Optical Doppler Effect

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I. Abstract

The following paper elaborates upon the optical Doppler effect. Sinusoidal wave forms that propagate through mediums such as sound and light can be stretched or shrunk while moving. This fluctuation in wave frequency is observed with a Doppler shift. A mathematical articulation can represent the scale of this shift. Due to light speed magnitudes, relativistic considerations must be deployed in calculations. Beat frequency can be utilized to decipher a Doppler shifted wave form from an original one. A moving reflector along a track resembles the moving source through which the Doppler shift can be shown. Through experimentation, we were able to confirm the Doppler phenomenon with velocity to frequency correlation. Analysis of the resultant data provided valuable understanding of wave mechanics and objects moving relative to different perspectives.

II. Introduction

The Doppler effect is a phenomenon in which wave frequencies shift relative to an observer. Any wave mechanism such as light and sound can undergo this effect. Christian Doppler first adopted this phenomenon in 1842 when he predicted color shift due to frequency in moving heavenly bodies. Moving sources are conducive for a shift in the observer findings for frequency. Most commonly, this effect is heard in moving vehicles driving toward or away from each other making a sound such as a horn or alarm. Although still apparent, light wave Doppler shift is more challenging to observe with high frequencies. This light wave frequency difference can be measured through beat frequency. The experiential methodology is outlined below for observation of this effect.

III. Theory

The Doppler shift can be modeled with the beat frequency as it is the difference between two frequencies. The two frequencies being the source and the observation frequency. The difference in the two frequencies expresses a shift due to some elongation or compression of sinusoidal waves. Any sinusoidal wave system can be compressed or stretched as the source of the system moves. Modeling a system for frequency shift denotes the frequency of the source as f_0 and the receiver as f_1. A wave emission with wavelength $\lambda = \left(\frac{v}{f_0}\right)$, and velocity v, propagates from the source with velocity relative to the observer.

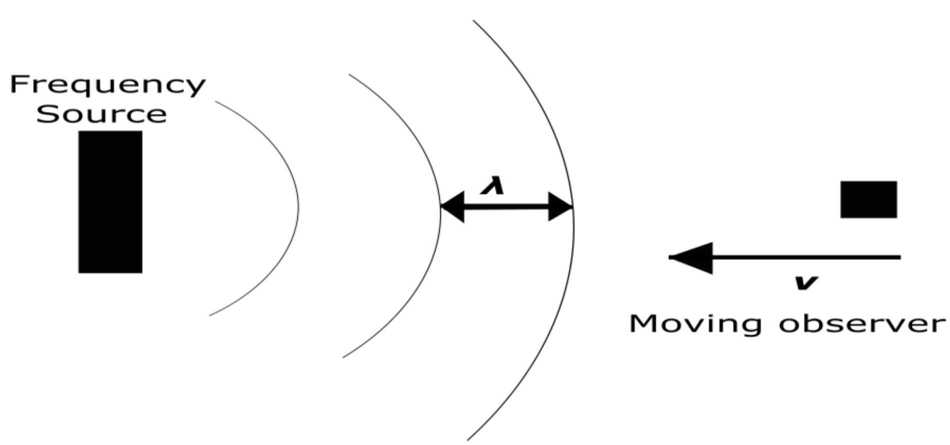

Fig. (1): A observer moving towards a wave producer sees an increase in frequency.

Therefore, a listener moving towards the wave source maintains a relative velocity greater than in a stationary state such that, $f_1 = \left(\frac{v+v_1}{\lambda}\right) = \left(\frac{v+1}{\frac{v}{f_0}}\right)$ or, $f_l = \left(\frac{v+v_1}{v}\right) f_0$ simplifying yields, $f_l = \left(1 + \left(\frac{1}{v}\right)\right) f_0$. Allowing for all situations of movement for source and observation yields the standard equation for the Doppler effect, $f_o = \left(\frac{v \pm v_1}{v \pm v_0}\right) f_0$. When considering electromagnetic waves, relativistic factors must account for length contraction and time dilation. The Lorentz factor ($\sqrt{1 - v^2/c^2}$), holds the relativistic assumptions needed for an electromagnetic wave propagating towards or away a receiver. Using this factor gives, $f = \frac{\frac{c}{c-v}(\sqrt{c^2-v^2}) f_0}{c} =$

$$\sqrt{\frac{c+v}{c-v}} f_0 = f_1 \text{ Eq. (1)}$$

Measuring the frequency shift with electromagnetic waves becomes nearly impossible as the frequency of light sources are much too high for any practical equipment to pick up. To account for extremely high frequency in light, the beat frequency can be utilized to observe this effect. Superimposing sinusoidal wave functions of equal amplitude and slightly different frequencies results in waves interfering constructively when in phase and destructively when out of phase. The resultant superposition wave form graph of two wavefunctions results in beats. These beats can be shown to be the difference in the two coinciding wavefunctions, $f_{beat} = |f_1 - f_2|$. Also modeled with

$$\Delta f = \frac{2v}{\lambda_0} \text{ Eq. (2)}$$

Thus, a doppler shift can be detected without directly measuring the incident frequencies. This technique is called heterodyning and requires the source and observed frequency to have a consistent phase shift. To account for uncertainty in beat frequency, quadrature uncertainty can be performed. This is defined with arbitrary values x, z, u and w as $q = \frac{(x)\ldots(z)}{((u)\ldots(w))}, \frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2}$. When applied to eq. (2),

$$\delta beat = \left(\frac{V\alpha}{\lambda_0}\right)\left(\sqrt{\left(\frac{\delta\alpha}{\alpha}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta\lambda_0}{\lambda_0}\right)^2}\right) \text{ Eq. (3)}$$

Where V is voltage, α is the voltage to velocity conversion factor, and λ_0 is the source wavelength of the laser.

IV. Experiment

1. Set up apparatus similar to Fig. (2).
2. Mark a specified length on the rail, measure the time for the rail car to pass each mark at varying voltages. Record measurements at several voltages. Make sure to use a voltage multimeter with the power generator to observe accurate values of voltages as the power generator fluctuates.
3. Plot data, and the resulting slope is the conversion factor from Voltage to velocity of the rail car.
4. Plug the photomultiplier light sensor into the oscilloscope for beat frequency readings. To acquire an accurate reading, use proper frequency scales that display proportional decipherable readings.
5. At each voltage, freeze the oscilloscope and record the beat frequency. Record the data, calculate expected values for the corresponding voltage values. Use uncertainty in voltage to approximate a resulting uncertainty for the beat frequency.
6. A Fast Fourier Transform (FFT)[3] can be utilized on the oscilloscope to convert the signal into individual components and receive a better visualization of the observed beat frequencies.

7. When observed values match up with corresponding expectations within uncertainty, the Doppler effect is being accurately reflected.

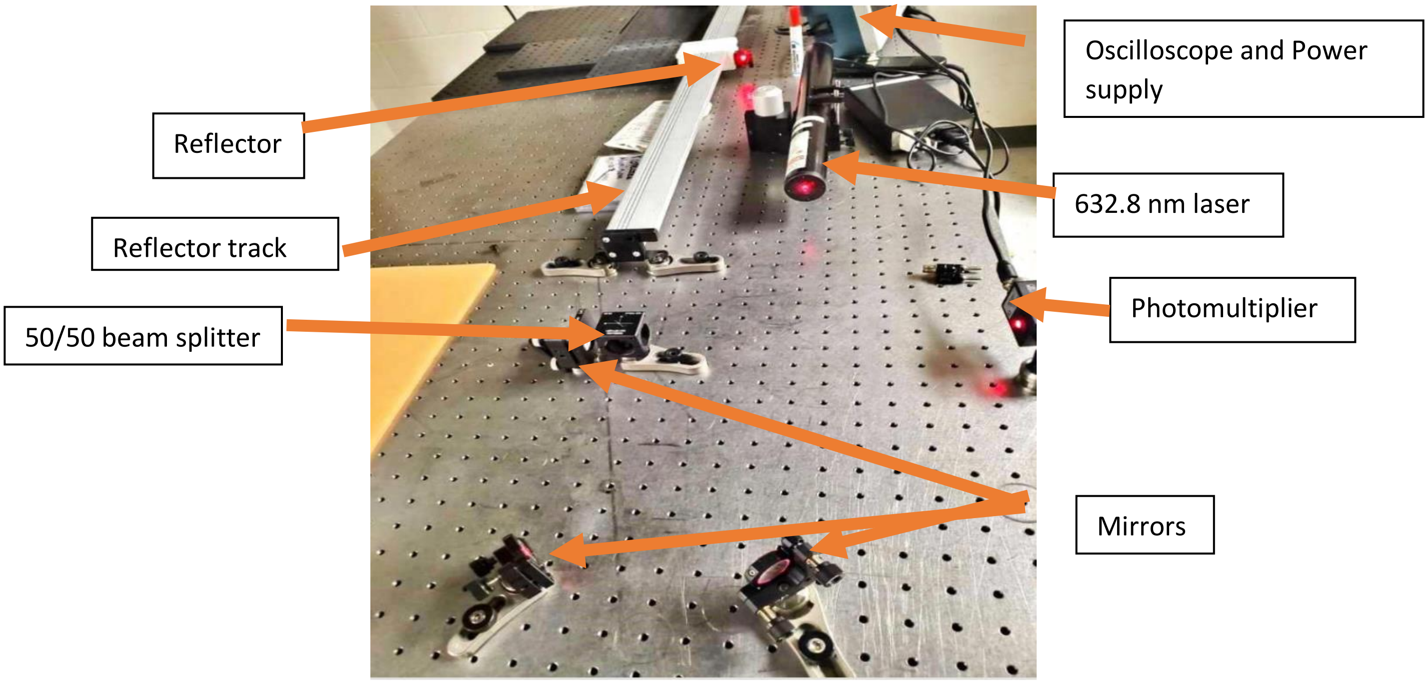

Fig. (2): apparatus for optical Doppler effect. Optical components are two mirrors angled to cooperate with reflector on track, and a 50% beam splitter such that an incident beam is reflected into the photomultiplier and the reflection track. Aligned so that the reflector track remains parallel with the incident beam and the reflector stays centralized.

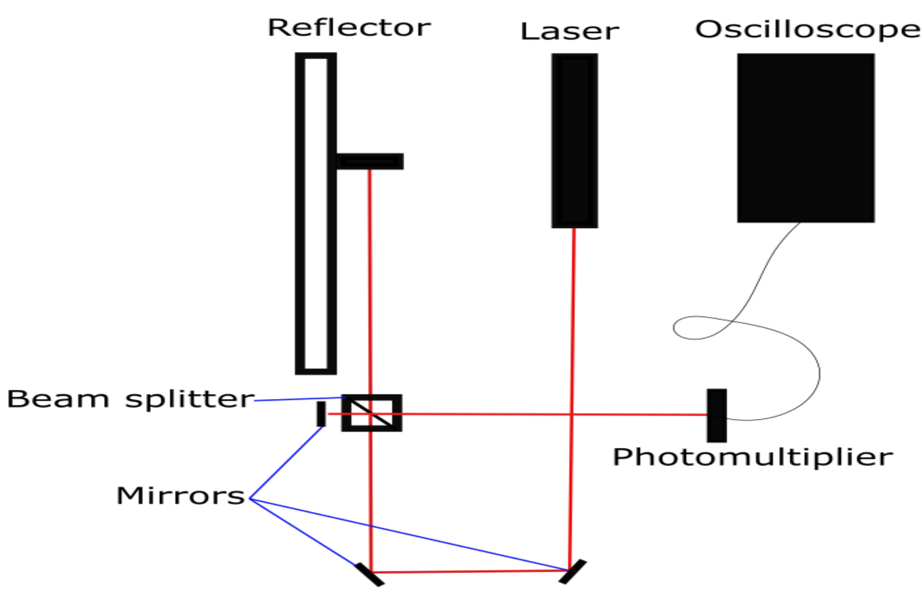

Fig (3): Overhead view of laser apparatus. The laser comes out of the lasing medium, reflects off two mirrors, 50% of the beam reflects into another mirror then into the photomultiplier, 50% transmits through splitter, the transmitted beam reflects off the moving reflector and reflects into the multiplier. This creates a doppler shifted beam and an original beam which both are incident upon the photomultiplier. These frequencies are then to be analyzed with beat frequency.

V. Results

With the experimental process expressed above, we observed verification of the Doppler effect. In calibration of our rail track, we evaluated velocities with different voltages from the power supply. This relation showcased a linear relation. A linear function can be placed through the data values yielding a slope conversion factor.
The conversion factor yields an A parameter of -2.38 +- .018 and a B parameter of 0.162 +- .054 $\frac{cm}{s}/Volt$. A relation can be presented for velocity as, $v = V_0 + \alpha V \rightarrow velocity = -.238 + .162V$. This voltage to velocity conversion factor was used to efficiently calculate velocity of the reflector along the track from the voltage power supply. The conversion value is denoted as α. The data below in fig. (4) displays strong linearity. The error bars being rather small due to minimal factory indicated error in voltage supply unit. The slight inconsistency in velocity production was due to varying voltage supply within the power generator. This linearity gives the best indication of a voltage supply to a resulting velocity along the track.

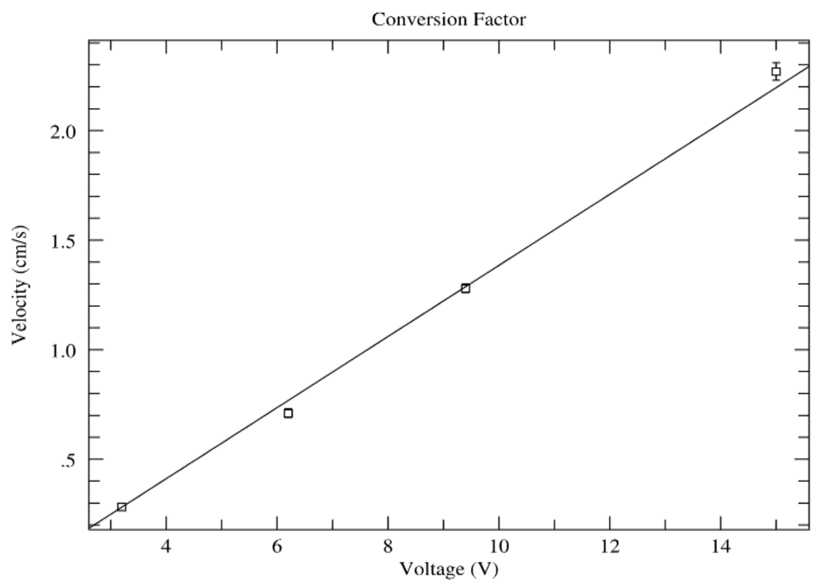

Fig. (4): Plot of conversion factor with voltage correlating to a measured velocity along the reflector track.

Gathering beat frequencies along different voltages, a linear plot was acquired (fig. (5)). The linear function expresses an increase in Doppler shift prevalence as velocity of the reflector increases. Each increase in velocity stretches the sinusoidal electromagnetic wave from the laser.

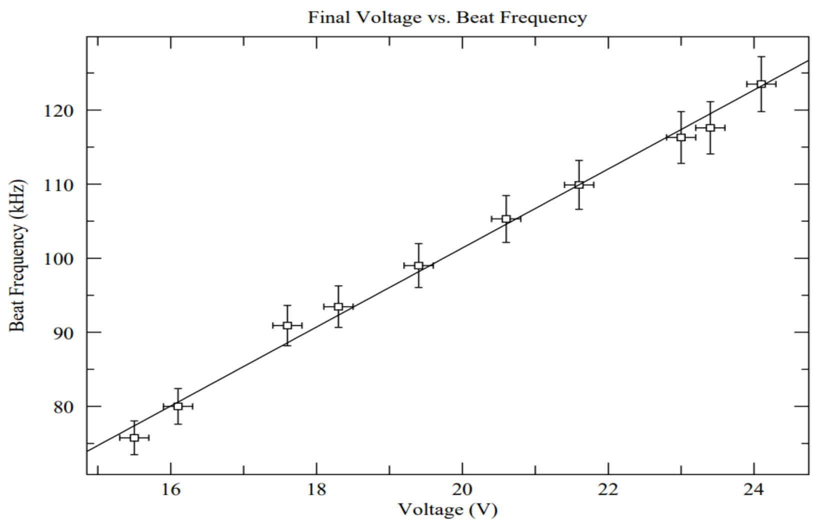

Fig (5): Final display of observed beat frequencies as a function of voltage

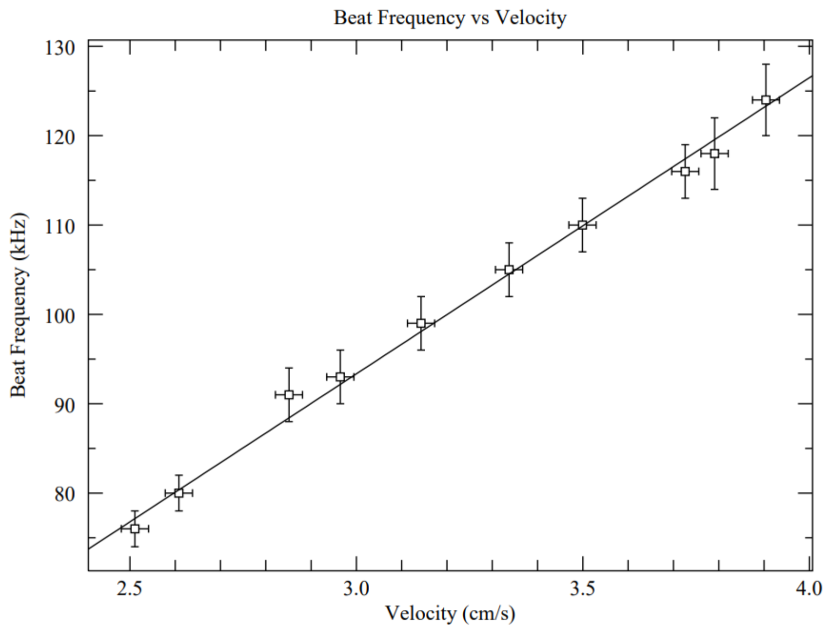

Fig (6): the conversion factor can be applied to the voltages yielding a relation of beat frequency as a function of velocity. This function produced a slope of 33.14 +- 0.86 kHz/(cm/s) and with some dimensional analysis, $\frac{33.14*10^3}{.01} = 3314000 \pm 86000\ \frac{Hz}{\frac{m}{s}} \rightarrow \frac{\frac{1}{s}}{\frac{m}{s}} \rightarrow \frac{1}{m}$. Eq. (2) predicts a function slope for beat frequency to be $\frac{2}{\lambda_0} = \frac{2}{632.8*10^{-9}} \approx 3200000\frac{1}{m}$. The accepted value falls $28000\ \frac{1}{m}$ or $\sim 0.875\%$ out of our observational value.

A function can be made to display our beat frequency values as a function of voltage compared to expected values. Our expected values were calculated using Eq. (2). This representation encapsulates our success in verification of the doppler shift equations and phenomenon. A linear best fit line is in place to display linearity among the expected values and observational.

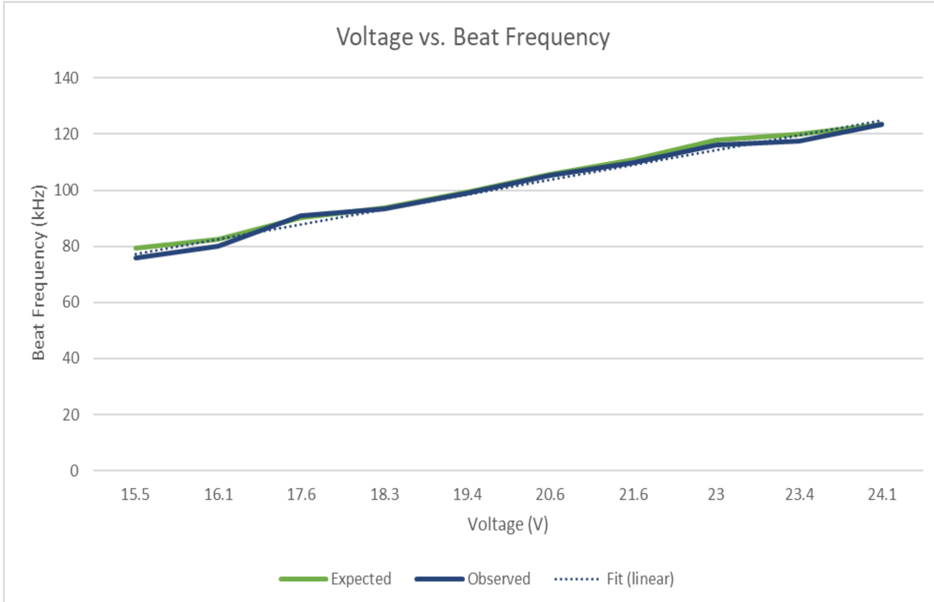

Fig. (7): Display of observational and expected beat frequency as a function of voltage.

Measured Data	Voltages (V)	Unc (V)	Beat f Observed (kHz)	Unc Beat (kHz)	Expected f KHz	Unc Expected (kHz)	Difference in Beat (Exp - Obs)
1	15.5	0.2	76	2	79.5	0.2	3.723412322
2	16.1	0.2	80	2	82.6	0.2	2.560189573
3	17.6	0.2	91	3	90.3	0.2	-0.657867299
4	18.3	0.2	93	3	93.8	0.2	0.381706161
5	19.4	0.2	99	3	99.5	0.2	0.472464455
6	20.6	0.2	105	3	105.6	0.2	0.336018957
7	21.6	0.2	110	3	110.8	0.2	0.863981043
8	23.0	0.2	116	3	117.9	0.2	1.643127962
9	23.4	0.2	118	4	120.0	0.2	2.394312796
10	24.1	0.2	124	4	123.6	0.2	0.083886256
Unc Expect	2*(W41*P37)*(SQRT(Q37/P37)^2+(X41/W41)^2+(0.0000000005/0.0000006328)^2)						
Unc Beat	(0.03*Y41)						
Expected Beat	((2*(0.001623*W41))/(633*10^-9))/1000						

Fig. (8): Final data table for ten increasing voltages. Expected values calculated using Eq. (2). The final data table outlines a voltage given to the reflector which creates a velocity at which the reflector moved down the track. Using the oscilloscope, beat frequencies were acquired. These values were then compared to theoretical values by taking the difference in the two.

VI. Discussion/Conclusions

The Doppler effect is an everyday phenomenon seen in police radar systems, meteorology, and diagnosing blood related issues. As an embodiment of sinusoidal source behavior, this effect can be seen every day. Complex issues can be modeled and better understood with physical demonstrations. The optical Doppler effect elaborated upon experimental understanding, relativity and beat frequencies. Utilization of beat frequency elaborated upon our knowledge of superimposing sinusoidal wave functions through Fourier analysis. The movement in a reflector mirror was responsible for the shift in incident frequency and observed frequency. This dynamic could be investigated further by using different laser wavelengths.

VII. References

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3. NTi. *FFT*, https://www.nti-audio.com/en/support/know-how/fast-fourier-transform-fft#:~:text=The%20%22Fast%20Fourier%20Transform%22%20(,frequency%20information%20about%20the%20signal.
4. CSBSJU physics 332 “Optical Doppler Effect”

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