Graph Theory winter term 2013

Problem sheet 2

Due date: **November 4, 12:00 am**. Discussion of solutions: November 6.

Problem 5. 5 points

Let G be a nonempty graph with minimum degree at least two. Show that there is a connected graph having the same degree sequence as G.

Problem 6. 5 points

Let T be a tree with an even number of vertices. Show that T has exactly one subgraph in which every vertex has odd degree.

Problem 7. 5 points

For any graph G let $\pi(G)$ denote the minimum number of walks in G so that every edge of G lies in exactly one walk.

Find an expression/formula for $\pi(G)$.

Problem 8. 5 points

A permutation matrix is a matrix of zeros and ones such that each row and each column contains exactly one 1.

Show that a square matrix A with nonnegative integer entries is a sum of k permutation matrices if and only if the sum of elements in each row and in each column of A is k.

Open Problem.

Let G be a k-regular graph with k odd. Is there always a set S of paths so that every edge of G is in exactly one path and every vertex of G is the endpoint of exactly one path in S?