Problem 13

Theorem 1.1. *If a graph has an ear-decomposition, then it is* 2*-connected.*

Proof. By *Menger's Theorem*, a graph G is k-connected if and only if for any two vertices a, b in G there exist k independent a-b-paths. We find those 2 paths for any ear-composable graph. □

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Problem 14

For $0 < l < m \leq d$, we will construct a graph $F(d, l, m)$ with

- $\delta(F(d, l, m)) = d$
- $\kappa(F(d, l, m)) = l$
- $\kappa'(F(d, l, m)) = m$

Let $F'(l, m) := K_{l,m}$ be the complete bipartite graph of l, m partition sizes. Thus,

$$\kappa(F'(l, m)) = \kappa(K_{l,m}) = \min\{l, m\} = l \tag{1}$$

and furthermore,

$$\kappa'(F'(l, m)) = \kappa'(K_{l,m}) = \max\{l, m\} = m \tag{2}$$

.

Now, we construct $F(d, l, m)$ by substituting each vertex of $F'(l, m)$ with the complete graph K_d. Hence, $\delta(F(d, l, m)) = d$. Since $\kappa(K_d) = d \geq m$ and $\kappa'(K_d) > l$ and since we do not modify the connections between those substituted vertices, the edge- and vertex-connectivity of $F'(l, m)$ are maintained in $F(d, l, m)$.

Problem 15

I will prove that any block-cut-vertex graph is a tree, by showing by contradiction that any block-cut-vertex graph is acyclic and connected.

Theorem 3.1. *The block-cut-vertex graph $G = (V, E)$ of any connected graph $G' = (V', E')$ is a tree.*

Proof. Let's assume for the sake of contradiction that G has a cycle $C = (b_1 b_2 ... b_1)$. Then, the C induced subgraph of G is 2-connected, because obviously each pair $b_i, b_j \in C$ is connected by two distinct paths. Let $u, v \in V'$ be two vertices with u being in the block represented by b_i and v being in the block represented by b_j.

□

Problem 16