Problem sheet 6

Due date: **December 4, 12:00 am**. Discussion of solutions: December 6.

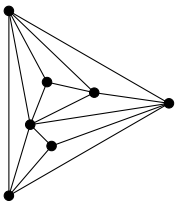
(Please prepare solutions for **at most three** problems.)

Problem 21. **5 points**

(a) Prove that every planar triangulation on at least four vertices contains a vertex whose neighborhood induces a cycle.

(b) Prove that every n-vertex planar graph has at most $3n - 8$ triangles.

Hint: Show with (a) that there is a vertex of degree 3.



A planar graph with 7 vertices and 13 triangles.

Problem 22. **5 points**

(a) Find the largest number of edges in an n-vertex TK_3-free graph.

(b) Prove that if G is 3-connected then $TK_4 \subseteq G$.

Problem 23. **5 points**

Use Kuratowski's theorem to prove that a graph is outerplanar if and only if it has no subdivision of K_4 or $K_{2,3}$.

Problem 24. **5 points**

Without using the 4-Color-Theorem, prove that every outerplanar graph is 3-colorable.

Open Problem.

Prove or disprove that for $n \geq 6$, the largest number of edges in an n-vertex graph with no TK_5 is $3n - 6$.