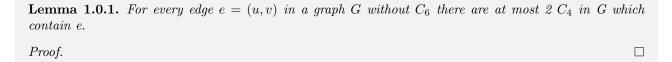
Graph Theory - Sheet 10 - January 13, 2014 J. Batzill (1698622), M. Franzen (1696933), J. Labeit (1656460)

Problem 37



By using the lemma we can simply construct H by removing edges from G.

Theorem 1.1. Every graph G without C_6 has a subgraph H with $|E(H)| \geq \frac{|E(G)|}{2}$, which contains C_4 .

Proof. Let G be such a graph. We know by using the lemma that evere edge is at most part of 2 C_4 s. Let's denote k with the number of C_4 s in G. $k \leq \frac{2}{4} * |E(G)| = \frac{|E(G)|}{2}$, because every edge is at most in 2 C_4 s and $|C_4| = 4$. Now we can construct H by removing an edge from every C_4 in G. It is easy to see that by removing an edge no new cycles are created, hence H contains no C_4 and $|E(H)| = |E(G)| - k \geq |E(G)| - \frac{|E(G)|}{2} \leq \frac{|E(G)|}{2}$.