Problem 9

Theorem 1.1. *A hypercube Q_n is Hamiltonian. It has a girth of 4, a diameter of n, an order of 2^n and a size of ?.*

Proof. $\square$

Theorem 1.2. *A bipartite complete graph $K_{m,n}$ is Hamiltonian iff $m = n$. It's girth is 4 for $m, n \geq 2$ and ∞ otherwise. It's diameter is 2. The graph's order is $m + n$ and it's size is $m \cdot n$.*

Proof. $\square$

Theorem 1.3. *The Petersen graph is Hamiltonian, it has a girth of 5, a diameter of 2, an order of 10 and a size of 15.*

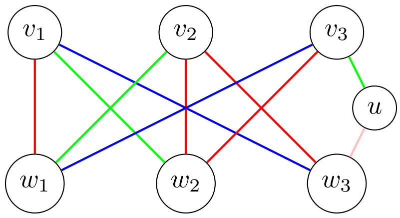
Proof. $\square$

Problem 10

Theorem 2.1. *For a natural number $n \geq 2$ let $G = (V_G, E_G)$ be the graph of order $2n+1$ obtained from $K_{n,n}$ by subdividing an edge by a vertex. Then, $\chi'(X) = \Delta(G) + 1 = n + 1$ but $\chi'(G - e) = \Delta(G - e)$ for any edge e of G.*

Proof. Let $V = \{v_1, ..., v_n\}$ and $W = \{w_1, ..., w_n\}$ denote the two partitions of $K_{n,n}$ and $\{c_k \mid k \in \mathbb{N}\}$ be a set of colors. Furthermore, let $u \in V_G$ be the vertex subdividing an edge between these two partitions.

First, we will prove that $\chi'(X) = \Delta(G) + 1 = n + 1$. We will argue, that any partial coloring of G will inevitably result in $n + 1$ colors.



For a partially colored G, let $v \in V_G$ be a vertex of G.

- **Case 1:** $v \neq u$: Since v has n adjacent vertices, we need n colors to color the edges incident to v. Of course,

□

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Problem 11

For each even integer $k > 1$, the complete graph $K_{(n+1)}$ is a k-regular graph with no 1-factor. For each odd integer $k > 1$ the graph must not be bipartite (*Hall's Theorem*). Furthermore, there must be a subset U such that the graph without U has at least $|U|$ connected components with an odd number of vertices (*Tutte Theorem*).

Problem 12

In the following I will show that any graph G with $2n$ vertices and all degrees at least n has a 1-factor. I will show that if we divide such a G into two parts we can remove edges until we get an bipartite 1-regular graph. Then using the corollary of Hall's theorem we know we can find a perfect matching.

Theorem 4.1. *Let $G = (V, E)$ be a graph with $|V| = 2n$ vertices with all degree atleast n, then G has a 1-factor.*

Proof. Because $|V| = 2n$ we can divide the vertices V into two subsets $A, B \subset V$ with $|A| = |B| = n$ and $A \cap B = \emptyset$

□