# Analyzing Quantum Many-Body Systems with ITensor and PastaQ

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github.com/mtfishman/ITensorTutorials.jl

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► PhD from Caltech with John Preskill and Steve White (UCI) in 2018.



mtfishman.github.io











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- ► Develop PastaQ with Giacomo Torlai (AWS).



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- ► We are hiring postdocs, full-time scientists, part-time and full-time software developers, interns, etc.





### SIMONS FOUNDATION

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- Find out more at: julialang.org.



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  - ► Tensor network-based quantum state and process tomography.



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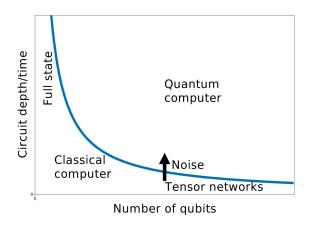


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  - Approximate circuit evolution and optimization with MPS/MPO, etc.
- ► Find out more: github.com/GTorlai/PastaQ.jl

▶ In my opinion, tensor networks are the best general purpose tool we have right now for simulating and analyzing quantum computers.



► Good for many sites or qubits but limited to shorter times/circuit depths (though better with noise).

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  - ► ITensor and PastaQ handle this seamlessly.
  - ► This is not the focus of ITensor and PastaQ at the moment, specialized libraries like Yao.jl may be faster.
- ➤ Tensor networks are a common, general language for reasoning about quantum many-body systems (for example, quantum circuits).

### What are tensor networks?



Matrix  $M_{ij}$ Order-2 tensor



Tensor  $T_{ijk}$ Order-3 tensor



### What are tensor networks?



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1. Download Julia: julialang.org/downloads

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- 2. Launch Julia:
  - 1 \$ julia

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- 2. Launch Julia:

```
1 $ julia
```

#### 3. Type the following commands:

```
julia > using Pkg
julia > Pkg.add("ITensors")
julia > Pkg.add("ITensors")
julia > Pkg.add("PastaQ")
julia > Pkg.add("PastaQ")
julia > Pkg.add("PastaQ")
```

Now you can use ITensors and PastaQ:

```
julia> using ITensors
julia> i = Index(2);
julia> A = ITensor(i);
```

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julia> using ITensors
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```

```
1  julia> using PastaQ
2
3  julia> gates = [("X", 1), ("CX", (1, 3))];
4
5  julia> ψ = runcircuit(gates);
```

```
1  using ITensors
2
3  i = Index(2)
4
5
```

#### Load ITensor

2-dimensional labeled Hilbert space (dim=2|id=510)

```
1     using ITensors
2
3     i = Index(2)
4
5
```



```
1     using ITensors
2
3     i = Index(2)
4
5
```

1 
$$Zp = ITensor(i)$$
  
2  $Zp[i=>1] = 1$ 

$$4 \quad Zp = ITensor([1, 0], i)$$

i

$$|Z+\rangle = \begin{bmatrix} 1\\0 \end{bmatrix}$$

Construct from a Vector

```
using ITensors
   i = Index(2)
5
   Zp = ITensor(i)
   Zp[i=>1] = 1
   Zp = ITensor([1, 0], i)
```

i

3 
$$Xp = ITensor([1, 1]/\sqrt{2}, i)$$

$$4 \quad Xm = \overline{ITensor}([1, -1]/\sqrt{2}, i)$$

$$\begin{split} |Z+\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad |Z-\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ |X+\rangle &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} / \sqrt{2}, \ |X-\rangle = \begin{bmatrix} 1 \\ -1 \end{bmatrix} / \sqrt{2} \end{split}$$

```
1 Zp = ITensor([1, 0], i)

2 Zm = ITensor([0, 1], i)

3 Xp = ITensor([1, 1]/\sqrt{2}, i)

4 Xm = ITensor([1, -1]/\sqrt{2}, i)
```

1 
$$Zp = ITensor([1, 0], i)$$
  
2  $Zm = ITensor([0, 1], i)$   
3  $Xp = ITensor([1, 1]/\sqrt{2}, i)$ 

 $Xm = \overline{ITensor}([1, -1]/\sqrt{2}, i)$ 

$$Xp = (Zp + Zm)/\sqrt{2}$$

$$dag(Zp) * Xp$$

$$inner(Zp, Xp)$$

$$|X+\rangle = (|Z+\rangle + |Z-\rangle)/\sqrt{2}$$

$$\langle Z + | X + \rangle \approx 1/\sqrt{2}$$

```
1 Zp = ITensor([1, 0], i)

2 Zm = ITensor([0, 1], i)

3 Xp = ITensor([1, 1]/\sqrt{2}, i)

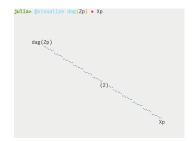
4 Xm = ITensor([1, -1]/\sqrt{2}, i)
```

$$\frac{1}{X+} = \frac{1}{Z+} + \frac{1}{Z-}$$

$$\sqrt{2}$$

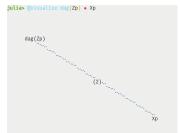
$$\overline{Z} + \overline{X} + = 1/\sqrt{2}$$

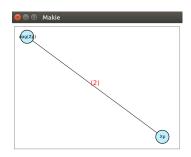
- 1 using ITensorUnicodePlots
- 2
  - @visualize dag(Zp) \* Xp



- 1 using ITensorUnicodePlots
- $\overline{3}$  @visualize dag(Zp) \* Xp

- 1 using ITensorGLMakie
- 2
  - @visualize dag(Zp) \* Xp





### Tutorial: One-site states

```
1 i = Index(2, "S=1/2")
2
3
4
5
6
7
8
```

```
(dim=2|id=25|"S=1/2")
```

"S=1/2" defines an operator basis

Additionally: "Qubit", "Qudit", "Electron", ...

#### Tutorial: One-site states

```
(dim=2|id=25|"S=1/2")

"S=1/2" defines an operator basis

Additionally:
"Qubit", "Qudit",
"Electron", . . .
```

```
1 Zp = state("Z+", i)

2 Zm = state("Z-", i)

3 Xp = state("X+", i)

4 Xm = state("X-", i)
```

```
1 ITensor([1 0], i)
2 ITensor([0 1], i)
3 ITensor([1 1]/√2, i)
4 ITensor([1 -1]/√2, i)
```

#### Tutorial: Custom one-site states

```
import ITensors: state

function state(
::StateName"iX-",
::SiteType"S=1/2"

return [im -im]/√2
end
```

Overload ITensors.jl behavior

Define a state with the name "iX-"

#### Tutorial: Custom one-site states

```
import ITensors: state

function state(
::StateName"iX-",
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)
return [im -im]/√2
end
```

```
1 iXm = state("iX-", i)
2
3
4 inner(Zm, iXm)
```

Overload ITensors.jl behavior

Define a state with the name "iX-"

$$|iX-\rangle = i|X-\rangle$$

$$\langle Z - | iX - \rangle = -i/\sqrt{2}$$

#### Tutorial: Custom one-site states

```
import ITensors: state

function state(
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return [im -im]/√2
end
```

```
1 iXm = state("iX-", i)
2
3
4 inner(Zm, iXm)
```

Overload ITensors.jl behavior

Define a state with the name "iX-"

$$iX+ = (iX+)*i$$

```
1 i = Index(2) (dim=2|id=837)

2 j = Index(2) (dim=2|id=839)

3 (dim=2|id=899)

4 i \neq j
```

```
 \begin{array}{ll} 1 & i = Index(2) \\ 2 & j = Index(2) \\ 3 & \\ 4 & i \neq j \end{array}
```

```
1 i = Index(2)

2 j = Index(2)

3 i \neq j
```

```
1  i = Index(2)
2
3  prime(i) == i'
4
5  i ≠ i'
6  noprime(i') == i
```

```
 \begin{array}{ll} 1 & i = Index(2) \\ 2 & j = Index(2) \\ 3 & \\ 4 & i \neq j \end{array}
```

```
1  i = Index(2)
2
3  prime(i) == i'
4
5  i ≠ i'
6  noprime(i') == i
```

$$\frac{\mathbf{i}}{\mathbf{i}}$$

$$\mathbf{rime}\left(\underline{\mathbf{i}}\right) = \underline{\mathbf{i}}'$$

$$\underline{\mathbf{i}} \quad \mathbf{1} \quad \underline{\mathbf{i}}'$$

- $1 \quad Z = ITensor(i', i)$
- Z[i'=>1, i=>1] = 1
- Z[i'=>2, i=>2] = -1

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- $1 \quad Z = ITensor(i', i)$
- 2 Z[i'=>1, i=>1] = 1
- Z[i'=>2, i=>2] = -1



```
1 Z = ITensor(i', i)
2 Z[i'=>1, i=>1] = 1
3 Z[i'=>2, i=>2] = -1
```

```
1 z = [
2 10
3 0-1
4 ]
5 Z = ITensor(z, i', i)
6
7
8 Z = op("Z", i)
```



Matrix representation Z

Convert to ITensor

Use predefined definition

```
 \begin{array}{lll} 1 & Z = op(\mbox{"}\mbox{Z"}, i) & Z \\ 2 & X = op(\mbox{"}\mbox{X"}, i) & X \\ 3 & & Zp = state(\mbox{"}\mbox{Z+"}, i) & |Z+\rangle \\ 5 & Zm = state(\mbox{"}\mbox{Z-"}, i) & |Z-\rangle \\ \end{array}
```

```
1 Z = op("Z", i)

2 X = op("X", i)

3 Zp = state("Z+", i)

5 Zm = state("Z-", i)
```

1 
$$Z = op("Z", i)$$
  
2  $X = op("X", i)$   
3  $Zp = state("Z+", i)$   
5  $Zm = state("Z-", i)$ 

$$X|Z+\rangle = |Z-\rangle$$

$$\begin{array}{ll} 1 & X*Zp == Zm' \\ 2 & \underset{}{\mathbf{noprime}}(X*Zp) == Zm \end{array}$$

$$i'$$
  $X$   $i$   $Z+ =  $i'$   $Z-$$ 

- $1 \quad \operatorname{dag}(\operatorname{Zm})' * X * \operatorname{Zp}$
- $2 \quad inner(Zm', X, Zp)$

$$\langle Z - |X|Z + \rangle \approx 1$$

- $1 \quad dag(Zm)' * X * Zp$
- 2 inner(Zm', X, Zp)



```
1 dag(Zm)' * X * Zp
2 inner(Zm', X, Zp)
```

$$Z - X Z + = 1$$

```
1 apply(X, Zp) ==
2 noprime(X * Zp)
3
4
5
6 inner(Zm, apply(X, Zp))
```

$$X|Z+\rangle$$

$$\langle Z - |X|Z + \rangle \approx 1$$

```
1 dag(Zm)' * X * Zp
2 inner(Zm', X, Zp)
```

```
1 apply(X, Zp) ==
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6 inner(Zm, apply(X, Zp))
```

$$Z - \frac{i'}{X} X = Z + = 1$$

apply 
$$\left(\frac{1}{X}, \frac{1}{X}, \frac{1}{X}, \frac{1}{X}\right)$$

$$= \operatorname{noprime}\left(\frac{1}{X}, \frac{1}{X}, \frac{1}{X}\right)$$

$$= \frac{1}{X}$$

```
import ITensors: op
    function op(
     ::OpName"iX",
    ::SiteType"S=1/2"
6
     return [
      0 \text{ im}
    im 0
10
```

Overload ITensors.jl behavior

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import ITensors: op
    function op(
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6
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10
```

$$\frac{i'}{X} = \frac{i'}{X} \times i$$

```
import ITensors: op
    function op(
     ::OpName"iX",
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    im 0
10
    end
```

$$\frac{\mathbf{i'} \quad \mathbf{iX} \quad \mathbf{i}}{\mathbf{iX} \quad \mathbf{i}} = \left( \frac{\mathbf{i'} \quad \mathbf{X} \quad \mathbf{i}}{\mathbf{X}} \right) * \mathbf{i}$$

```
1 op("iX", i)
```

```
import ITensors: op
   function op(
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6
     return [
      0 im
    im 0
10
```

$$\frac{\mathbf{i}' \mathbf{i} \mathbf{X} \cdot \mathbf{i}}{\mathbf{i}} = \left(\frac{\mathbf{i}' \mathbf{X} \cdot \mathbf{i}}{\mathbf{X}}\right) * \mathbf{i}$$

```
1 op("iX", i)
```

```
1 X * im
```

```
1 i1 = Index(2, "S=1/2")

2 i2 = Index(2, "S=1/2")

3 (dim=2|id=505|"S=1/2")

(dim=2|id=576|"S=1/2")

(dim=2|id=576|"S=1/2")

(dim=2|id=576|"S=1/2")

7 ZpZm = ITensor(i1, i2)

7 ZpZm[i1=>1, i2=>2] = 1 |Z+\rangle_1|Z-\rangle_2 = |Z+Z-\rangle
```

```
1 i1 = Index(2, "S=1/2")

2 i2 = Index(2, "S=1/2")

3 

4 i1 \neq i2

5 

6 ZpZm = ITensor(i1, i2)

7 ZpZm[i1=>1, i2=>2] = 1
```

```
1 i1 = Index(2, "S=1/2")

2 i2 = Index(2, "S=1/2")

3

4 i1 ≠ i2

5

6 ZpZm = ITensor(i1, i2)

7 ZpZm[i1=>1, i2=>2] = 1
```

$$\begin{aligned} |Z+\rangle_1 \\ |Z+\rangle_2 \\ |Z-\rangle_1 \\ |Z-\rangle_2 \\ |Z+Z-\rangle &= |Z+\rangle_1 |Z-\rangle_2 \end{aligned}$$

```
1 i1 = Index(2, "S=1/2")

2 i2 = Index(2, "S=1/2")

3

4 i1 ≠ i2

5

6 ZpZm = ITensor(i1, i2)

7 ZpZm[i1=>1, i2=>2] = 1
```

```
1 Zp1 = state("Z+", i1)

2 Zp2 = state("Z+", i2)

3 

4 Zm1 = state("Z-", i1)

5 Zm2 = state("Z-", i2)

6 

7 ZpZm = Zp1 * Zm2
```

$$\begin{array}{ccc} \underline{\mathbf{i}_1} & \underline{\mathbf{i}_2} & \mathbf{Z} + \\ \underline{\mathbf{i}_2} & \mathbf{Z} - \end{array}$$



1 
$$\psi = \text{ITensor}(i1, i2)$$
  
2  $\psi[i1=>1, i2=>2] = 1/\sqrt{2}$   
3  $\psi[i1=>2, i2=>1] = 1/\sqrt{2}$   
4  $\psi = (\text{Zp1} * \text{Zm2} + \text{Zm1} * \text{Zp2})/\sqrt{2}$ 

$$(|Z+\rangle|Z-\rangle + |Z-\rangle|Z+\rangle)/\sqrt{2}$$

From single-site states

```
1 \psi = \text{ITensor}(i1, i2)

2 \psi[i1=>1, i2=>2] = 1/\sqrt{2}

3 \psi[i1=>2, i2=>1] = 1/\sqrt{2}

4

5 \psi = (\text{Zp1} * \text{Zm2} + \text{Zm1} * \text{Zp2})/\sqrt{2}
```

$$\frac{1}{12} \psi = \frac{1}{12} \frac{Z + \frac{1}{12} Z - \frac{1}{12} Z + \frac$$

```
1 \psi = \text{ITensor}(i1, i2)

2 \psi[i1=>1, i2=>2] = 1/\sqrt{2}

3 \psi[i1=>2, i2=>1] = 1/\sqrt{2}

4 \psi = (\text{Zp1} * \text{Zm2} + \text{Zm1} * \text{Zp2})/\sqrt{2}
```

```
1 inner(ZpZm, \psi) == 1/\sqrt{2}

2 3

4 U, S, V = svd(ZmZp, i1)

5 diag(S) == [1, 0]

6 7 U, S, V = svd(\psi, i1)

8 diag(S) == [1/\sqrt{2}, 1/\sqrt{2}]
```

$$\frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \Psi = \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \mathbf{Z} - \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{2}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{3}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{4}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{5}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{5}}{\mathbf{i}$$

$$svd\left(\frac{i_1}{Z+Z-\frac{i_2}{2}}\right)$$

$$=\frac{i_1}{U}\frac{U}{S}\frac{V}{V}\frac{i_2}{0}$$

$$=\frac{1}{0}$$

```
1 \psi = \text{ITensor}(i1, i2)

2 \psi[i1=>1, i2=>2] = 1/\sqrt{2}

3 \psi[i1=>2, i2=>1] = 1/\sqrt{2}

4 \psi = (\text{Zp1} * \text{Zm2} + \text{Zm1} * \text{Zp2})/\sqrt{2}
```

```
inner(ZpZm, \psi) == 1/\sqrt{2}

U, S, V = svd(ZmZp, i1)

diag(S) == [1, 0]

U, S, V = svd(\psi, i1)

diag(S) == [1/\sqrt{2}, 1/\sqrt{2}]
```

$$\frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \Psi = \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \mathbf{Z} + \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \mathbf{Z} + \sqrt{2}$$

$$svd\left(\frac{i_{1}}{Z+Z-\frac{i_{2}}{2}}\right)$$

$$=\frac{i_{1}}{U} \underbrace{U} \underbrace{S}^{V} \underbrace{V}^{i_{2}}$$

$$\underbrace{U}^{U} \underbrace{S}^{V} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$svd\left(\frac{i_{1}}{U} \underbrace{V}^{i_{2}}\right)$$

$$=\frac{i_{1}}{U} \underbrace{U} \underbrace{S}^{V} \underbrace{V}^{i_{2}}$$

$$\underbrace{U}^{U} \underbrace{S}^{V} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

```
1 H = ITensor(i1', i2', i1, i2)
2 H[i1'=>2, i2'=>1,
```

$$[i1=>2, i2=>1]=-1$$

$$[11=>2, 12=>1]=-1$$

4 # ...

Make a Hamiltonian: Transverse field Ising (n = 2)

$$H = -\sum_{j=1}^{n-1} Z_j Z_{j+1} + h \sum_{j=1}^{n} X_j$$

```
\begin{array}{lll} 1 & H = ITensor(i1',\,i2',\,i1,\,i2) \\ 2 & H[i1'=>2,\,i2'=>1, \\ 3 & i1=>2,\,i2=>1] = -1 \\ 4 & \# \ \dots \end{array}
```

```
1 Id1 = op("Id", i1)
2 Z1 = op("Z", i1)
3 X1 = op("X", i1)
4 # ...
5
6 ZZ = Z1 * Z2
7 XI = X1 * Id2
8 IX = Id1 * X2
9
10 h = 0.5
11 H = -ZZ + h * (XI + IX)
```

Make a Hamiltonian: Transverse field Ising (n = 2)

$$H = -\sum_{j}^{n-1} Z_j Z_{j+1} + h \sum_{j}^{n} X_j$$

$$\frac{i_{1}}{i_{2}}, H \frac{i_{1}}{i_{2}} = -\frac{i_{1}}{i_{2}}, \frac{Z}{Z} \frac{i_{1}}{i_{2}} + h \frac{i_{1}}{i_{2}}, \frac{Z}{X} \frac{i_{1}}{i_{2}} + \frac{i_{1}}{i_{2}}, \frac{Z}{X} \frac{i_{1}}{i_{2}}$$

$$1 \quad \underline{inner}(Zp' * Zp', H, Zp * Zp)$$

$$\langle Z+Z+|H|Z+Z+\rangle$$

$$1 \quad \underline{inner}(Zp'*Zp', H, Zp*Zp)$$

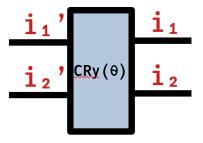
$$\begin{bmatrix}
 Z + \frac{i_1}{i_2} \\
 Z + \frac{i_2}{i_2}
 \end{bmatrix}
 H
 \begin{bmatrix}
 i_1 \\
 i_2
 \end{bmatrix}
 Z + = -1$$

$$\begin{array}{ll} 1 & D,\,U=\mathrm{eigen}(H) \\ 2 & \mathrm{diag}(D)==[-\sqrt{2},-1,\,1,\,\sqrt{2}] \end{array}$$

```
import ITensors: op
2
    function op(
     ::OpName"CRy",
     ::SiteType"S=1/2":
6
     c = \cos(\theta/2)
     s = \sin(\theta/2)
   return
10
11 1 0 0 0
12 0 1 0 0
  00c-s
13
14 	 0.0 s c
15
16
    end
```

```
Controlled-Ry (CRy) rotation gate CRy(\theta)
```

```
import ITensors: op
    function op(
     ::OpName"CRy".
     ::SiteType"S=1/2";
     c = \cos(\theta/2)
     s = \sin(\theta/2)
    return
10
11
    1000
12
   0\ 1\ 0\ 0
13
   0~0~\mathrm{c} -s
14 	 0.0 s c
15
16
    end
```



1 CH = op("CRy", i1, i2; 
$$\theta = \pi/2$$
)

Controlled-Hadamard gate  $CH = CRy(\theta = \pi/2)$ 

1 CH = op("CRy", i1, i2; 
$$\theta = \pi/2$$
)

$$\frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'}_{GRy(n/2)} \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} = \frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'}_{GH} \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}$$

# Tutorial: Custom two-site operators

1 CH = op("CRy", i1, i2; 
$$\theta = \pi/2$$
)

$$\frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}, \frac{\mathbf{i}_{1}}{\mathbf{cry}(\mathbf{n}/2)} = \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}, \mathbf{CH} = \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}$$

$$\begin{array}{ccc} 1 & \text{apply}(CH,\, Zp*Zm) == \\ 2 & Zp1*Xm2 \end{array}$$

$$\mathrm{CH}|\mathrm{Z+Z-}\rangle = |\mathrm{Z+X-}\rangle$$

# Tutorial: Custom two-site operators

1 CH = op("CRy", i1, i2; 
$$\theta = \pi/2$$
)

1 
$$apply(CH, Zp * Zm) ==$$

$$\begin{array}{ccc} & \text{apply}(\text{CH}, \text{Zp} * \text{ZH}) \\ & \text{Zp1} * \text{Xm2} \end{array}$$

$$\frac{1_1}{1_2}$$
,  $\frac{1_1}{1_2}$  =  $\frac{1_1}{1_2}$ , CH  $\frac{1_1}{1_2}$ 

$$\begin{array}{c} \underline{i_1}' \\ \underline{i_2}' \end{array}$$
 CH  $\begin{array}{c} \underline{i_1} \\ \underline{i_2} \end{array}$  Z+  $\begin{array}{c} \underline{i_1} \\ \underline{i_2} \end{array}$  X+

```
1 function E(\psi)

2 \psi H \psi = inner(\psi', H, \psi)

3 \psi \psi = inner(\psi, \psi)

4 return \psi H \psi / \psi \psi

5 end
```

$$\mathrm{E}(\psi) = \langle \psi | \mathrm{H} | \psi \rangle / \langle \psi | \psi \rangle$$

```
1 function E(\psi)

2 \psi H \psi = inner(\psi', H, \psi)

3 \psi \psi = inner(\psi, \psi)

4 return \psi H \psi / \psi \psi

5 end
```



```
1 function E(\psi)

2 \psi H \psi = inner(\psi', H, \psi)

3 \psi \psi = inner(\psi, \psi)

4 return \psi H \psi / \psi \psi

5 end
```

```
1 function minimize(f, \partial f, x;

2 nsteps, \gamma)

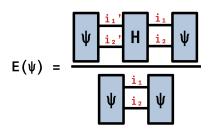
3 for n in 1:nsteps

4 x = x - \gamma * \partial f(x)

5 end

6 return x

7 end
```



Gradient descent.

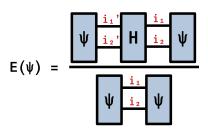
f(x): function to minimize.

 $\partial f(x)$ : gradient of f.

 $\gamma$ : step size.

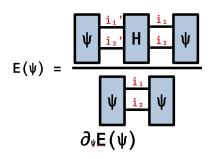
```
function E(\psi)
\psi H \psi = inner(\psi', H, \psi)
\psi \psi = inner(\psi, \psi)
return \psi H \psi / \psi \psi
send
```

```
function minimize(f, \partial f, x;
nsteps, \gamma)
for n in 1:nsteps
x = x - \gamma * \partial f(x)
end
return x
```



$$_{\psi}^{\mathsf{min}}\mathsf{E}(\psi)\qquad\partial_{\psi}\mathsf{E}(\psi)$$

```
1 \psi_0 = (\mathrm{Zp1} * \mathrm{Zm2} + \mathrm{Zm1} * \mathrm{Zp2})/\sqrt{2}
3 4 \mathrm{E}(\psi_0) == -1
5 6 using Zygote # autodiff
7 8 \partial \mathrm{E}(\psi) = \mathrm{gradient}(\mathrm{E}, \psi)[1]
9 \mathrm{norm}(\partial \mathrm{E}(\psi_0)) == 2
```



```
1 \psi_0 = ({\rm Zp1} * {\rm Zm2} + {\rm Zm1} * {\rm Zp2})/\sqrt{2}

3 4 {\rm E}(\psi_0) == -1

6 using Zygote # autodiff

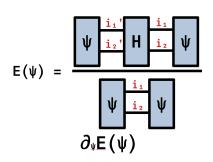
7 \partial {\rm E}(\psi) = {\rm gradient}({\rm E}, \psi)[1]

9 {\rm norm}(\partial {\rm E}(\psi_0)) == 2
```

```
1 \psi = \underset{\text{minimize}(E, \partial E, \psi_0;}{\text{msteps}=10, \gamma=0.1}

3 4 E(\psi_0) == -1

5 E(\psi) == -1.4142131 \approx -\sqrt{2}
```



$$\min_{\psi} E(\psi) = \min_{\psi} \langle \psi | H | \psi \rangle / \langle \psi | \psi \rangle$$

```
# Circuit as a vector of gates:
     U(\theta, i1, i2) = [
      op("Ry", i1; \theta = \theta[1]),
       op("Ry", i2; \theta = \theta[2]),
     op("CX", i1, i2),
       op("Ry", i1; \theta = \theta[3]),
      op("Ry", i2; \theta = \theta[4]),
 8
 9
10
11
```

$$\frac{\mathbf{i}_{1}''}{\mathbf{i}_{2}''} | \mathbf{u}(\theta_{1}) | \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} = \frac{\mathbf{i}_{1}''}{\mathbf{i}_{2}'}, \mathbf{CX} | \frac{\mathbf{i}_{1}''}{\mathbf{i}_{2}'} | \mathbf{g}_{y(\theta_{1})} | \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}$$

$$\frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'} | \mathbf{u}(\theta_{1}) | \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}} \rightarrow \frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'} | \mathbf{U}(\theta) | \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}$$

```
# PastaQ notation:
     \mathbf{u}(\theta) = [
     ("Ry", 1, (; \theta = \theta[1])),
     ("Ry", 2, (; \theta = \theta[2])),
    ("CNOT", 1, 2),
     ("Ry", 1, (; \theta = \theta[3])),
     ("Ry", 2, (; \theta = \theta[4])),
 8
 9
10
      U(\theta, i1, i2) =
        buildcircuit(u(\theta), [i1, i2])
11
```

$$\frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'}, \mathbf{u}_{(\theta_{1})} = \frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}}, \mathbf{CX} = \frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'}, \mathbf{Ry}_{(\theta_{1})} = \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}$$

$$\frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}}, \mathbf{u}_{(\theta_{1})} = \frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}}, \mathbf{CX} = \frac{\mathbf{i}_{1}'}{\mathbf{i}_{2}'}, \mathbf{U}_{(\theta_{1})} = \frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}$$

```
1 \psi_0 = \mathrm{Zp1} * \mathrm{Zp2}

2 3 4 function \mathrm{E}(\theta)

5 \psi_\theta = \mathrm{apply}(\mathrm{U}(\theta, \mathrm{i1}, \mathrm{i2}), \psi_0)

6 return \mathrm{inner}(\psi_\theta, \mathrm{H}, \psi_\theta)

end
```

```
References state:

|0\rangle = |Z+Z+\rangle

\min_{\theta} E(\theta)

= \min_{\theta} \langle 0|U(\theta)^{\dagger} H U(\theta)|0\rangle

= \min_{\theta} \langle \theta|H|\theta\rangle
```

```
1 \psi_0 = \mathrm{Zp1} * \mathrm{Zp2}

2 3 4 function \mathrm{E}(\theta)

5 \psi_\theta = \mathrm{apply}(\mathrm{U}(\theta,\,\mathrm{i1},\,\mathrm{i2}),\,\psi_0)

6 return \mathrm{inner}(\psi_\theta',\,\mathrm{H},\,\psi_\theta)

7 end
```

$$E(\theta) = \begin{bmatrix} \mathbf{i}_{1} \\ \mathbf{i}_{2} \end{bmatrix} + \begin{bmatrix} \mathbf{i}_{1$$

```
1 \psi_0 = \mathrm{Zp1} * \mathrm{Zp2}

2 3 4 function \mathrm{E}(\theta)

5 \psi_\theta = \mathrm{apply}(\mathrm{U}(\theta, \mathrm{i1, i2}), \psi_0)

6 return \mathrm{inner}(\psi_\theta, \mathrm{H}, \psi_\theta)

7 end
```

```
1 \theta_0 = [0, 0, 0, 0]

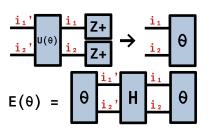
2 \partial \mathbf{E}(\theta) = \text{gradient}(\mathbf{E}, \theta)[1]

3 \theta = \text{minimize}(\mathbf{E}, \partial \mathbf{E}, \theta_0;

4 \text{nsteps=}40, \gamma = 0.5)

5 \mathbf{E}(\theta_0) == -1

7 \mathbf{E}(\theta) == -1.4142077 \approx -\sqrt{2}
```



$$\frac{\min}{\theta} E(\theta) \qquad \partial_{\theta} E(\theta)$$

# Tutorial: Two-site fidelity optimization

```
1 \psi_0 = \mathrm{Zp1} * \mathrm{Zp2}

2 \psi = (\mathrm{ZpZp} + \mathrm{ZmZm}) / \sqrt{2}

3 4 5 6 function F(\theta)

7 \psi_\theta = \mathrm{apply}(\mathrm{U}(\theta, \mathrm{i1}, \mathrm{i2}), \psi_0)

8 return -abs(inner(\psi, \psi_\theta))^2

9 end
```

Reference state: 
$$\begin{aligned} |0\rangle &= |Z+Z+\rangle \\ \text{Target state:} \\ |\psi\rangle &= (|Z+Z+\rangle + |Z-Z-\rangle)/\sqrt{2} \\ \min_{\theta} &F(\theta) = \\ \min_{\theta} &-|\langle \psi | U(\theta) | 0 \rangle|^2 \end{aligned}$$

# Tutorial: Two-site fidelity optimization

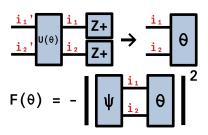
```
1 \psi_0 = \mathrm{Zp1} * \mathrm{Zp2}

2 \psi = (\mathrm{ZpZp} + \mathrm{ZmZm}) / \sqrt{2}

3 4 5 6 function F(\theta)

7 \psi_\theta = \mathrm{apply}(\mathrm{U}(\theta, \mathrm{i1}, \mathrm{i2}), \psi_0)

8 return -abs(inner(\psi, \psi_\theta))^2 end
```



# Tutorial: Two-site fidelity optimization

```
1 \psi_0 = \mathrm{Zp1} * \mathrm{Zp2}

2 \psi = (\mathrm{ZpZp} + \mathrm{ZmZm}) / \sqrt{2}

3 4 5 6 function F(\theta)

7 \psi_\theta = \mathrm{apply}(\mathrm{U}(\theta, \mathrm{i1}, \mathrm{i2}), \psi_0)

8 return -abs(inner(\psi, \psi_\theta))^2 end
```

```
1 \theta_0 = [0, 0, 0, 0]

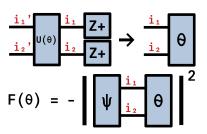
2 \partial F(\theta) = \text{gradient}(F, \theta)[1]

3 \theta = \text{minimize}(F, \partial F, \theta_0; \theta)

4 \text{nsteps}=50, \gamma=0.1

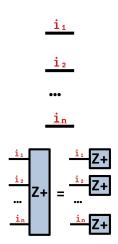
5 F(\theta_0) == -0.5

7 F(\theta) == -0.9938992 \approx -1
```



```
n = 30
                                   n-site state
    i = [Index(2, "S=1/2")]
                for j in 1:n
5
    Zp = MPS(i, "Z+")
8
    maxlinkdim(Zp) == 1 #
       product state
```

$$|Z+Z+\ldots Z+\rangle$$



1 
$$\psi = (\mathrm{Zp} + \mathrm{Zm})/\sqrt{2}$$
  
2 3 4  $\mathrm{maxlinkdim}(\psi) == 2 \#$   
entangled state

$$\begin{array}{l}(|Z+Z+\ldots Z+\rangle+\\|Z-Z-\ldots Z-\rangle)/\sqrt{2}\end{array}$$

```
1 \psi = (\mathrm{Zp} + \mathrm{Zm})/\sqrt{2}

2 3 4 \mathrm{maxlinkdim}(\psi) == 2 \#

entangled state
```

1 
$$\psi = (\mathrm{Zp} + \mathrm{Zm})/\sqrt{2}$$
  
2 3 4  $\mathrm{maxlinkdim}(\psi) == 2 \ \#$  entangled state

inner(Zp, 
$$\psi$$
) ==  $1/\sqrt{2}$ 

$$\langle Z + | Z + \rangle = 1$$

$$\langle Z + | \psi \rangle = 1/\sqrt{2}$$

1 
$$\psi = (\text{Zp} + \text{Zm})/\sqrt{2}$$
  
2 3 4  $\text{maxlinkdim}(\psi) == 2 \#$   
entangled state

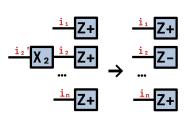
$$\begin{array}{c} \underbrace{\frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}}_{\mathbf{i}_{2}} \psi = \underbrace{\frac{\frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}}{\mathbf{i}_{2}}}_{\mathbf{i}_{2}} \underbrace{\frac{\mathbf{i}_{1}}{\mathbf{i}_{2}}}_{\mathbf{i}_{2}} \underbrace{\frac{\mathbf{$$

inner(Zp, Zp) == 1

inner(Zp, 
$$\psi$$
) ==  $1/\sqrt{2}$ 

$$X_j|Z+Z+\ldots Z+\rangle = |Z+Z+\ldots Z-\ldots Z+\rangle$$

$$|Z+Z+\ldots Z-\ldots Z+\rangle$$



1  $\max_{j} \operatorname{maxlinkdim}(X_{j}Zp) == 1$ 2  $\operatorname{inner}(Zp, X_{j}Zp) == 0$ 

### Tutorial: n-site operators with MPO

```
function ising(n; h)
     H = OpSum()
    for j in 1:(n - 1)
     H = "Z", j, "Z", j + 1
5
     end
6
    for j in 1:n
    H += h, "X", j
     end
     return H
    end
10
   h = 0.5
   H = MPO(ising(n; h=h), i)
    maxlinkdim(H) == 3
14
```

$$n$$
 sites 
$$H = -\sum_{j}^{n-1} Z_{j} Z_{j+1} + h \sum_{j}^{n} X_{j}$$

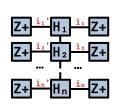
# Tutorial: n-site operators with MPO

```
function ising(n; h)
     H = OpSum()
     for j in 1:(n - 1)
     H = "Z", j, "Z", j + 1
     end
5
    for j in 1:n
    H += h, "X", j
     end
     return H
    end
   h = 0.5
   H = MPO(ising(n; h=h), i)
   maxlinkdim(H) == 3
14
```

# Tutorial: n-site operators with MPO

```
function ising(n; h)
     H = OpSum()
     for j in 1:(n - 1)
      H = "Z", i, "Z", i + 1
     end
    for j in 1:n
     H += h, "X", j
     end
     return H
    end
   h = 0.5
   H = MPO(ising(n; h=h), i)
    \max \lim \dim(H) == 3
14
```

```
1 Zp = MPS(i, "Z+")
2 inner(Zp', H, Zp) == -(n-1)
```



```
1 \psi_0 = \text{MPS}(i, "Z+")

2 \psi = \text{minimize}(E, \partial E, \psi_0;

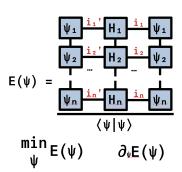
4 \text{nsteps}=50, \gamma=0.1,

5 \text{maxdim}=10, \text{cutoff}=1\text{e-}5)

6 E_{dmrg}, \psi_{dmrg} = \text{dmrg}(H, \psi_0;

8 \text{nsweeps}=10,

9 \text{maxdim}=10, \text{cutoff}=1\text{e-}5)
```



```
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6 E_{dmrg}, \psi_{dmrg} = \text{dmrg}(H, \psi_0; \text{nsweeps}=10, \text{maxdim}=10, \text{cutoff}=1e-5)
```

```
1 \max_{\text{linkdim}}(\psi_0) == 1

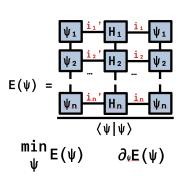
2 \max_{\text{linkdim}}(\psi) == 3

3 \max_{\text{linkdim}}(\psi_{dmrg}) == 3

4 E(\psi_0) == -29

5 E(\psi) == -31.0317917

6 E(\psi_{dmrg}) == -31.0356110
```



```
1 Ry_layer(\theta, i) = [op("Ry", i[j]; \theta=\theta[j]) for j in 1:n]
2 CX_layer(i) = [op("CX", i[j], i[j+1]) for j in 1:2:(n-1)]
```

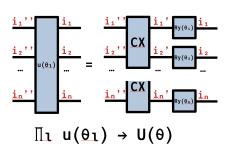
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```

```
\begin{array}{ll} \textbf{function} \ U(\theta, \, i; \, nlayers) \\ \textbf{2} & n = length(i) \\ \textbf{3} & \textbf{for} \ l \ in \ 1: (nlayers - 1) \\ \textbf{4} & \theta_l = \theta[(1:n) \ .+ \ l \ * \ n] \\ \textbf{5} & U_\theta = [U_\theta; \ Ry\_layer(\theta_l, \, i)] \\ \textbf{6} & U_\theta = [U_\theta; \ CX\_layer(i)] \\ \textbf{7} & \textbf{end} \\ \textbf{8} & \textbf{return} \ U_\theta \\ \textbf{9} & \textbf{end} \end{array}
```

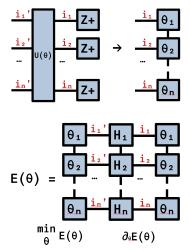
```
U(\theta) = Ry_1(\theta_1) \dots Ry_n(\theta_n)
* CX_{1,2} \dots CX_{n-1,n}
* Ry_1(\theta_{n+1}) \dots Ry_n(\theta_{2n})
* ...
```

```
1 Ry_layer(\theta, i) = [op("Ry", i[j]; \theta=\theta[j]) for j in 1:n]
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```
\begin{array}{ll} \textbf{1} & \textbf{function} \ U(\theta, \, i; \, nlayers) \\ 2 & n = length(i) \\ 3 & \textbf{for} \ l \ in \ 1: (nlayers - 1) \\ 4 & \theta_l = \theta[(1:n) \ .+ \ l * \ n] \\ 5 & U_\theta = [U_\theta; \ Ry\_layer(\theta_l, \, i)] \\ 6 & U_\theta = [U_\theta; \ CX\_layer(i)] \\ 7 & \textbf{end} \\ 8 & \textbf{return} \ U_\theta \\ 9 & \textbf{end} \end{array}
```



```
\psi_0 = MPS(i, "Z+")
       nlayers = 6
       function E(\theta)
         \psi_{\theta} = \text{apply}(U(\theta, i; \text{nlayers}), \psi_0)
         return inner(\psi_{\theta}', H, \psi_{\theta})
       end
       \theta_0 = \text{zeros}(\text{nlayers} * \text{n})
       \theta = \text{minimize}(E, \partial E, \theta_0;
              nsteps=20, \gamma=0.1)
10
```



```
1 \psi_0 = \text{MPS}(i, "Z+")

2 \text{nlayers} = 6

3 \text{function E}(\theta)

4 \psi_\theta = \text{apply}(U(\theta, i; \text{nlayers}), \psi_0)

5 \text{return inner}(\psi_\theta', H, \psi_\theta)

6 \text{end}

7 \theta_0 = \text{zeros}(\text{nlayers} * n)

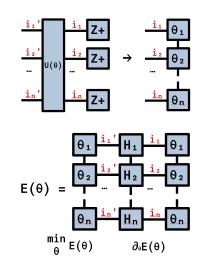
9 \theta = \text{minimize}(E, \partial E, \theta_0; \text{nsteps} = 20, \gamma = 0.1)
```

```
1 \max_{\theta} \min_{\theta} (\psi_{0}) = 1

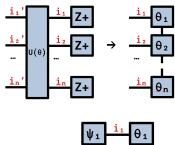
2 \max_{\theta} \min_{\theta} (\psi_{\theta}) = 3

3 E(\theta_{0}) == -29

4 E(\theta) == -31.017062
```



```
\psi_0 = MPS(i, "Z+")
       nlayers = 6
       function F(\theta)
       \psi_{\theta} = \frac{\text{apply}}{\text{U}(\theta, i; \text{nlayers})}, \psi_0
         return -abs(inner(\psi', \psi_{\theta}))^2
       end
       \theta_0 = \text{zeros}(\text{nlayers} * \text{n})
       \theta = \text{minimize}(F, \partial F, \theta_0);
              nsteps=20, \gamma=0.1)
10
```



$$F(\theta) = -\begin{bmatrix} \psi_1 & \vdots & \vdots & \vdots & \vdots \\ \psi_2 & \vdots & \vdots & \vdots & \vdots \\ \psi_n & \vdots & \vdots & \vdots & \vdots \\ \psi_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots & \vdots & \vdots \\ \theta_n & \vdots & \vdots$$

#### Tutorial: n-site states with MPS

```
\psi_0 = MPS(i, "Z+")
       nlayers = 6
       function F(\theta)
         \psi_{\theta} = \text{apply}(U(\theta, i; \text{nlayers}), \psi_0)
         return -abs(inner(\psi', \psi_{\theta}))^2
       end
       \theta_0 = \text{zeros}(\text{nlayers} * \text{n})
       \theta = \text{minimize}(F, \partial F, \theta_0);
              nsteps=20, \gamma=0.1)
10
       \operatorname{maxlinkdim}(\psi_0) == 1
       \operatorname{maxlinkdim}(\psi_{\theta}) == 2
       \mathbf{F}(\theta_0) == -0.556066
       \mathbf{F}(\theta) = -0.995230
```

$$F(\theta) = - \begin{bmatrix} \frac{1}{1} & \frac{1$$

 $\partial_{\theta} F(\theta)$ 

► Noisy state and process simulation (PastaQ). AD/optimization support in progress.

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- ▶ More that I am probably forgetting... Ask Giacomo and me for more details.



▶ More AD, make ITensor fully differentiable (have some work to do, like tensor decompositions and general network contractions, more MPS/MPO functions. You will find bugs, please contribute gradient definitions!).

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- More HPC with multithreaded and multiprocessor parallelism and GPUs.
- ► Improved gradient optimization: higher order derivatives, preconditioners, isometrically constrained gradient optimization, etc.

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- ▶ I'm interested in building general tools and algorithms, and I'm looking for people with problems to solve. Also, we need help, code to contributions are welcome!

