

Problem 13

Theorem 1.1. *If a graph has an ear-decomposition, then it is 2-connected.*

Proof. By *Menger's Theorem*, a graph G is k -connected if and only if for any two vertices a, b in G there exist k independent a - b -paths. We find those 2 paths for any ear-composable graph. \square

Problem 14

For $0 < l < m \leq d$, we will construct a graph $F(d, l, m)$ with

- $\delta(F(d, l, m)) = d$
- $\kappa(F(d, l, m)) = l$
- $\kappa'(F(d, l, m)) = m$

Let $F'(l, m) := K_{l, m}$ be the complete bipartite graph of l, m partition sizes. Thus,

$$\kappa(F'(l, m)) = \kappa(K_{l, m}) = \min\{l, m\} = l \quad (1)$$

and furthermore,

$$\kappa'(F'(l, m)) = \kappa'(K_{l, m}) = \max\{l, m\} = m \quad (2)$$

Now, we construct $F(d, l, m)$ by substituting each vertex of $F'(l, m)$ with the complete graph K_d . Hence, $\delta(F(d, l, m)) = d$. Since $\kappa(K_d) = d \geq m$ and $\kappa'(K_d) > l$ and since we do not modify the connections between those substituted vertices, the edge- and vertex-connectivity of $F'(l, m)$ are maintained in $F(d, l, m)$.

Problem 15

Problem 16