Problem sheet 11

Due date: **January 22, 12:00 am**.   Discussion of solutions: January 24.

(Please prepare solutions for **at most three** problems.)

Problem 41.   **5 points**
Show that an ϵ-regular partition of a graph G is also an ϵ'-regular partition of its complement $\overline{G}$ for some ϵ'.

Problem 42.   **5 points**
Let G be any graph of chromatic number t. Show that $\mathrm{ex}(n, G) \geq \mathrm{ex}(n, K_t)$ and that equality holds only if there is an edge e such that $\chi(G - e) < t$.

Problem 43.   **5 points**
Show that for any graph $G = (V, E)$, $|V| = n$, there exists a set S, $|S| \leq \lfloor n^2/4 \rfloor$, and for each vertex $v \in V$ a subset $S_v \subseteq S$ such that $S_v \cap S_w \neq \emptyset$ if and only if $vw \in E$.

Hint: Try to cover the edges of G with cliques.

Problem 44.   **5 points**
Determine the smallest number n such that the following holds. Whenever one colors the edges of K_n with two colors, red and blue, one creates a red triangle or a blue K_4.

Open Problem.
Prove or disprove that if G is a triangle-free graph on n vertices, then there is a set of at most $n^2/25$ edges in G whose deletion destroys all odd cycles in G.