Problem sheet 1

Due date: **October 30, 12:00 am**. Discussion of solutions: November 1.

Problem 1. **5 points**

Show that any tree T has at least $\Delta(T)$ leaves.

Problem 2. **5 points**

We define the following properties for any graph G.

(P1) G is acyclic.

(P2) The removal of any edge in G disconnects the graph.

(P3) Adding an edge between any two non-adjacent vertices in G introduces a cycle.

(P4) Any two vertices are joined by a unique path in G.

Show each of the following implications directly.

(i) (P2) $\implies$ (P1)

(ii) (P1) and (P3) $\implies$ (P4)

(iii) (P4) $\implies$ (P2)

Problem 3. **5 points**

Prove that either a graph or its complement is connected.

Problem 4. **5 points**

Prove or disprove that if u and v are the only vertices of odd degree in G then there is a u-v-path in G.

Open Problem.

In a connected graph, pick any three paths of maximum length. Is there always a vertex that lies on all of them?