J. Batzill (1698622), M. Franzen (1696933), J. Labeit (1656460)

Problem 13

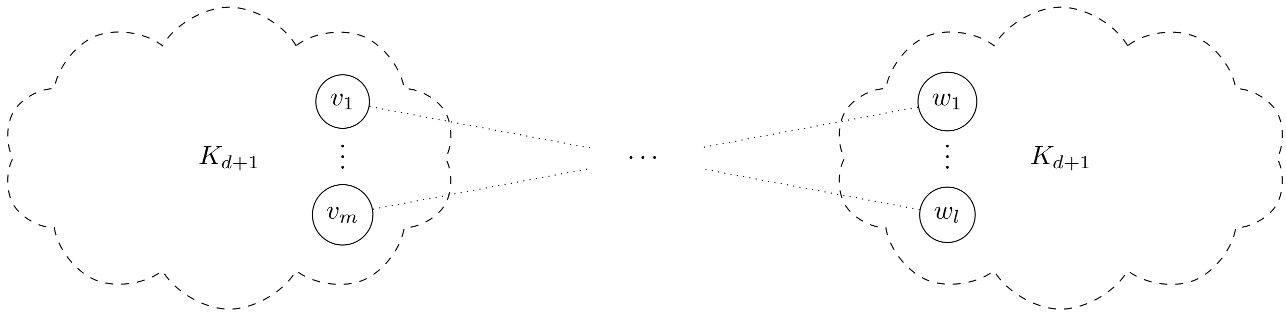
Theorem 1.1. *If a graph has an ear-decomposition, then it is* 2*-connected.*

Proof. By *Menger's Theorem*, a graph G is k-connected if and only if for any two vertices a, b in G there exist k independent a-b-paths. We find those 2 paths for any ear-composable graph. □

Problem 14

For $0 < l < m \leq d$, we will construct a graph $F(d, l, m)$ with

- $\delta(F(d, l, m)) = d$
- $\kappa(F(d, l, m)) = l$
- $\kappa'(F(d, l, m)) = m$



Problem 15

I will prove that any block-cut-vertex graph is a tree, by showing by contradiction that any block-cut-vertex graph is acyclic and connected.

Theorem 3.1. *The block-cut-vertex graph $G = (V, E)$ of any connected graph $G' = (V', E')$ is a tree.*

Proof. Let's assume for the sake of contradiction that G has a cycle $C = (b_1 b_2 ... b_1)$. Let's denote the subgraphs $B_1, B_2, ...B_n$ of G' which are the 2-connected components and bridges corresponding to the nodes $b_1, b_2, ...b_n$ of G. Let B_1 and B_2 be as stated above two different subgraphs of G'. Because the corresponding nodes b_1 and b_2 are adjacent in G, B_1 and B_2 have to share a vertex $x \in V(B_1) \cap V(B_2)$. We can use the same argument for each pair $B_i, B_{i+1)}$. Additionally, we know because each component B_j is either 2-connected or a bridge. Thus we can find a circle through all the components $B_1, B_2 ... B_n$ which is 2-connected. this is a contradiction to $B_1, B_2 ... B_n$ being the blocks of an block-cut-vertex graph, because by definition these blocks are either bridges or maximal 2-connected components. $\square$

Problem 16