

## Problem 13

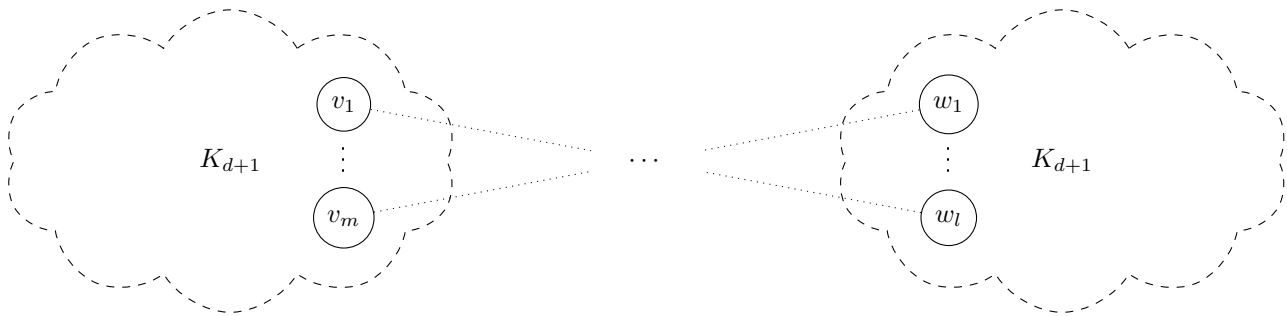
**Theorem 1.1.** *If a graph has an ear-decomposition, then it is 2-connected.*

*Proof.* By *Menger's Theorem*, a graph  $G$  is  $k$ -connected if and only if for any two vertices  $a, b$  in  $G$  there exist  $k$  independent  $a$ - $b$ -paths. We find those 2 paths for any ear-composable graph.  $\square$

## Problem 14

For  $0 < l < m \leq d$ , we will construct a graph  $F(d, l, m)$  with

- $\delta(F(d, l, m)) = d$
- $\kappa(F(d, l, m)) = l$
- $\kappa'(F(d, l, m)) = m$



## Problem 15

I will prove that any block-cut-vertex graph is a tree, by showing by contradiction that any block-cut-vertex graph is acyclic and connected.

**Theorem 3.1.** *The block-cut-vertex graph  $G = (V, E)$  of any connected graph  $G' = (V', E')$  is a tree.*

*Proof.* Let's assume for the sake of contradiction that  $G$  has a cycle  $C = (b_1 b_2 \dots b_1)$ . Then, the  $C$  induced subgraph of  $G$  is 2-connected, because obviously each pair  $b_i, b_j \in C$  is connected by two distinct paths.  $\square$

## Problem 16