

# MTH229 Notes

Professor Kevin O'Bryant

Spring 2025

Spring 2025

Student:

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# First Day Checklist

Lab Rules: Attendance is required, each and every lab period for the full lab period. You are not to use a phone in the lab for texting, calling, nor browsing. No goofing off, that is.

## **Checklist for the bare minimum first day work.**

- ☐ Log into the campus network SLAS.
- ☐ Log into Brightspace and see if there are any announcements for this lab class.
- ☐ Make sure the email address that Brightspace has is one that works for you.
- ☐ Make sure the email address that CUNYFirst has for you is one that you monitor.
- ☐ Take a look at the Syllabus, also located on Brightspace.
- ☐ WeBWorK is the service that we will use to assign and grade homework. Log in to WeBWorK at <https://www.math.csi.cuny.edu/webwork2/> and selecting your professor's name or by going to the math department website <https://www.math.csi.cuny.edu/>, finding the "WeBWorK" link at the lower right, and then selecting our class. You will find your username and initial password described on that page (it is your first initial, last name, and 4 digits from ID). If it doesn't work, your professor can help.
- ☐ Change WeBWorK password. WeBWorK is not secure: you should not re-use a password from a system you care about. It's under "User Settings", as is an email address that will be used by WeBWorK. Change it, too, if you want. Course related emails will go through the CUNYFirst address or the Brightspace address, but exchanges about specific homework problems will go through WeBWorK. If you ever forget your password, your professor can reset it.
- ☐ Contact someone else in the classroom by sending them a chat message, or a text, or just using your old-fashioned voice. Say hello, exchange email addresses, etc. It's okay to do this with several people.
- ☐ Open "Problem 1" of the first homework set, "01-calculator".

- ☐ Use the “Juliabox” button at the top of the problem. Your login name and Juliabox password are both set to your WeBWorK username. It works best if you are using Chrome.
- ☐ Open the “01-calculator.ipynb” file. This opens our textbook. Read through this material. The first line is “Read about this material here” with a link. That link will take you to a *more* detailed and in-depth explanation — I advise you to skip it at least until you are stuck on a problem. After giving this material a read-through, you will come to a blank input line at the bottom. Look at the WeBWorK problem. Try to do it in the blank input line at the bottom. If you can’t, get in touch with your instructor.
- ☐ When you have the value for (a), copy-and-paste it into the WeBWorK blank. Press the “Submit Answers” button at the bottom of the WeBWorK page. This saves your answer, and also tells you if it was correct or incorrect. You have unlimited attempts for most problems, including this one.
- ☐ You are expected to do your own work, but you are not expected to do it alone. The best experience for this class is to work in parallel with someone, as it helps to have someone to bounce ideas around and have fresh eyes, occasionally. Do not split the work in half: everyone needs to do and understand every problem. There’s plenty of time.
- ☐ You should do several problems from this project on the first day. Some people are able to finish it, even. It should definitely be completed by (at the latest) the end of the second lab period. Note that the due dates shown in WeBWorK are deadlines: they will not be postponed under (almost) any circumstances. You are expected to finish the projects a week or more *ahead* of the deadlines.
- ☐ At the end of lab period, navigate to the first problem that you have not completed (or the last problem, if you’ve completed them all), and press the “email instructor” button. In the message, include the names of the people sitting next to you. This email will count as your attendance at the first lab period.

# Installing Julia

It is in principle easy to install Julia on any computer, including laptops. This is sometimes broken, as there are too many types of computers with too many possible combinations of software on them.

The best advice I can give is to check out [www.julialang.org](http://www.julialang.org) and download the version that seems most appropriate for your situation.

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# Chapter 1

## 01-calculator

1. Problem 1: Title: “Using Julia like a calculator”

(a) Summary of the problem:

evaluating simple expressions

(b) What I was supposed to learn:

adding, subtracting, multiplying in Julia

When I want to multiply, I should use \*

I should put parentheses around numerators and denominators if they have more than one term.

(c) Date started, finished, and time spent

started 25-2-27, finished 25-2-27, 10 minutes

(d) Difficulty:

1 out of 10

2. Problem 2: Title: "Precedence (PEMDAS)"

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

7. Problem 7: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

8. Problem 8: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

# Chapter 2

## 02-functions

Define the function

$$f(x) = \frac{(x-1)^2}{x^2+3x+2} + \tan^{-1}(x) + \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

and use your function to find  $f(0), f(1), f(-3/2)$ .

**Solution:** We input the following:

```
1 f(x) = (x-1)^2 / (x^2+3x+2) + atan(x) + 1/sqrt(2*pi)*exp(-x^2/2)
2 f(0),f(1),f(-3/2)
```

and the output is the following:

```
1 (0.8989422804014326, 1.0273688879165916, -25.853276127581438)
```

Thus,

$$\begin{aligned}f(0) &\approx 0.8989422804014326 \\f(1) &\approx 1.0273688879165916 \\f(-3/2) &\approx -25.853276127581438\end{aligned}$$

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



2. Problem 2:                      Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

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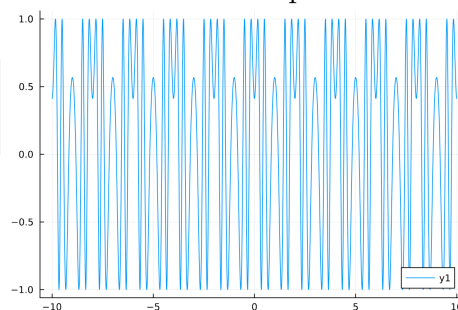
# Chapter 3

## 03-graphics

Make a plot of the periodic function  $f(x) = \sin(9 \cos(2 \sin(\frac{\pi x}{2})))$  over one period.

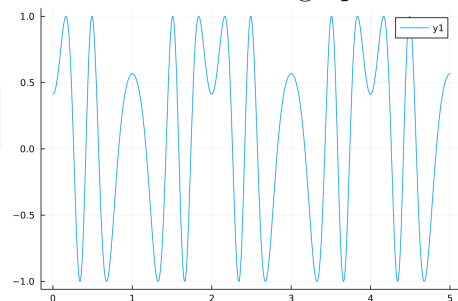
**Solution:** We input the code on the left and are rewarded with the picture on the right.

```
1 using MTH229, Plots
2 f(x) = sin(9*cos(2*sin(pi*x/2)))
3 plot(f,-10,10)
```



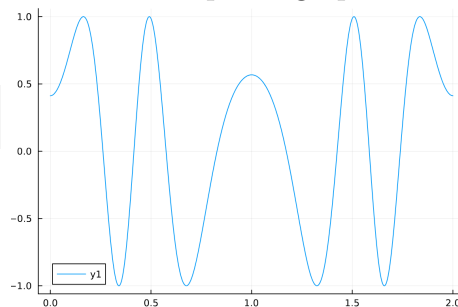
From that image, we count 9 or 10 periods. We decide to look at the graph over the domain  $[0, 5]$

```
1 plot(f,0,5)
2
```



We graphically conclude that the period is 2, and make the required graph.

```
1 plot(f,0,2)
2
```



1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



2. Problem 2: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

7. Problem 7: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

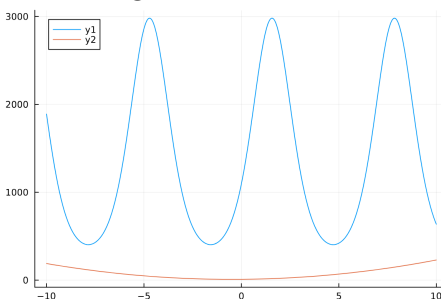
# Chapter 4

## 04-zeros

Solve the equation  $\exp(7 + \sin x) = 2x^2 + 2x + 9$ .

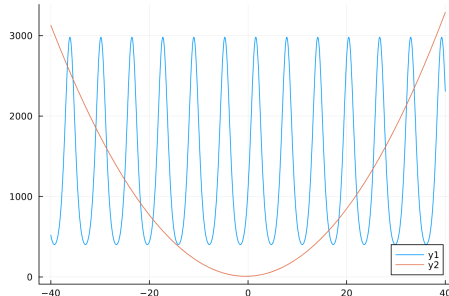
**Solution:** First, we define the functions and plot them together.

```
1 using MTH229, Plots
2 left(x) = exp(7+sin(x))
3 right(x) = 2x^2 + 2x + 9
4 plot(left, -10, 10)
5 plot!(right)
```



We think: As  $-1 \leq \sin x \leq 1$ , it must be that  $e^6 \leq \exp(7 + \sin x) \leq e^8$ . Also, we know that  $y = 2x^2 + 2x + 9$  is a parabola. We visualize it arcing up to where it crosses above  $y = e^8$ . With that in mind, we make a picture over  $[-100, 100]$ , and from that picture decide to instead use  $[-50, 50]$ . From that picture, we decide to use  $[-40, 40]$ .

```
1 plot(left, -40, 40)
2 plot!(right)
```



Now we can see 14 solutions, and there's a close call around  $x \approx -16$  that we can't see visually if the graphs intersect or not. Now, we can use `fzeros` to find the solutions.

```
1 aux(x) = left(x) - right(x)
2 fzeros(aux, -40, 40)
```

The output is a list of all 14 solutions. There is no solution close to  $-16$ , which we double-check by plotting over the domain  $[-18, -13]$ .

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



2. Problem 2:                      Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

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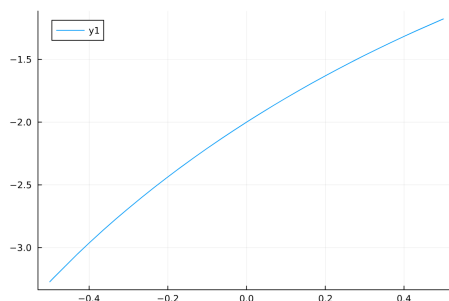
# Chapter 5

## 05-limits

Compute  $\lim_{x \rightarrow 0} \frac{\tan(x)(\sin(x) - \cos(x) - 1)}{\exp(x) - 1}$  three different ways.

**Solution:** First, we define a function, and then we plot it on an interval around 0 (because it is a limit as  $x \rightarrow 0$ ):

```
1 using MTH229, Plots
2 top(x) = tan(x) * (sin(x) - cos(x) - 1)
3 bot(x) = exp(x) - 1
4 f(x) = top(x) / bot(x)
5 plot(f, -0.5, 0.5)
```



From the image, we conclude that the limit is about  $-2$ .

Alternatively, we could make a table showing inputs getting close to 0 and see what the outputs are getting close to.

```
1 lim(f, 0 )
2
```

0.100000	-1.8080220187207678
0.010000	-1.980083002344296
0.001000	-1.9980008330001504
0.000100	-1.999800008332135
0.000010	-1.9999800001027366
0.000001	-1.9999980000767992
⋮	⋮
c	L?
⋮	⋮
-0.000001	-2.000002000032556
-0.000010	-2.0000200000821873
-0.000100	-2.00020000833364
-0.001000	-2.0020008336669637
-0.010000	-2.0200836690448423
-0.100000	-2.208692087969323

We see in the second column that  $L$  wants to be  $-2$ .

Alternatively, we could use SymPy, which is loaded with the MTH229 package.

```
1 @syms x
2 limit(f(x), x => 0 )
```

which outputs a simple “ $-2$ ”.

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



2. Problem 2:                      Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

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# Chapter 6

## 06-derivatives

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



2. Problem 2: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

7. Problem 7: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

8. Problem 8: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

9. Problem 9: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



# Chapter 7

## 07-newton

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

2. Problem 2: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

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# Chapter 8

## 08-first second derivatives

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

2. Problem 2: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

7. Problem 7: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



# Chapter 9

## 09-extrema

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

2. Problem 2: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

7. Problem 7: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



8. Problem 8: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

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# Chapter 10

## 10-integration

1. Problem 1: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

2. Problem 2: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

3. Problem 3: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

4. Problem 4: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

5. Problem 5: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:



6. Problem 6: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

7. Problem 7: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

8. Problem 8: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

9. Problem 9: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

10. Problem 10: Title: \_\_\_\_\_

(a) Summary of the problem

(b) What I was supposed to learn

(c) Date started, finished, and time spent

(d) Difficulty:

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# Appendix A

## Various Functions

### A.1 Trigonometric Functions

Julia Function	Description
<code>sin(x)</code>	Sine of $x$
<code>cos(x)</code>	Cosine of $x$
<code>tan(x)</code>	Tangent of $x$
<code>cot(x)</code>	Cotangent of $x$
<code>sec(x)</code>	Secant of $x$
<code>csc(x)</code>	Cosecant of $x$

### A.2 Inverse Trigonometric Functions

Julia Function	Description
<code>asin(x)</code>	Inverse sine (arcsine) of $x$
<code>acos(x)</code>	Inverse cosine (arccosine) of $x$
<code>atan(x)</code>	Inverse tangent (arctangent) of $x$
<code>atan(y, x)</code>	Inverse tangent (arctangent) of $\frac{y}{x}$
<code>acot(x)</code>	Inverse cotangent (arccotangent) of $x$
<code>asec(x)</code>	Inverse arcsecant (arcsecant) of $x$
<code>acsc(x)</code>	Inverse arccosecant (arccosecant) of $x$

### A.3 Hyperbolic Functions

Julia Function	Description
<code>sinh(x)</code>	Hyperbolic sine of $x$
<code>cosh(x)</code>	Hyperbolic cosine of $x$
<code>tanh(x)</code>	Hyperbolic tangent of $x$
<code>coth(x)</code>	Hyperbolic cotangent of $x$
<code>sech(x)</code>	Hyperbolic secant of $x$
<code>csch(x)</code>	Hyperbolic cosecant of $x$

## A.4 Inverse Hyperbolic Functions

Julia Function	Description
<code>asinh(x)</code>	Inverse hyperbolic sine of $x$
<code>acosh(x)</code>	Inverse hyperbolic cosine of $x$
<code>atanh(x)</code>	Inverse hyperbolic tangent of $x$
<code>acoth(x)</code>	Inverse hyperbolic cotangent of $x$
<code>asech(x)</code>	Inverse hyperbolic secant of $x$
<code>acsch(x)</code>	Inverse hyperbolic cosecant of $x$

## A.5 Exponential and Logarithmic Functions

Julia Function	Description
<code>exp(x)</code>	$e^x$
<code>exp2(x)</code>	$2^x$
<code>exp10(x)</code>	$10^x$
<code>expm1(x)</code>	$e^x - 1$
<code>log(x)</code>	Natural logarithm of $x$
<code>log2(x)</code>	Base-2 logarithm of $x$
<code>log10(x)</code>	Base-10 logarithm of $x$
<code>log1p(x)</code>	$\log(1 + x)$

## A.6 Root Functions

Julia Function	Description
<code>sqrt(x)</code>	Square root of $x$
<code>cbrt(x)</code>	Cube root of $x$
<code>hypot(x, y)</code>	$\sqrt{x^2 + y^2}$

## A.7 Power Functions

Julia Function	Description
<code>x^y</code>	$x$ raised to the power of $y$
<code>x^2</code>	Square of $x$
<code>x^3</code>	Cube of $x$
<code>x^(-1)</code>	reciprocal of $x$



## A.8 Special Functions

Julia Function	Description
<code>abs(x)</code>	Absolute value of $x$
<code>abs2(x)</code>	Squared absolute value
<code>sign(x)</code>	Sign of $x$
<code>factorial(n)</code>	Factorial of $n$
<code>binomial(n, k)</code>	Binomial coefficient
<code>gamma(x)</code>	Gamma function at $x$
<code>lgamma(x)</code>	Log-gamma function at $x$

## A.9 Rounding Functions

Julia Function	Description
<code>round(x)</code>	Round to nearest integer
<code>ceil(x)</code>	Round up to integer
<code>floor(x)</code>	Round down to integer
<code>trunc(x)</code>	Truncate to integer

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Student:

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# Appendix B

## SymPy

### B.1 Setup and Basic Usage

```
1  # Installing SymPy
2  using Pkg
3  Pkg.add("SymPy")
4
5  # Loading the package
6  using SymPy
7
8  # Creating symbolic variables
9  @vars x y z
10 @syms a b c real=true # Declaring real variables
11 @syms n::Integer      # Integer variable
```

### B.2 Algebraic Manipulation

```
1  # Basic operations
2  expr = x^2 + 2*x + 1
3  expanded = expand((x + 1)^2)
4  factored = factor(x^2 + 2*x + 1)
5
6  # Substitution
7  expr = x^2 + y
8  subs(expr, x => 2)          # Replace x with 2
9  subs(expr, Dict(x => 2, y => 3)) # Multiple substitutions
10
11 # Simplification
12 simplified = simplify((x^2 + 2*x + 1) / (x + 1))
13 trigsimp(sin(x)^2 + cos(x)^2) # Trigonometric simplification
14 collect(x^2*y + x*y^2 + x^2 + x, x) # Collect terms with x
15
16 # Polynomial operations
17 p1 = Poly(x^2 + 2*x + 1, x)
18 p2 = Poly(x + 1, x)
19 p1 + p2      # Addition
20 p1 * p2      # Multiplication
21 div(p1, p2)  # Division
```

## B.3 Calculus

```
1  # Differentiation
2  diff(x^2 + 2*x + 1, x)      # First derivative
3  diff(x^2 + 2*x + 1, x, 2)   # Second derivative
4  diff(sin(x)*exp(x), x)      # Product rule automatically applied
5
6  # Partial differentiation
7  f = x^2 + 2*y^2 + 3*x*y
8  diff(f, x)                  # Partial with respect to x
9  diff(f, y)                  # Partial with respect to y
10 diff(f, x, y)               # Mixed partial derivative
11
12 # Integration
13 integrate(x^2 + 2*x + 1, x)   # Indefinite integration
14 integrate(x^2 + 2*x + 1, (x, 0, 1)) # Definite integration
15
16 # Limits
17 limit(sin(x)/x, x => 0)
18 limit((1 + 1/x)^x, x => oo)   # Limit as x approaches infinity
19
20 # Series expansion
21 series(sin(x), x, 0, 5)      # Taylor series around x=0 up to x^5
```

## B.4 Solving Equations

```
1  # Solving a single equation
2  solve(x^2 - 4, x)           # Solve x^2 - 4 = 0
3  solve(sin(x) - cos(x), x)   # Trigonometric equation
4
5  # Systems of equations
6  eqs = [x + y - 2, x - y - 0]
7  solve(eqs, [x, y])
8
9  # Solving inequalities
10 solve_univariate_inequality(x^2 - 4 < 0, x)
11
12 # Differential equations
13 @vars y(x)
14 diffeq = Eq(diff(y(x), x, 2) + y(x), sin(x))
15 dsolve(diffeq, y(x))
```

## B.5 Linear Algebra

```
1  # Creating matrices
2  A = [x 1; 1 1] # 2x2 symbolic matrix
3  B = Matrix{Sym}([1 2; 3 4]) # Matrix from numeric values
4
5  # Matrix operations
6  A + B          # Addition
7  A * B          # Matrix multiplication
8  A^2            # Matrix power
9  transpose(A)   # Transpose
10 inv(A)         # Inverse
11
12 # Determinant and eigenvalues
13 det(A)          # Determinant
14 eigenvals(A)    # Eigenvalues
15 eigenvects(A)   # Eigenvectors
16
17 # Solving linear systems
18 sols = solve_linear_system(A, [1, 2], [x, y])
```

## B.6 Numerical Evaluation

```
1  # Converting symbolic to numeric
2  N(pi)           # Default precision
3  N(pi, 50)       # 50 digits of precision
4
5  # Evaluating expressions
6  expr = sin(pi/3)
7  float(expr)     # Convert to floating point
8  complex(expr)   # Convert to complex
9
10 # Using with standard Julia
11 f(x) = 2*x^2 + 3*x + 1
12 sym_f = lambdify(f(x)) # Convert to Julia function
13 sym_f(2)          # Evaluate at x = 2
```

## B.7 Plotting with SymPy and Plots.jl

```
1  using Plots
2
3  # Converting symbolic expressions for plotting
4  f(x) = sin(x) * exp(-0.1*x)
5  sym_f = lambdify(f(x))
6
7  # Create plot
8  x_range = 0:0.1:10
9  plot(x_range, sym_f.(x_range),
10     title="Symbolic Function Plot",
11     label="sin(x)*exp(-0.1x)",
12     xlabel="x", ylabel="f(x)")
```

## B.8 Common Functions and Constants

```
1  # Constants
2  PI      # pi = 3.14159...
3  E       # Euler's number e = 2.78182828...
4  oo      # Infinity
5  I       # Imaginary unit
6
7  # Functions
8  sin(x), cos(x), tan(x)      # Trigonometric
9  asin(x), acos(x), atan(x)  # Inverse trigonometric
10 sinh(x), cosh(x), tanh(x)   # Hyperbolic
11 exp(x)                      # Exponential
12 log(x), log(x, b)           # Natural log, log base b
13 sqrt(x)                    # Square root
14 factorial(n)                # Factorial
15 binomial(n, k)              # Binomial coefficient
```

## B.9 Tips and Tricks

```
1  # Assumption handling
2  @vars x real=true positive=true
3  simplify(sqrt(x^2)) # Returns x, not |x|
4
5  # Comparing expressions
6  a = (x + 1)^2
7  b = x^2 + 2*x + 1
8  a == b # Structural equality
9  simplify(a - b) == 0 # Mathematical equality
10
11 # Converting to Julia expressions
12 expr = x^2 + sin(y)
13 convert(Expr, expr) # Convert to Julia Expr
14
15 # Function for numerical calculations with uncertainties
16 @vars x y
17 f = x^2 + y^2
18 subs(f, Dict(x => 3 +/- 0.1, y => 2 +/- 0.2))
```





# Appendix C

## Extra Credit

These instructions step you through using Julia<sup>1</sup> to create a good looking scientific document. You will enter the questions, solve them, and print your solutions. You will email the completed file, which should have extension “.ipynb” and filename your name without spaces. The project is due before your final exam. Your grade will be based, in part, on the beautiful appearance of your document.

Your professor should provide a value of  $m$  specific to you with  $m \in [3, 10) \setminus \{\pi\}$ .

Note that this is a take-home outside-the-lab experience. You are allowed to use your notes, past homework and exams, and to google questions. You may *not* use any live-answer site like Chegg, StackExchange, Slader, et cetera. In fact, anyone who would use such a site, or would ask you for help, is totally lost and should not be allowed to skew the curve that applies to students who have actually done the work, like you. Please report any such activity you are aware of; you will be rewarded (if there’s evidence) and you will be doing the right thing.

1. Start Julia.
2. Open a new notebook using Julia.
3. Type “MTH229 Final Exam” into the cell. Then hit the escape button, then 1, which will make the cell into the title. Then use escape-m, and press shift-enter. If these directions do not work with how you are using Julia, contact your instructor immediately for discussion and advice.
4. Insert a cell below that, and type your name, followed by a comma, followed by your student ID number, followed by a comma, followed by the value of  $m$  you will receive when you email your professor. Then use escape-2 to make this cell a subtitle, escape-m to make it markdown, and shift-enter to evaluate it. Don’t put off getting your value of  $m$  until the last minute!
5. In a new cell, type “using MTH229, Plots”, and use shift-enter to evaluate the cell.

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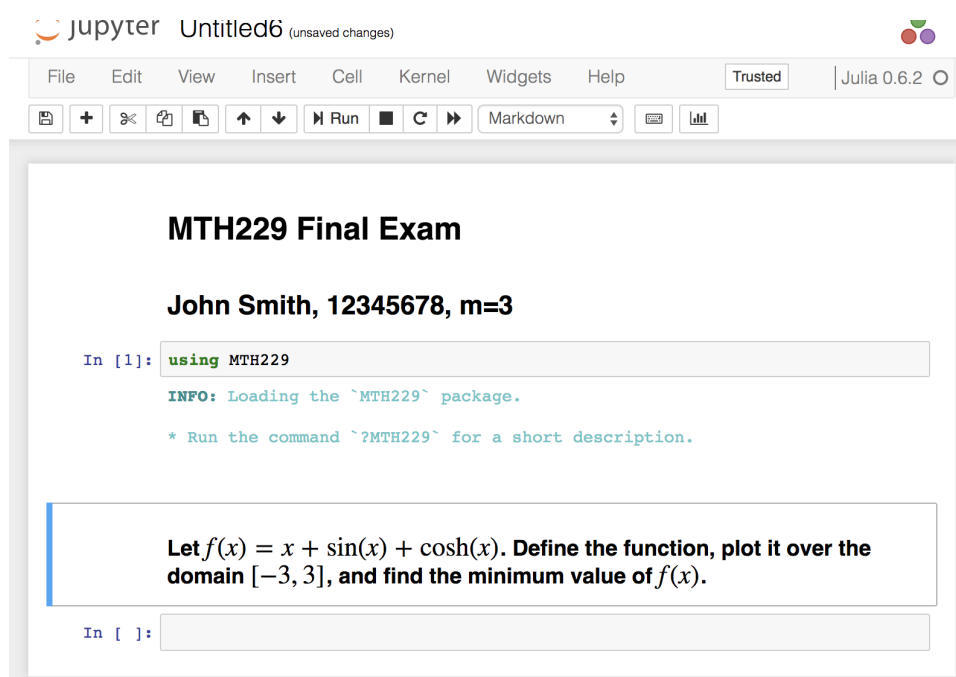
<sup>1</sup>Actually, Jupyter notebooks.

6. **FIRST PROBLEM:** In a new cell, type

```
1   Let  $f(x) = x+4\sin(x)+\cosh(x)$ . Define the function, plot it over
2   the domain  $[-3,3]$ , and find the maximum value of  $f(x)$ .
3
```

Include all of the dollar signs and backslashes *exactly* as above. Use escape-m to convert the cell to markdown (which means it's text, not code). Then escape-3 to make it bold, and then shift-enter the cell to render it.

7. Insert a new cell below this. Your screen should look similar to mine:



8. This is a good moment to figure out how to save your work (use your name as the file name), and how to locate the `.ipynb` file. Especially if you are using Binder, make sure you have work saved where it won't disappear.
9. Solve the problem. Don't be shy about creating your own markdown cells (but don't use escape-3 to make them bold) to explain what you are doing, or what you are concluding. Real mathematicians use complete sentences.
10. In a cell below your solution, enter

\*\*\*

11. Save your work.

12. **SECOND PROBLEM:** In a new cell, type (replacing  $m$  with the value assigned to you).

```
1   Let  $g(x) = \sin(x)\cos(mx)\sin(\pi x)$ . Graph  $g$  on the interval
    $[0, 2]$ , and also the tangent line at  $x = \sqrt{2}$ . Compute the roots
   of  $g$  in the interval  $[0, 2]$ . Find the minimum and maximum values of
    $g$  on  $[0, 2]$ .
```

Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

13. Solve the problem. Don't be shy about creating your own markdown cells to explain what you are doing, and what you have concluded. Below your solution, create a markdown cell with "\*\*\*", which gives a horizontal line.

14. **THIRD PROBLEM:** In a new cell, type (replacing  $m$  with the value assigned to you).

```
1   Let  $g(x) = \sin(x)\cos(mx)\sin(\pi x)$ . Compute  $\int_0^{1/\pi} g(x)$ 
    $\, dx$ .
```

Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

15. Solve the problem. Don't be shy about creating your own markdown cells to explain what you are doing, and what you have concluded. Below your solution, create a horizontal line. Save your work.

16. **FOURTH PROBLEM:** In a new cell, type (replacing  $m$  with the value assigned to you).

```
1   Find all solutions to the equation  $\sqrt[3]{x} = \frac{x^2}{m} - 229$ .
```

Warning: that's a cube root, not a square root. Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

17. Solve the problem.

18. **BONUS PROBLEM:** In a new cell, type (replacing  $m$  with the value assigned to you).

```
1   Let  $r(x) = \frac{m/\pi}{\log(1 + \sqrt{m})} \frac{\tan^{-1} x}{x(m+x^2)}$ . Compute the derivative  $r'(x)$  symbolically. Compute  $\int_{-1}^1 r(x) \, dx$  numerically. Find estimates for the inflection
   points of  $r(x)$ .
```

Convert the cell to markdown (escape-m), and bold (escape-3), and typeset (shift-enter).

19. Solve the problem. Don't be shy about creating your own markdown cells to explain what you are doing, and what you have concluded. Below your solution, create a horizontal line. Save your work.

## C.1 The problems

**FIRST:** Let  $f(x) = x + 4\sin(x) + \cosh(x)$ . Define the function, plot it over the domain  $[-3, 3]$ , and find the maximum value of  $f(x)$ .

**SECOND:** Let  $g(x) = \sin(x) \cos(mx) \sin(\pi x)$ . Graph  $g$  on the interval  $[0, 2]$ , and also the tangent line at  $x = \sqrt{2}$ . Compute the roots of  $g$  in the interval  $[0, 2]$ . Find the minimum and maximum values of  $g$  on  $[0, 2]$ .

**THIRD:** Let  $g(x) = \sin(x) \cos(mx) \sin(\pi x)$ . Compute  $\int_0^{1/\pi} g(x) dx$ .

**FOURTH:** Find all solutions to the equation  $\sqrt[3]{x} = \frac{x^2}{m} - 229$ .

**BONUS:** Let  $r(x) = \frac{m/\pi}{\log(1 + \sqrt{m})} \frac{\tan^{-1} x}{x(m + x^2)}$ . Compute the derivative  $r'(x)$  symbolically. Compute  $\int_{-1}^1 r(x) dx$  numerically. Find estimates for the inflection points of  $r(x)$ .