



# Procedural cloud shader

## Project documentation

### Project 2

The goal of this project is to research and implement a procedural, volumetric cloud shader. The following document reveals the process of creating such a shader from both a technical and mathematical perspective, considering different algorithms for techniques like noise generation and raymarching.

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Date:	April 10, 2020
Version:	0.4

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# 1 General

## 1.1 Purpose

During this project, all gathered information and knowledge about the researched algorithms and techniques are written down in this document.

## 1.2 Revision History

Version	Date	Name	Comment
0.1	March 21, 2020	Matthias Thomann	Initial draft
0.2	March 29, 2020	Matthias Thomann	Added first research results
0.3	April 01, 2020	Matthias Thomann	Added Unity prototype environment
0.4	April 03, 2020	Matthias Thomann	Added further research results
0.4	April 08, 2020	Matthias Thomann	Added further research results

## 2 Natural clouds

### 2.1 Formation

Clouds, as seen in nature, consist of a visible body of tiny water droplets and frozen crystals. In their natural occurrence, clouds are mostly generated from a nearby source of moisture, usually in the form of water vapor. This composition of particles creates the pleasant look of a white-grayish "fluffy" mass, floating in the sky.

Due to certain factors like altitude or water source, different types of cloudscapes can be formed. They vary in shape, convection, density and more. That makes different cloudscapes highly unique in terms of appearance.

For now, those factors are regarded as nature's randomness. However, an approximation of randomness will be covered in subsection 4.1.

### 2.2 Types of clouds

Cloudscapes are classified in multiple groups, mainly differing in altitude, meaning the distance from the earth's surface to the cloud formation. The following four cloud genera stand out due to their distinctiveness. A realistic simulation of a cloud system would consist of a combination of these types, which is why they are displayed here.



**Figure 1:** Photographic reference of stratus clouds[5].



**Figure 2:** Photographic reference of cirrus clouds[6].



**Figure 3:** Photographic reference of an altocumulus cloud formation[7].



**Figure 4:** Photographic reference of stratuscumulus cloudscape[8].

## 2.3 Clouds in games

Depicted in Figure 3 and Figure 4 of subsection 2.2 are clouds of the genus *cumulus*, which translated to English means *heap* or *pile*. Their distinctive cotton-like look makes them easy to recognize, which is also why they are often used in games as a reference for "normal" clouds.

In games, the formation as well as the natural composition of clouds are irrelevant, as they are essentially only used for cinematic ambience or as a medium to enhance the atmosphere. This leaves just the rendering technique and performance to worry about.

### 2.3.1 Skyboxes

A widespread solution for representing clouds in games is not rendering them separately at all, but instead using a set of polar sky dome images, also known as the skybox. This is a six-sided cube which is rendered around the whole game world. On each inward looking face of the cube, one of the sky dome images is displayed, creating a seamless sky around the inner side of the box.



**Figure 5:** The skybox cube as it is used in games.



**Figure 6:** The polar sky dome images, folded out.

Besides rendering the sky, this of course allows clouds to be drawn right into the background. Also, in terms of performance, this is extremely cheap and efficient. On the other hand, it removes the ability for the clouds to move. They also have no volumetric body and no way of interaction with the game world whatsoever.

This method does indeed give the scenery a more cloudy look, but what is missing is the "feel", or in other words the motion, interaction and lifelikeness of the clouds.

### 2.3.2 Billboards

Similar to the approach with the skybox, this technique also only uses 2D images of clouds. They are rendered individually and are always facing the camera. This is called *billboarding*. Now that each cloud is represented by its own game object, having a position in world space as well as a scale and many other properties, it is possible to animate the clouds. For example, by moving the game objects in a circle around the world, the clouds seemingly "pass by".



**Figure 7:** A collection of 2D cloud billboards facing the camera.



**Figure 8:** The rendered result of the image to the left.

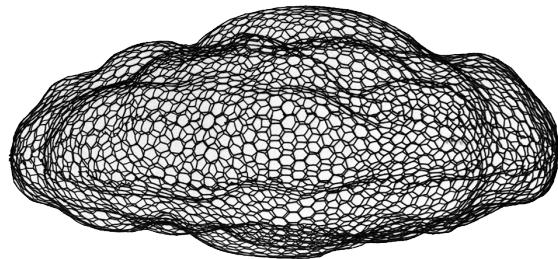
Due to billboarding, the orientation is already given, making the overall time and effort of this technique quite advantageous to others.

The major flaw of using billboards is of course that they are still 2D images, meaning they cannot really change appearance and therefore, do not evolve at all. Still, for many games, this technique suffices in the required diversity of background scenery and does not exceed the allowed performance share for such a task.

### 2.3.3 Mesh-based objects

It is imaginable to simply use a polymesh shaped like a cloud and render that like any other game object. By adding a texture, this would make for some decent looking clouds.

However, the level of detail of such a polymesh is directly connected to the amount of vertices and faces that have to be processed every frame. As seen in Figure 9, there are hundreds of polygons required to merely represent the basic shape of a realistic cloud. If a similarly complex mesh is to be used for every cloud, a massive overhead is generated for objects that usually only contribute to the background of a game.

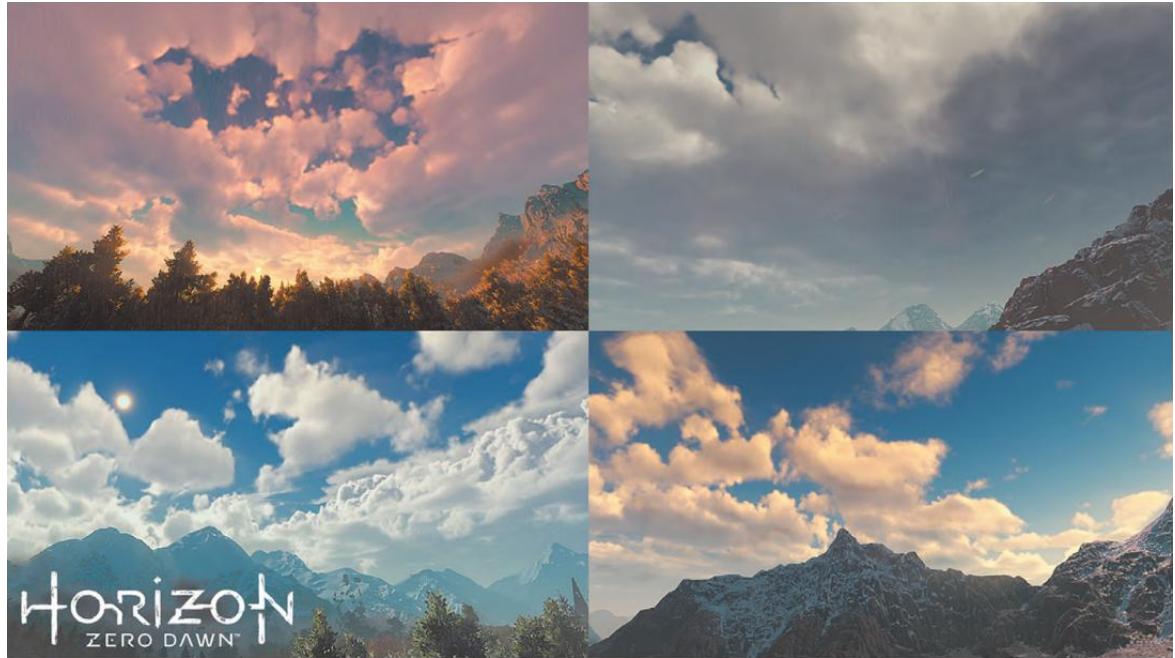


**Figure 9:** A polymesh in the shape of an altocumulus cloud[9].

Apart from the performance impact, this method offers a volumetric, possibly interactable object just like any other 3D model does. When massively decreasing the polygon count and therefore relinquishing the realistic look, mesh-based objects may be a viable solution for some low poly games. Otherwise, it is not reasonable to use this method.

#### 2.3.4 Volumetric clouds

Finally, clouds can be rendered via a technique called *volumetric rendering*. The image below shows volumetric cloudscapes as seen in popular AAA titles. The method itself is explained in detail in subsection 3.1.



**Figure 10:** Several volumetric cloudscapes from the game *Horizon: Zero Dawn*, drawn in real time[10].

## 3 Rendering techniques

### 3.1 Volumetric rendering

#### 3.1.1 Definition

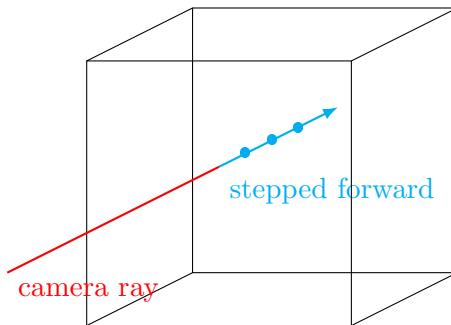
Volumetric rendering describes a technique for generating a visual representation of data that is stored in a 3D volume. This especially comes to use for visual effects that are volumetric in nature, like fluids, clouds, fire, smoke, fog and dust, which all are extremely difficult or even impossible to model with geometric primitives.

In addition to rendering such effects, volumetric rendering has become essential to scientific applications like medical imaging, for which a typical 3D data volume is a set of 2D slice images acquired by a CT (computed tomography) or MRI (magnetic resonance imaging) scanner.

The data volume is also called a *scalar field* or *vector field*, which associates a scalar value, or *voxel*, to every point in the defined space. This can be imagined like a 3D grid, where each point holds a single number. This number could, for example, represent the density of a cloud at that very point. A voxel may also consist of more than just a single value, but rather a set, like an RGB color.

#### 3.1.2 Ray Marching with constant step

To actually render the volume data, a method called *ray marching* is used. With it, the surface distance of the volumetric data is approximated by creating a ray from the camera to the object for each fragment processed in the fragment shader. The ray is then extended into the volume of the object and stepped forward until the surface is reached.

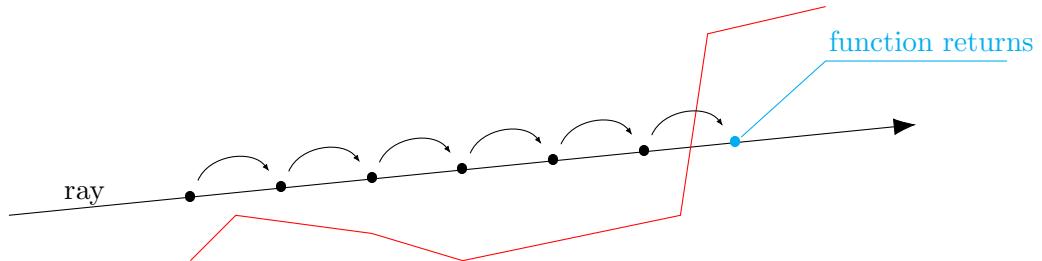


**Figure 11:** A ray is created from the camera to the object. From there, it is extended into the volume.

In ray marching, the algorithm only knows when it has reached the surface, or to be precise when it is inside the actual object volume.

With this information, it is only possible to extend the ray in steps of a predefined length until the inside of the object is reached. With a constant step, the approximation of the surface distance is exactly as precise as the size of the constant step.

Once the ray is inside the actual volume, the function returns the distance for this ray.



**Figure 12:** Traditional ray marching, the precision of which being directly dependent on the step size.

An implementation of this algorithm can be seen in Listing 2. Note that the volume to be rendered in this example is just a simple sphere. So in order to check if the ray is inside the volume, the function `sphereHit()` is used.

```

1 bool sphereHit(float3 position) {
2     float4 sphere = float4(0, 1, 0, 1);
3     return distance(sphere.xyz, position) < sphere.w;
4 }
```

**Listing 1:** Implementation of a volume distance function for a sphere.

With that given, the raymarch function is implemented like so:

```

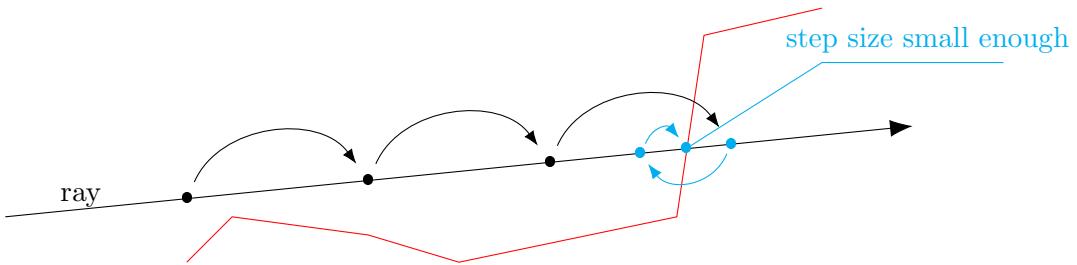
1 fixed4 raymarch(float3 position, float3 direction)
2 {
3     for (int i = 0; i < MAX_STEPS; i++)
4     {
5         if (sphereHit(position))
6             return fixed4(1,0,0,1);
7
8         position += normalize(direction) * STEP_SIZE;
9     }
10
11    return fixed4(0,0,0,1);
12 }
```

**Listing 2:** Ray march function with constant step.

### 3.1.3 Traditional Ray Marching

It is obvious to see that, for a constant step ray march to result in an accurate approximation of the surface distance, the step size is required to be relatively small. This has a direct impact on performance and thus, is not a viable solution for the problem.

In traditional ray marching, an optimization for that has been developed. The algorithm does not blindly step forward, but instead tries to get as close to the real distance as possible. After the volume is reached, the step size is decreased and the ray steps out of the volume again. It then tries to approximate the surface distance by stepping back and forward repeatedly in continuously smaller steps, thus converging towards the exact intersection. Once the step size falls below a certain threshold, the distance approximation is assumed to be precise enough and the value is returned for that ray march.



**Figure 13:** Traditional ray marching.

As clearly visible, the traditional ray marching ends up with a more precise result and the amount of steps per ray could be relatively lower, ultimately saving performance.

However, there is still an issue. The algorithm may jump in and out of the volume, even if it would already be precise enough, essentially taking unnecessary steps.

```

1 fixed4 raymarch(float3 position, float3 direction)
2 {
3     float stepSize = STEP_SIZE;
4     float dirMultiplier = 1;
5     for (int i = 0; i < MAX_STEPS; i++)
6     {
7         if (stepSize < MINIMUM_STEP_SIZE)
8             return fixed4(1,0,0,1);
9
10        if (sphereHit(position)) {
11            // reduce step size by half and invert marching direction.
12            stepSize /= 2;
13            dirMultiplier = -1;
14        } else {
15            dirMultiplier = 1;
16        }
17
18        position += normalize(direction) * stepSize * dirMultiplier;
19    }
20
21    return fixed4(0,0,0,1);
22 }
```

**Listing 3:** Traditional ray march function with converging surface distance approximation.

### 3.1.4 Sphere Tracing

An even better approach to approximate the intersection of the ray and the volume is called *sphere tracing*. Instead of evaluating if the ray is inside the volume or not, an exact distance to the scene is measured. This distance is the minimum amount of space the algorithm can march along its ray without colliding with anything. For that, a function group called *signed distance functions* is used.

#### 3.1.4.1 Signed Distance Functions

A signed distance function (SDF) returns the shortest distance from that a given point in space to some surface. The sign of the returned value indicates whether that point is inside the surface or outside, hence the name.

For example, the signed distance function  $f(p)$  for a point  $p = (p_1, p_2, p_3)$  to the surface of a sphere  $s = (s_1, s_2, s_3)$  with radius  $R$  looks like this:

$$f(p) = \sqrt{(s_1 - p_1)^2 + (s_2 - p_2)^2 + (s_3 - p_3)^2} - R$$

This translates into a simple code snippet, mostly identical to the function `sphereHit()` in Listing 1, except the distance is returned instead of a boolean.

```
1 float sceneSDF(float3 position) {
2     float4 sphere = float4(0, 0, 0, 1);
3     return distance(sphere.xyz, position) - sphere.w;
4 }
```

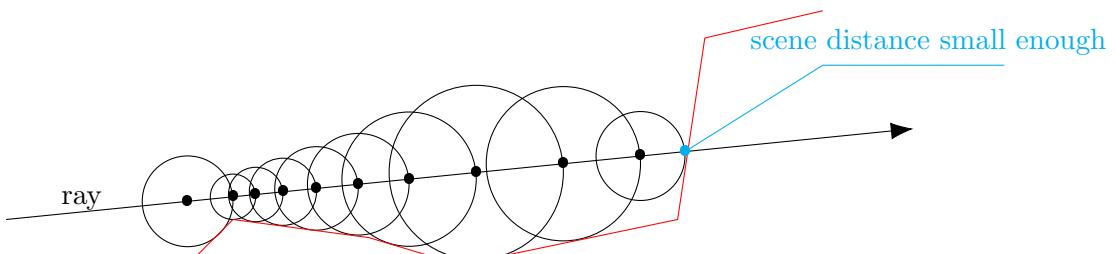
**Listing 4:** Implementation of a signed distance function for a sphere.

With the sphere in the example being at the origin and having  $R = 1$ , a positive distance is returned for points outside the sphere and a negative distance if the point is inside the sphere.

```
1 float d1 = sceneSDF(float3(2, 0, 0)); // d1 = 1.0
2 float d2 = sceneSDF(float3(0, 0.5, 0)); // d2 = -0.5
3 float d3 = sceneSDF(float3(5, -5, 5)); // d3 = 7.66
```

#### 3.1.4.2 Sphere Tracing with SDFs

If the distance to the scene can be calculated with a signed distance function, the algorithm becomes rather straight forward. The distance to the scene is evaluated at the start, then one can freely march along the ray for that amount of distance. Once arrived at the new point, the process is repeated until the SDF returns a small enough value.



**Figure 14:** Ray marching with SDF-based sphere tracing.

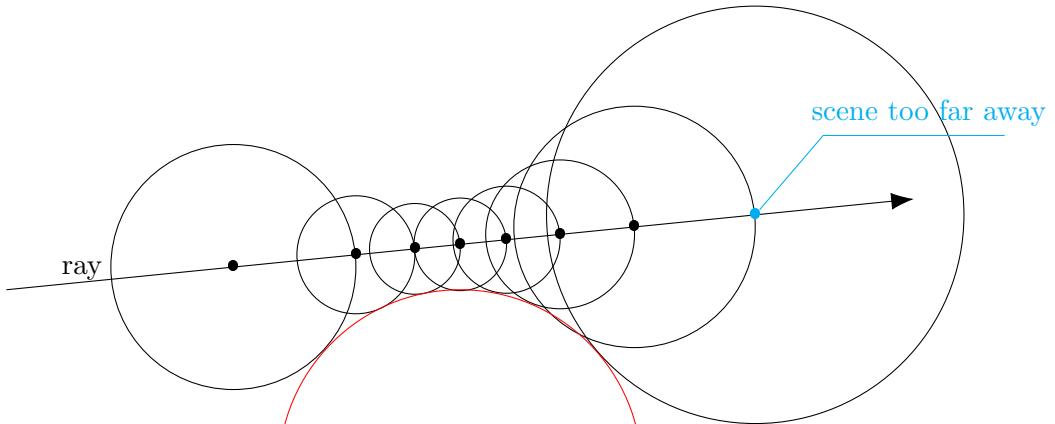
As seen in Figure 14, the result is highly accurate. For the previous example with just one single sphere as a volume, the algorithm can be implemented like in Listing 5.

```

1 float raymarch (float3 position, float3 direction)
2 {
3     float dOrigin = 0.0;
4     for (int i = 0; i < MAX_STEPS; i++)
5     {
6         float dScene = sceneSDF(position + dOrigin * direction);
7         if (dScene < SURFACE_DISTANCE || dScene > MAX_DISTANCE)
8             break;
9
10        dOrigin += dScene;
11    }
12    return dOrigin;
13 }
```

**Listing 5:** Implementation of ray marching with sphere tracing.

In order to save on performance, it is imperative to break the loop when `distanceScene` exceeds `MAX_DISTANCE`. This way, the distance evaluation for that ray can be stopped earlier than waiting for the loop to complete. Another example why this check is important can be seen in the next figure. The ray is terminated early, because it does not collide and never reaches the minimum surface distance.



**Figure 15:** Ray marching with SDF-based sphere tracing, without collision.

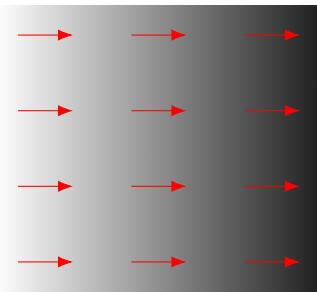
### 3.1.5 Surface Normals and Lighting

As it is the case for many other lighting models, the surface normals are used to calculate lighting in volumetric rendering. If the object is defined with a polymesh, the surface normals are usually specified for each vertex. The normals for any given point on the surface can then be calculated by interpolating the adjacent vertex normals.

Since there is no polymesh in volumetric rendering, another solution has to be found for calculating the surface normals for a scene defined by signed distance functions. Because of that, it is not possible to explicitly calculate the normals and therefore, an approximation is used.

#### 3.1.5.1 Surface Normal Estimation

To approximate the normal vectors in a 3D data volume, the *gradient* is used. The gradient represents the direction of greatest change of a scalar function. In Figure 16, the red arrows indicate the gradient for the points at the start of the arrows.



**Figure 16:** Gradient in a 2D scalar field.

Mathematically, the gradient of a function  $f$  at point  $p = (x, y, z)$  defines the direction to move in from  $p$  to most rapidly increase the value of  $f$ . It is written as  $\nabla f$ .

$$\nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

Instead of calculating the real derivative of the SDF, an approximation is used to estimate the normal vectors. As previously declared, the signed distance function returns zero for a point on the surface, greater than zero if the point is outside and less than zero if it is inside the volume. Therefore, the direction at the surface which will go from negative to positive most quickly will be orthogonal to the surface.

The estimation  $\vec{n}$  is done by sampling some points around the point on the surface and take their difference, the result of which is the approximate surface normal.

$$\vec{n} = \begin{bmatrix} f(x + \epsilon, y, z) - f(x - \epsilon, y, z) \\ f(x, y + \epsilon, z) - f(x, y - \epsilon, z) \\ f(x, y, z + \epsilon) - f(x, y, z - \epsilon) \end{bmatrix}$$

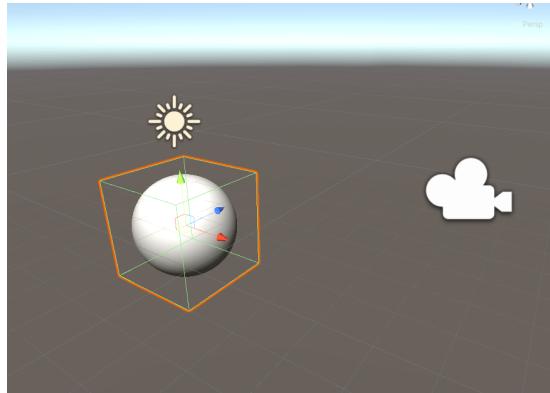
The implementation of surface normal estimation looks like this:

```

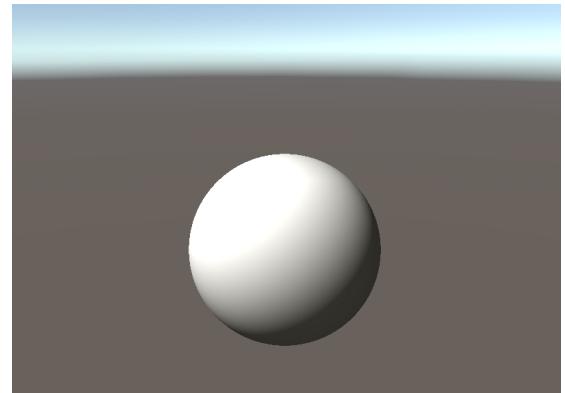
1 float3 estimateNormal(float3 p) {
2     return normalize(float3(
3         sceneSDF(p + float3(EPSILON, 0, 0)) - sceneSDF(p - float3(EPSILON, 0, 0)),
4         sceneSDF(p + float3(0, EPSILON, 0)) - sceneSDF(p - float3(0, EPSILON, 0)),
5         sceneSDF(p + float3(0, 0, EPSILON)) - sceneSDF(p - float3(0, 0, EPSILON)),
6     ));
7 }
```

**Listing 6:** Implementation of surface normal estimation.

Now that the normal vectors can be calculated for the volume, the object can be shaded. In this example, the Phong Illumination Model[11] is used.



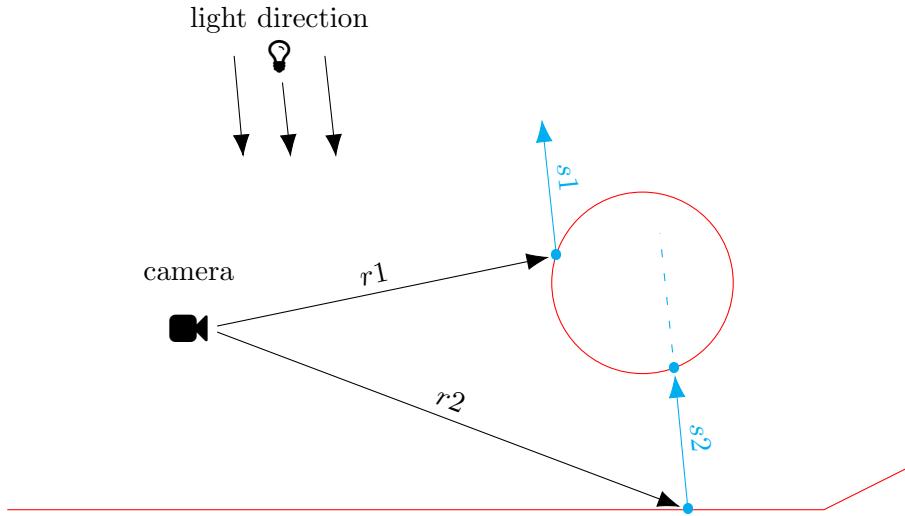
**Figure 17:** A 3D cube with a volumetric shader.



**Figure 18:** The shaded sphere rendered volumetrically.

### 3.1.6 Shadow Casting

In ray marching, adding shadows to the scene is not difficult. As defined in Listing 5, the `raymarch()` function moves along the given ray and returns the distance to the nearest intersection of ray and volume. Therefore, when a surface point has been determined, a second ray march can be started from the newly found point in the opposite direction the primary light source is facing. If anything is hit on the way, the surface point lies in the shadow of the second hit object and should be darkened.



**Figure 19:** Shadow casting in ray marching.

As seen in the figure above, the ray  $r_1$  hits the volumetric sphere, then checks if anything is between the ray intersection and the negative light source direction. In this case,  $s_1$  does not collide with anything and the surface is shaded normally. For the other ray  $r_2$  however, the shadow ray march returns a distance  $s_2 > 0$  and  $s_2 < \text{MAX\_DISTANCE}$ , meaning some object is inbetween the hit point and the light source, casting a shadow.

```

1 float hardshadow(float3 position, float3 direction, float dMin, float dMax)
2 {
3     float dOrigin = dMin;
4     for (int i = 0; i < MAX_STEPS; i++) {
5         float dScene = sceneSDF(position + direction * dOrigin);
6         if (dScene < SURFACE_DISTANCE)
7             return 0.0;
8         if (dScene > dMax)
9             return 1.0;
10        dOrigin += dScene;
11    }
12    return 1.0;
13 }
```

**Listing 7:** Implementation of hard shadow casting.

It is very clearly similar to SDF-based sphere tracing, except that only 0 or 1 is returned instead of the distance. The final color is then multiplied by this output. For 0, this results in a total black, hence the name *hard* shadows.

### 3.1.6.1 Soft Shadows

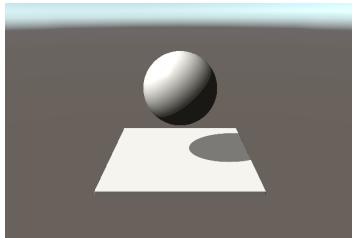
The method described in Figure 19 evaluates only if any given point is directly covered by any other object. It does not account for diffuse shadows with soft edges, called *penumbra* or simply *soft* shadows. But there is an easy and also cost-effective solution to that problem. Instead of strictly returning 0 when an object is covered by another, the shortest distance to scene qualified by some factor  $k$  is returned.

```

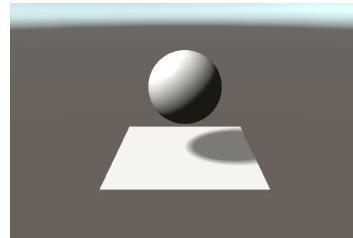
1 float softshadow(float3 position, float3 direction, float dMin, float dMax,
                  float k)
2 {
3     float result = 1.0;
4     float d0origin = dMin;
5     for (int i = 0; i < MAX_STEPS; i++) {
6         float dScene = sceneSDF(position + direction * d0origin);
7         if (dScene < SURFACE_DISTANCE)
8             return 0;
9         if (d0origin > dMax)
10            return result;
11
12         result = min(result, k * dScene / d0origin);
13         d0origin += dScene;
14     }
15     return result;
16 }
```

**Listing 8:** Implementation of hard shadow casting.

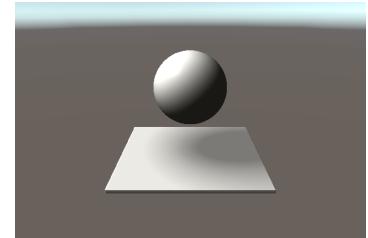
Those are the resulting renders with a sphere and a flat box as the volumetric scene.



**Figure 20:** Hard shadows only.



**Figure 21:** Soft shadows with  $k = 7.0$ .



**Figure 22:** Soft shadows with  $k = 1.2$ .

## 4 Common Algorithms

### 4.1 Noise Generation

This section is empty.

## Glossary

**Billboard** A 2D image always facing towards the main camera. 3

**Convection** Convection describes the transfer of heat from movement of liquid or gas. 2

**Gradient** The *gradient* denotes the direction of the greatest change of a scalar function. 11

**Low poly** A 3D polymesh with a relatively low count of polygons. 4

**Penumbra** The partially shaded outer region of diffuse shadows. Also described as soft edges.. 14

**Polymesh** A polymesh is a 3D model composed of polygons or triangles. 4, 11, 18

**Ray marching** Ray marching is a type of method to approximate the surface distance of a volumetric object, where a ray is cast into the volume and stepped forward until the surface is reached. 6, 7, 8, 13

**Scalar field** A scalar field describes a typically three-dimensional grid of elements called *voxels*, each containing a scalar value. 6

**Signed distance function** A signed distance function, short SDF, returns a positive distance if the origin is outside the volume and a negative distance if it is inside the volume. 9, 11

**Sphere tracing** Sphere tracing describes an optimized algorithm of ray marching by using signed distance functions to approximate the surface distance of the volume. 9

**Surface normal** A *surface normal* or *normal* is a vector which is perpendicular to a given geometry, like a triangle or polygon. 11, 12

**Vector field** It is the same as a scalar field, except the voxels are vector values. 6

**Volumetric rendering** This describes a technique which takes a 3D volume of data and projects it to 2D. It is mostly used for transparent effects stored as a 3D image. 6, 11

**Voxel** Short for *volume element*, a voxel is a value (either a number or a vector) on a scalar or vector field . 6

**World space** Coordinates defined with respect to a global Cartesian coordinate system. 3

## References

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