
  power = .9
)
```

```
##
##      Chi squared power calculation
##
##               w = 0.1853
##               N = 368.5317
##              df = 2
##       sig.level = 0.05
##           power = 0.9
##
## NOTE: N is the number of observations
```

The results suggest that

- a study with 369 participants
  - will be adequate to detect
  - a relationship between ethnicity and promotion
- given the
  - effect size,
  - power,
  - and significance level specified.

```

# Sample sizes for detecting significant effects in a one-way ANOVA
library(pwr)
es <-
  seq(.1, .5, .01) # Sets the range of correlations and power values
nes <- length(es)

samsize <-
  NULL
for (i in 1:nes) {  # Obtains sample sizes

  result <- pwr.anova.test(
    k = 5,
```

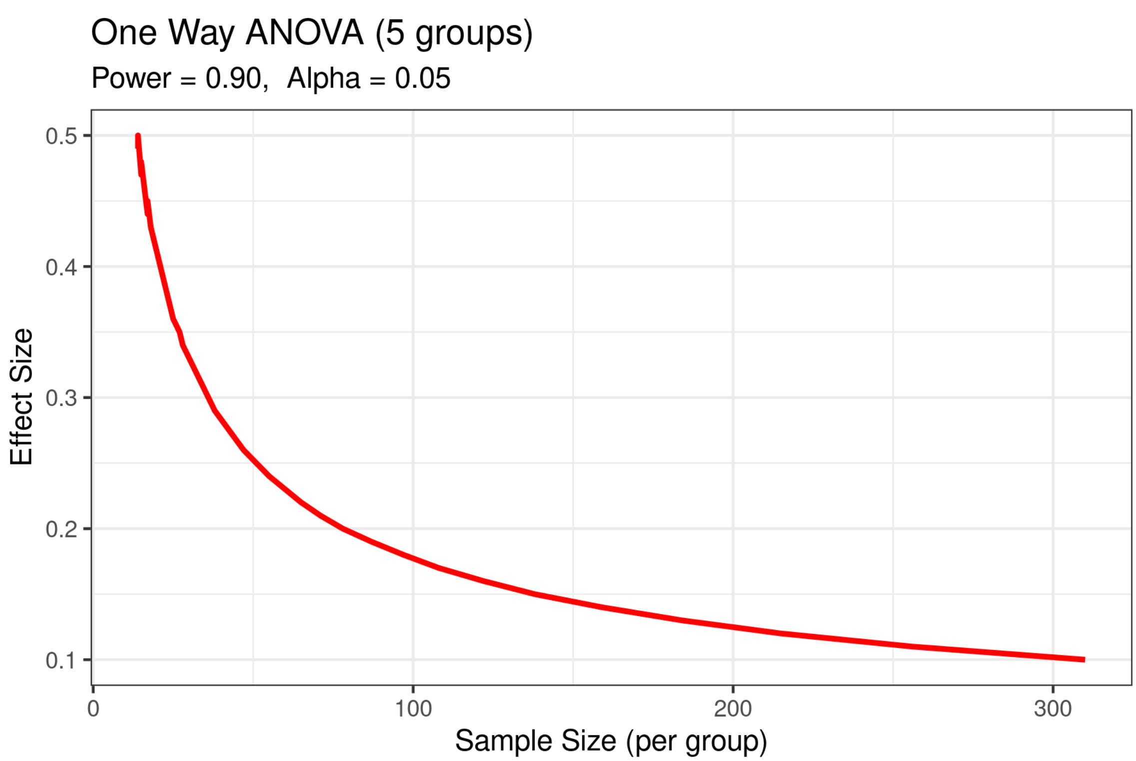
```
    f = es[i],
    sig.level = .05,
    power = .9
  )
  samsize[i] <- ceiling(result$n)
}

plotdata <- data.frame(es, samsize)

library(ggplot2)  # plot power curves
ggplot(plotdata, aes(x = samsize, y = es)) +
  geom_line(color = "red", size = 1) +
  theme_bw() +
  labs(
    title = "One Way ANOVA (5 groups)",
    subtitle = "Power = 0.90,  Alpha = 0.05",
    x = "Sample Size (per group)",
    y = "Effect Size"
  )
```

10.2.3.3.7 Samples Sizes in Anova Analysis

```
## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use `linewidth` instead.
```



10.2.3.4 Creating Power Analysis Plots

Let's look at a more involved graphing example.

Suppose you'd like to see the sample size necessary

- to declare a correlation coefficient statistically significant
- for a range of
  - effect sizes
  - and power levels.

You can use the `pwr.r.test()` function

- and for loops to accomplish this task

Here we use the `seq()` function

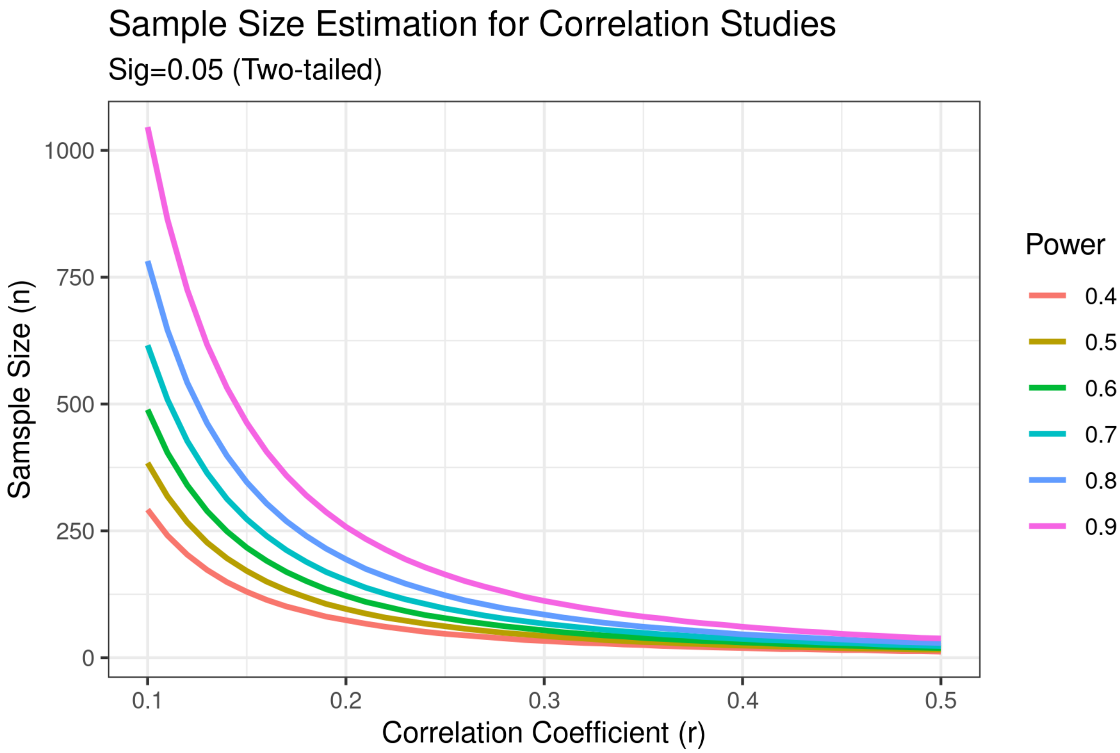
- to generate a range of effect sizes `r`
  - (correlation coefficients under H 1)
- and power levels `p`.
- The `expand.grid()` function is used
  - to create a data frame
  - with every combination of these two variables.
- A for loop then cycles through rows of the data frame,
  - calculating the sample size (`n`)
  - for that row's correlation and power level,
  - and saving the result.
- The ggplot2 package is then used
  - to plot a sample size vs. correlation curve for each power level.

```
# Sample size curves for detecting correlations of various sizes
library(pwr)
r <- seq(.1, .5, .01) #  Sets the range of correlations and power values
p <- seq(.4, .9, .1)

df <- expand.grid(r, p)
colnames(df) <- c("r", "p")

for (i in 1:nrow(df)) { # obtain sample sizes
  result <- pwr.r.test(
    r = df$r[i],
    sig.level = .05,
    power = df$p[i],
    alternative = "two.sided"
  )
  df$n[i] <- ceiling(result$n)
}

library(ggplot2) # plot power curves
ggplot(data = df,
       aes(x = r, y = n, color = factor(p))) +
  geom_line(size = 1) +
  theme_bw() +
  labs(
    title = "Sample Size Estimation for Correlation Studies",
    subtitle = "Sig=0.05 (Two-tailed)",
    x = "Correlation Coefficient (r)",
    y = "Samsple Size (n)",
    color = "Power"
  )
```



As you can see from the graph,

- you'd need a sample size of approximately 75
  - to detect a correlation of 0.20
  - with 40% confidence.
- You'd need approximately 185 additional observations
  - (n = 260)
  - to detect the same correlation with 90% confidence.
- With simple modifications, the same approach can be used
  - to create sample size and power curve graphs
  - for a wide range of statistical tests.

10.2.3.5 Links

Robert I. Kabacoff, R in Action, 3rd Edition, Manning Publications