Problem sheet 10

Due date: **January 15, 12:00 am**. Discussion of solutions: January 17.

(Please prepare solutions for **at most three** problems.)

Hint: You may use the fact that $\sum_{v\in V}\binom{\deg(v)}{2} \geq \frac{m}{n}(2m-n)$ *for any* n*-vertex,* m*-edge graph* $G=(V,E)$*. This inequality will be proven in the problem class.*

Problem 37. **5 points**

Show that every graph G without C_6 has a subgraph H with $|E(H)| \geq |E(G)|/2$, which contains no C_4.

Problem 38. **5 points**

Show that any graph on n vertices and at least $\lfloor \frac{n^2}{4} \rfloor + 1$ edges contains at least $\lfloor \frac{n}{2} \rfloor$ triangles.

Problem 39. **5 points**

Let $G=(V,E)$ be a graph on n vertices and m edges. For $i=0,1,2,3$ let t_i denote the number of vertex triples of G inducing exactly i edges.

(a) Prove that $t_0 + t_3 = \binom{n}{3} - (n-2)m + \sum_{v\in V}\binom{\deg(v)}{2}$.

(b) Conclude with (a) that $t_3 \geq \frac{m}{3n}(4m - n^2)$.

Problem 40. **5 points**

Prove that any graph G with $\delta(G) \geq k$ contains all trees on k edges as a subgraph.

Open Problem.

Prove or disprove that for all trees T with k edges $\mathrm{ex}(n,T) \leq \frac{n(k-1)}{2}$.