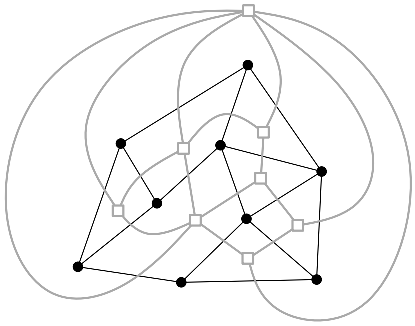
Problem sheet 5

Due date: **November 27, 12:00 am.** Discussion of solutions: November 29.

Let G be a plane embedded graph. Further assume that G is 3-connected. Then we define the *plane dual of* G, denoted by G^*, as the graph whose

- vertices correspond to the faces of G, and
- edges correspond to pairs of faces that share an edge of G.



A plane graph (black) and its plane dual (gray).

Problem 17. **5 points**

In a planar triangulation let n_i be the number of vertices of degree i. Prove that

$$\sum_{i \in \mathbb{N}} (6-i)n_i = 12.$$

Problem 18. **5 points**

Prove or disprove each of the following for every natural number n.

(a) There exists a connected 4-regular planar graph with at least n vertices.

(b) There exists a connected 5-regular planar graph with at least n vertices.

(c) There exists a connected 6-regular planar graph with at least n vertices.

Justify your answers!

Problem 19. **5 points**

Prove that if a plane embedded graph on n vertices has no triangular face, then it has at most $2n-4$ edges.

Problem 20. **5 points**

Let G be a plane embedded graph on n vertices such that $G^* \cong G$. Prove that G has $2(n-1)$ edges. For all $n \geq 4$ find such a graph.

Open Problem.

Without using the 4-Color-Theorem, prove that every n-vertex planar graph has an independent set of size at least $n/4$.