Problem sheet 4

Due date: **November 20, 12:00 am**. Discussion of solutions: November 22.

Problem 13. **5 points**
Prove that if a graph has an ear-decomposition, then it is 2-connected.

Problem 14. **5 points**
For all natural numbers ℓ, m, d with $0 < \ell \leq m \leq d$ construct a graph with minimum degree d that is ℓ-connected (and not $(\ell+1)$-connected) and m-edge-connected (and not $(m+1)$-edge-connected).

Justify your answer.

Problem 15. **5 points**
Prove that the block-cut-vertex graph of any connected graph is a tree.

Problem 16. **5 points**
Prove each of the following statements for any graph G.

(a) If G is 3-regular, then $\kappa'(G) = \kappa(G)$.

(b) If G is 4-regular, then $\kappa'(G) \leq \kappa(G) + 2$.

(c) if G is the d-dimensional hypercube, then $\kappa'(G) = \kappa(G) = d$.

Open Problem.
Let G be an $(a+b+2)$-edge-connected graph. Does there exist a partition $\{A, B\}$ of $E(G)$ so that (V, A) is a-edge-connected and (V, B) is b-edge-connected?