

## Problem 13

**Theorem 1.1.** *If a graph has an ear-decomposition, then it is 2-connected.*

*Proof.* By *Menger's Theorem*, a graph  $G$  is  $k$ -connected if and only if for any two vertices  $a, b$  in  $G$  there exist  $k$  independent  $a$ - $b$ -paths. We find those 2 paths for any ear-composable graph.  $\square$

## Problem 14

For  $0 < l < m \leq d$ , we will construct a graph  $F(d, l, m)$  with

- $\delta(F(d, l, m)) = d$
- $\kappa(F(d, l, m)) = l$
- $\kappa'(F(d, l, m)) = m$

Let  $F'(l, m) := K_{l, m}$  be the complete bipartite graph of  $l, m$  partition sizes. Thus,

$$\kappa(F'(l, m)) = \kappa(K_{l, m}) = \min\{l, m\} = l \quad (1)$$

and furthermore,

$$\kappa'(F'(l, m)) = \kappa'(K_{l, m}) = \max\{l, m\} = m \quad (2)$$

Now, we construct  $F(d, l, m)$  by substituting each vertex of  $F'(l, m)$  with the complete graph  $K_d$ . Hence,  $\delta(F(d, l, m)) = d$ . Since  $\kappa(K_d) = d \geq m$  and  $\kappa'(K_d) > l$  and since we do not modify the connections between those substituted vertices, the edge- and vertex-connectivity of  $F'(l, m)$  are maintained in  $F(d, l, m)$ .

## Problem 15

## Problem 16