

Problem 9

Theorem 1.1. *A hypercube Q_n is Hamiltonian. It has a girth of 4, a diameter of n , an order of 2^n and a size of $n \cdot 2^{n-1}$.*

Proof.

□

Theorem 1.2. *A bipartite complete graph $K_{m,n}$ is Hamiltonian iff $m = n$. Its girth is 4 for $m, n \geq 2$ and ∞ otherwise. Its diameter is 2. The graph's order is $m + n$ and its size is $m \cdot n$.*

Proof.

□

Theorem 1.3. *The Petersen graph is Hamiltonian, it has a girth of 5, a diameter of 2, an order of 10 and a size of 15.*

Proof.

□

Problem 10

Problem 11

For each even integer $k > 1$, the complete graph $K_{(n+1)}$ is a k -regular graph with no 1-factor. For each odd integer $k > 1$ the graph must not be bipartite (*Hall's Theorem*). Furthermore, there must be a subset U such that the graph without U has at least $|U|$ connected components with an odd number of vertices (*Tutte Theorem*).

Problem 12