Problem sheet 12

Due date: **January 29, 12:00 am**. Discussion of solutions: January 31.

(Please prepare solutions for **at most three** problems.)

Problem 45. **5 points**

Suppose that the edges of K_n are colored with $k \geq 2$ colors in such a way that

- every color appears at least once,
- every triangle in K_n has all edges colored the same or no two edges colored the same.

Find for given k the maximum n for which such a k-edge coloring of K_n exists.

Problem 46. **5 points**

If T is a tree on m vertices, show that in any red-blue coloring of the edges of $K_{(m-1)(n-1)+1}$ there is either a red copy of T or a blue copy of K_n.

Problem 47. **5 points**

Consider a complete graph K_{17} with vertex set $V = \{0, 1, \ldots, 16\}$. Color an edge xy red if there exists a number $a \in V$ with $x - y = a^2 \bmod 17$, and blue otherwise. Show that this red-blue edge coloring of K_{17} does not contain a monochromatic K_4.

Hint: Note that xy is red if and only if $x - y \equiv \pm 1, \pm 2, \pm 4,$ or $\pm 8 \pmod{17}$.

Problem 48. **5 points**

Let $0 < a_1 < a_2 < \cdots < a_{mn+1}$ be $mn+1$ integers. Prove that you can select either $m+1$ of them no one of which divides any other, or $n+1$ of them each dividing the following one.

Open Problem.

Determine $R(5, 5)$.