Investment Society Session: Option Pricing and Trading Strategies

Michael Ellington

University of Liverpool Management School

A Brief Introduction

Calls and Puts

European Call Options:

- Provide the right, but not the obligation, to buy an asset at a predetermined price on a specific date.

European Put Options:

- Provide the right, but not the obligation, to sell an asset at a predetermined price on a specific data.

Calls and Puts II

Call	
Buyer	**Seller**
Right to buy asset at price X at date T	If call buyer exercises, must sell asset at X to call buyer at date T
Pays premium, $-C$	Receives premium, $+C$
Put	
Buyer	**Seller**
Right to sell asset at price X at date T	If put buyer exercises, must buy asset at X from put buyer at date T
Pays premium, $-P$	Receives premium, $+P$

Profit Calculations

- Long Call:

$$\Pi = \max(S_T - X, 0) - C$$

- Long Put:

$$\Pi = \max(X - S_T, 0) - P$$

- Short Call:

$$\Pi = C - \max(S_T - X, 0)$$

- Long Put:

$$\Pi = P - \max(X - S_T, 0)$$

Options 'Moneyness'

- The intrinsic value of a call option is $\max(S_T - X, 0)$.
- The intrinsic value of a put option is $\max(X - S_T, 0)$.
- We can classify options in terms of their 'moneyness'.
  1. **In the money:** When a call option's exercise price, X, is less than the underlying price. When a put option's exercise price, X, is greater than the underlying asset price.
  2. **At the money:** When an option's exercise price, X, is equal to the underlying asset price.
  3. **Out of the money:** When a call option's exercise price, X, is above the underlying asset price. When a put option's exercise price, X, is less than the underlying asset price.

Trading Strategies

Popular Strategies

- **Long Call Spread:** Buy call at strike A and sell call at strike B, with B $>$ A.
- **Long Put Spread:** Buy a put at strike B and sell put at strike A, with B $>$ A.
- **Long Straddle:** Buy 1 ATM put and 1 ATM Call (i.e. same strike price)
- **Short Butterfly:** Sell put (or call) at strike A, buy 2 puts (or calls) at strike B, sell put or call at strike C, with A $<$ B $<$ C.

Valuing Calls and Puts

How do we get C and P?

- To price options, we use the Black Scholes Option Pricing Model. To price a European Call the equation is:

$$
\begin{aligned}
C &= SN(d_1) - Xe^{-R_f T} N(d_2) \\
d_1 &= \frac{\ln(S/X) + (R_f + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

C=call premium, S=underlying asset price, X=exercise price, R_f= risk-free rate, T=time to expiration in years, $N(\cdot)$=cumulative Normal probability, and σ=annualised stock return volatility.

Black Scholes Option Pricing Model

- We are given the following data and are required to calculate the price of a 6 month call option on Blackrock PLC's stock:
- The underlying asset price is \$38, the exercise price of the call is \$42, the risk free rate is 1.5% per annum and the stock return volatility is 33% per annum.

Table: Data on Blackrock PLC

S	\$38.00
X	\$42.00
R_f	1.50%
σ	33%
T	0.5

Blackrock PLC

- Given our data, let's break down the calculation into steps:
  1. Find d_1, d_2, then $N(d_1)$, $N(d_2)$.
  2. Calculate $SN(d_1)$, $Xe^{-R_f T}N(d_2)$.
  3. Apply the formula.
- Step 1:

$$
\begin{aligned}
d_1 &= \frac{\ln(S/X) + (R_f + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
&= \frac{-0.10 + 0.035}{0.233} = -0.2789 \approx -0.28 \\
d_2 &= d_1 - \sigma\sqrt{T} \\
d_2 &= -0.28 - 0.233 = -0.513 \approx -0.51
\end{aligned}
$$

UNIVERSITY OF LIVERPOOL

Blackrock PLC

- Step 1 cont: using the **NORMSDIST()** (or **NORM.DIST()**) function in Excel, we can find $N(d_1)$, $N(d_2)$. Specifically, $N(d_1) = N(-0.28) = 0.3897$, and $N(d_2) = N(-0.51) = 0.305$.
- Step 2:

$$\begin{aligned} SN(d_1) &= 38(0.3897) = 14.8086 \\ Xe^{-R_f T} N(d_2) &= 42e^{-0.015(0.5)}(0.305) = 12.7142 \end{aligned}$$

- Step 3:

$$\begin{aligned} C &= SN(d_1) - Xe^{-R_f T} N(d_2) \\ C &= 14.8086 - 12.7142 = 2.0944 \approx \$2.09 \end{aligned}$$

Put-Call-Parity

- We can value the price of a put with an equivalent X using put call parity:

$$P = C - S + Xe^{-R_f T}$$

Recall:

$$C = SN(d_1) - Xe^{-R_f T} N(d_2)$$

Plugging into put call parity we have:

$$\begin{aligned} P &= SN(d_1) - Xe^{-R_f T} N(d_2) - S + Xe^{-R_f T} \\ P &= -S(1 - N(d_1)) + Xe^{-R_f T}(1 - N(d_2)) \end{aligned}$$

UNIVERSITY OF LIVERPOOL

Put-Call-Parity

- Following our example using Blackrock:

$$
\begin{aligned}
P &= C - S + Xe^{-R_f T} \\
P &= 2.09 - 38 + 42e^{-0.015(0.5)} \\
P &= 2.09 - 38 + 41.686 \\
P &= 5.776 \approx \$5.78
\end{aligned}
$$

- Or we could use our put formula:

$$
\begin{aligned}
P &= -S(1 - N(d_1)) + Xe^{-R_f T}(1 - N(d_2)) \\
P &= -38(1 - 0.3897) + 42e^{-0.015(0.5)}(1 - 0.305) \\
P &= -23.1914 + 28.9718 \\
P &= 5.7804 \approx \$5.78
\end{aligned}
$$

Class Task I

Dolan's stock price is currently £55. You believe that over the next 3 months, the stock price will be extremely volatile, with annualised volatility at 20% per annum. Using a risk-free rate of 1.25% per annum construct a strategy that, if your expectation is correct, means you will profit from both upward and downward price fluctuations.

Class Task II

Required In Excel:

- Calculate the price of an at the money call and put
- Graph the profit profile of your strategy
- Compute the break-even points

$$
\begin{aligned}
C &= SN(d_1) - Xe^{-R_f T} N(d_2) \\
d_1 &= \frac{\ln(S/X) + (R_f + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T} \\
P &= C - S + Xe^{-R_f T}
\end{aligned}
$$