1. $2/N$, 37, $\sqrt{N}$, N, $N * \log(\log(N))$, $N * \log(N)$, $N * \log(N^2)$, $N * \log^2(N)$, $N^{1.5}$, N^2, N^3, N^N, $2^{N/2}$, 2^N

$N * \log(N) = \Theta(N * \log(N^2))$

2. .5 ms for input size of 100

a. Linear

Input size $1 = (.5)/100 = .005$ ms

$.005 \times 500 = 2.5$ ms

b. N * log(N)

Size $1 = .5 / N * \log(N)$

$= .5/(100 * \log(100))$

$= .0025$ ms

Size $500 = .0025 * (500 * \log(500)) =$

$.0025 * (1349.49) = 3.3737$ ms

c. Quadratic

Size $1 = .5/100^2 = .00005$ ms

Size $500 = .00005 * (500^2) = 12.5$ ms

d. <u>Cubic</u>

Size 1 $= .5/100^3 = .0000005$

Size 500 $= .0000005 * (500^3) = 62.5$ ms

3. .5ms for input size 100

$1\text{min} = 60{,}000\text{ms}$

a. Linear

$$C = 100/.5 = 200$$

$$200 * 60{,}000 = 12{,}000{,}000$$

$$N = 12{,}000{,}000$$

b. N * log(N)

$$C = ((100 * \log(100))/.5 = 400$$

$$400 * 60{,}000 = 24{,}000{,}000$$

$$N * \log(N) = 24{,}000{,}000$$

c. <u>Quadratic</u>

$$C = 100^2 / .5 = 20{,}000$$

$$20{,}000 * 60{,}000 = 1{,}200{,}000{,}000$$

$$N^2 = 1{,}200{,}000{,}000$$

$$N = 34641.016$$

d. <u>Cubic</u>

$$C = 100^3 / .5 = 2{,}000{,}000$$

$$2{,}000{,}000 * 60{,}000 = 1.2E11$$

$$N^3 = 1.2E11$$

$$N = 4932.4241$$

4.

```
sum = 0                 1
for i = 1 to n          n
  for j = 1 to i * i    n*n = n^2
    for k = 1 to j      n^2
      sum++             2
```

$$1 + \sum_{i=1}^{n}\left(\sum_{j=1}^{i^2}\left(\sum_{k=1}^{j}(2)\right)\right)$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i^2}\left(\sum_{k=1}^{j}(2)\right)\right)$$

$$\cancel{2}\sum_{i=1}^{n}\left(\sum_{j=1}^{i^2}\left(\sum_{k=1}^{j}(1)\right)\right)$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i^2}\left(\sum_{k=1}^{j}(1)\right)\right)$$

$$\sum_{k=1}^{j}(1) = 1 \cdot j = j$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i^2}(j)\right)$$

$$\sum_{k=1}^{n} = k = \frac{n(n+1)}{2} \quad \text{So,} \quad \sum_{j=1}^{i^2} j = \frac{i^2(i^2+1)}{2}$$

$$\sum_{i=1}^{n}\left(\frac{i^2(i^2+1)}{2}\right) = \sum_{i=1}^{n}\frac{1}{2}\left(i^2(i^2+1)\right)$$

$$= \frac{1}{2}\sum_{i=1}^{n} i^2(i^2+1)$$

$$= \frac{1}{2}\sum_{i=1}^{n} i^4 + i^2$$

$$= \frac{1}{2}\sum_{i=1}^{n} i^4 + \sum_{i=1}^{n} i^2$$

$$\frac{1}{2}\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\frac{1}{2}\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

According to the sum formula

$$\frac{1}{2}\cdot\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} + \frac{n(n+1)(2n+1)}{6}$$

$$\frac{1}{2}\cdot\frac{n(n+1)(2n+1)(3n^2+3n-1)+(5n(n+1)(2n+1))}{30}$$

$$\frac{1}{2}\cdot\frac{6n^5+15n^4+10n^3-n+5n(n+1)(2n+1)}{30}$$

$$\frac{6n^5+15n^4+10n^3-n+10n^3+15n^2+5n}{60}$$

$$\frac{6n^5 + 15n^4 + 20n^3 + 15n^2 + 4n}{60}$$

$$6n^5 + \cancel{15n^4} + \cancel{20n^3} + \cancel{15n^2} + \cancel{4n}$$

$$O(n^5)$$

$n = 2$

5.

```
sum = 0                      1
for i = 1 to n               n
  for j = 1 to i * i     }
    if (j mod i) == 0    }   i * i(i-1)/2
      for k = 1 to j         n^2
        sum++                2
```

$j \bmod i == 0$ when $j = 0, i, 2i, 3i$

$$1 + \sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} \left(i * \frac{i(i-1)}{2} \right) \right)$$

$$\frac{i(i-1)}{2} = i^2$$

$$\cancel{1} + \sum_{i=1}^{n} \left(i^3 \right) = \frac{n^2(n+1)^2}{4}$$

$$\frac{n^2(n^2+n+1)}{4}$$

$$\frac{n^4 + \cancel{n^3} + \cancel{n}}{4}$$

$$O(n^4)$$

6.

```
long long expo (long long x, unsigned int n)
{
  if (n == 0)
    return 1;
  if (n == 1)
    return x;
  if ((n % 2) == 0)
    return expo(x*x, n/2);
  else
    return expo(x*x, n/2) * x;
}
```

10^2 = 1 operation

10^3 = 2 operations

10^4 = 2 operations

10^5 = 3 operations

10^6 = 3 operations

10^7 = 4 operations

10^8 = 3 operations

15^9 = 4 operations

10^{14} = 5 operations

$14 = 1110$

$= (((10^2)10)^2 10)^2$

After starting from the second bit from the left, for every 1 in the bit representation of n, two multiplication operations occur. For every 0 in the bit representation of n, 1 multiplication occurs. The value of x doesn't matter for the number of multiplications.