

Problem sheet 7Due date: **December 11, 12:00 am.**

Discussion of solutions: December 13.

(Please prepare solutions for **at most three** problems.)**Problem 25.****5 points**

Let G be a graph whose odd cycles are pairwise intersecting, i.e., every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$ and find such a graph G with $\chi(G) = 5$.

Problem 26.**5 points**

Prove or disprove each of the following.

- (a) Every graph G with $\chi(G) = k$ has a proper k -coloring in which one color class has size at least $\alpha(G)$.
- (b) $\chi(G) \leq |V(G)| - \alpha(G) + 1$.
- (c) If $G = F \cup H$, then $\chi(G) \leq \chi(F) + \chi(H)$.

Problem 27.**5 points**

Show that for every graph G on n vertices we have

$$\chi(G) + \chi(\overline{G}) \geq 2\sqrt{n}.$$

Problem 28.**5 points**

Find the list-chromatic number of $K_{4,4}$. Justify your answer.

Open Problem.

Prove or disprove that for every graph G we have

$$MK_{\chi(G)} \subseteq G.$$