Project 3

MATH 339

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Introduction

Understanding the intricate balance of natural ecosystems is a complex yet vital endeavor, particularly when examining the dynamics between predator and prey species. The interplay between deer and wolf populations presents a classic example of such ecological interactions, where the survival and proliferation of each species are tightly interwoven with the other. This report delves into the heart of these relationships through the lens of discrete mathematical models, which serve as powerful tools for simulating and predicting population trends over time.

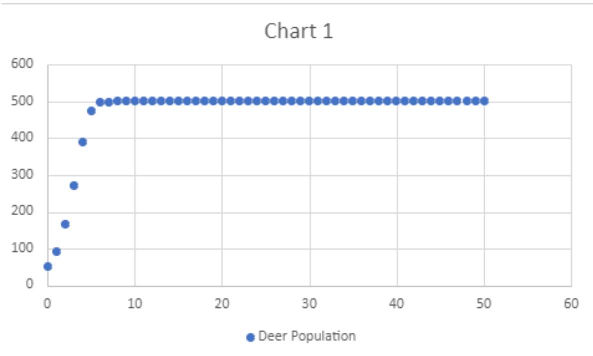
Through a series of simulations, we explore various scenarios, including the impact of changing growth rates and hunting practices on the population stability of both species.

Note: The abbreviation *IP* stands for Initial Population. This means that the initial conditions are different from the baseline plot. The abbreviation *SA* stands for Sensitivity Analysis. This means the conditions are different to compute a sensitivity analysis. Please see the attached Excel Spreadsheet for more information.

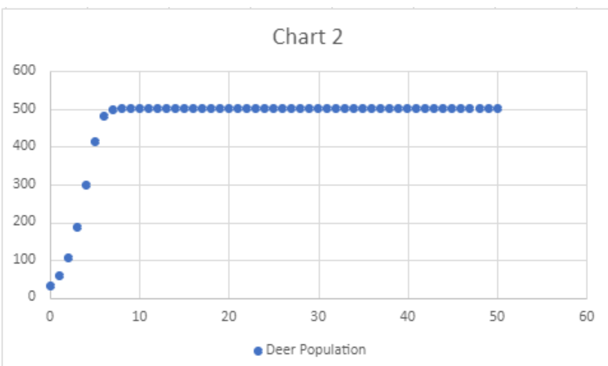
Single Species Model

Our first models represented the deer population without hunting or natural predators. The current models have the deer population exploding to the carrying capacity after a few years, regardless of the initial population. We hypothesize this is due to the lack of hunting, competition, and natural predators. For more information, see spreadsheet “Question1”.

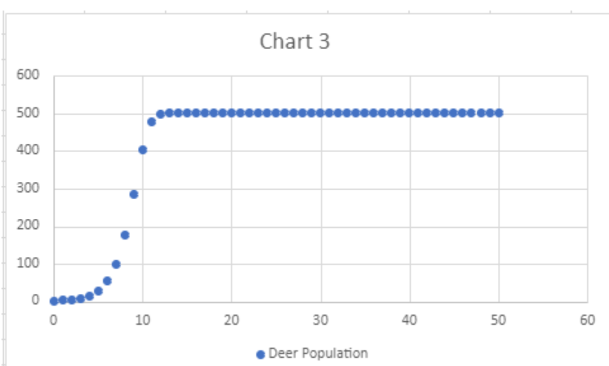
Initial Population: 50



Initial Population: 30



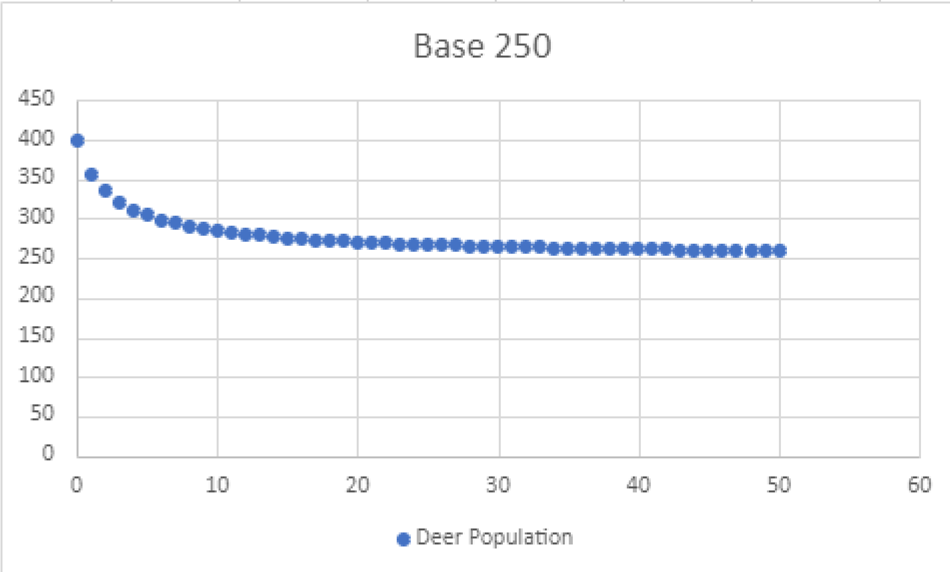
Initial Population: 1



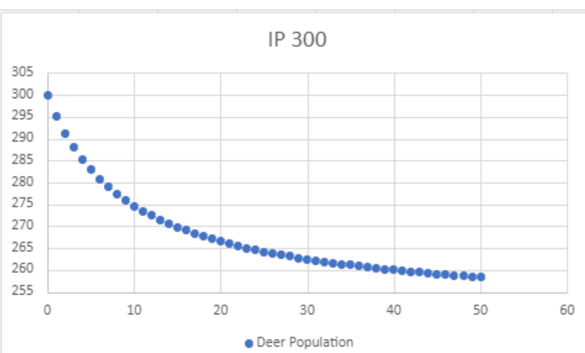
Single Species Model with Constant Harvest Amount

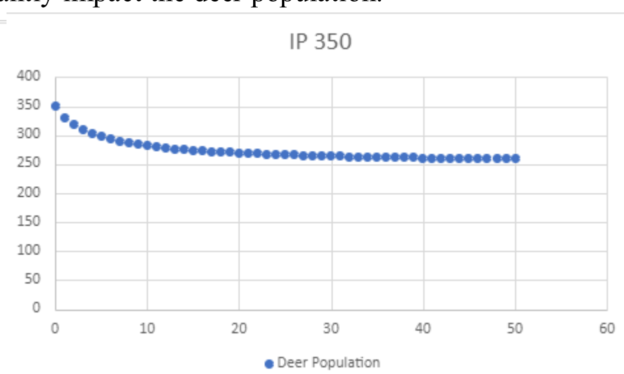
Our next models represent the deer population considering the effect of hunting. We modeled hunting by using a fixed harvesting amount. For more information, see spreadsheet “Question2”.

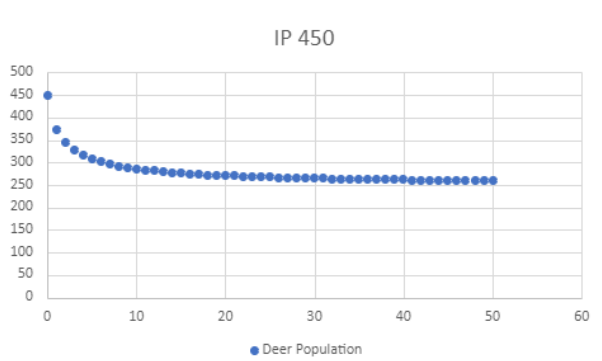
We found the best number of deer to hunt to get to half the carrying capacity (250 deer) was by hunting 120 deer every year.

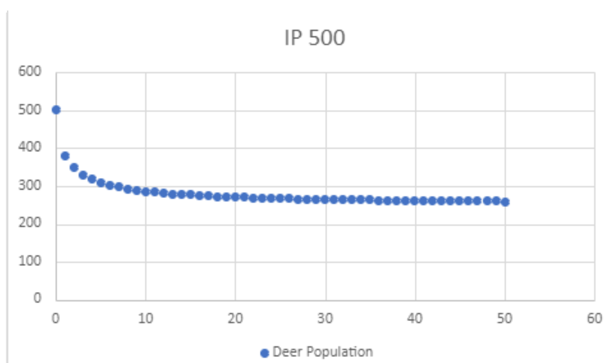


We found that the deer population is not extremely sensitive to the initial population. Changing the deer population by plus or minus 100 does not significantly impact the deer population.

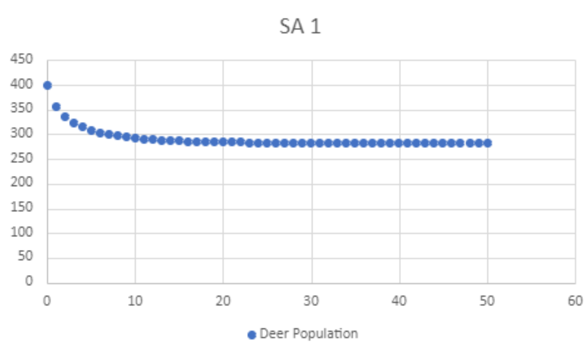


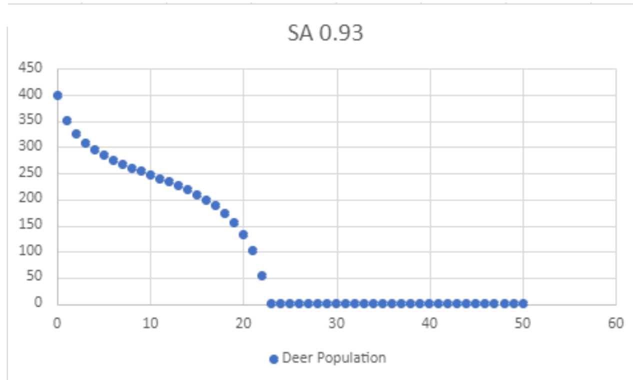


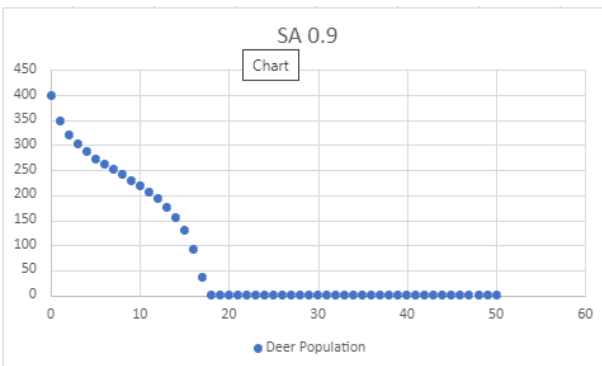




The deer population was extremely sensitive to the growth rate. Changing the growth rate from 0.96 to 1.00 resulted in the population halting at around 281 instead of 250. Decreasing the growth rate to 0.93 resulted in the deer population dying off entirely in 23 years. Decreasing the growth rate of 0.9 resulted in the deer population dying off entirely in 18 years.



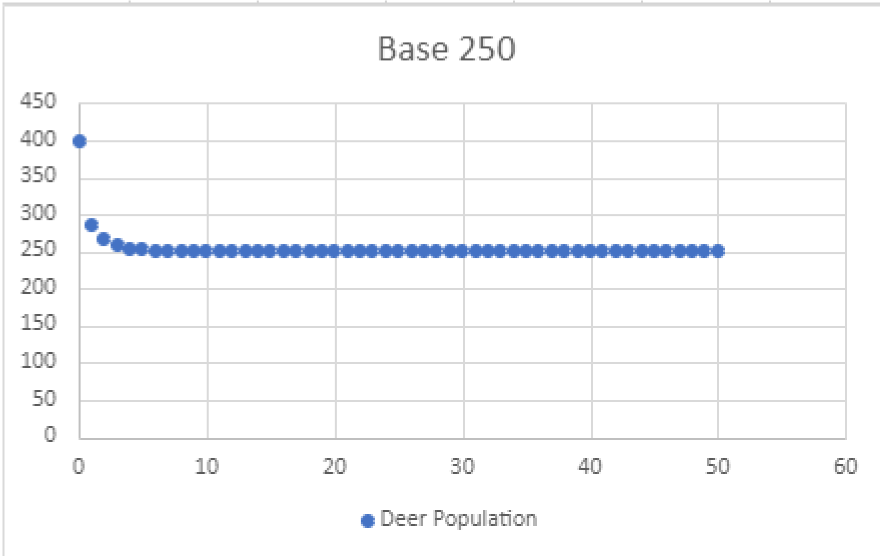




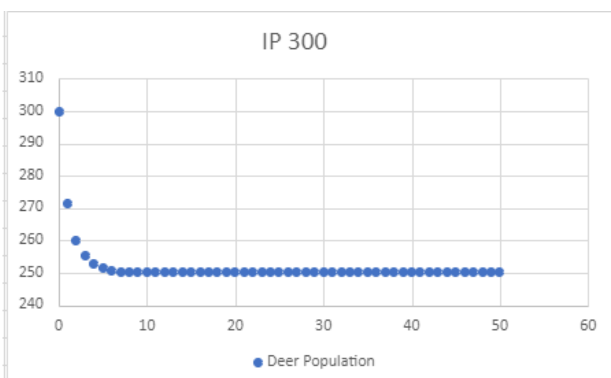
Single Species Model with Percentage Harvest Amount

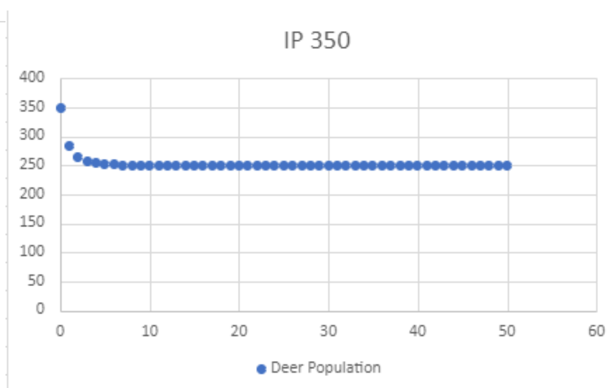
Our next models represent the deer population considering the effect of hunting. We modeled hunting by using a percentage harvesting amount. For more information, see spreadsheet “Question3”.

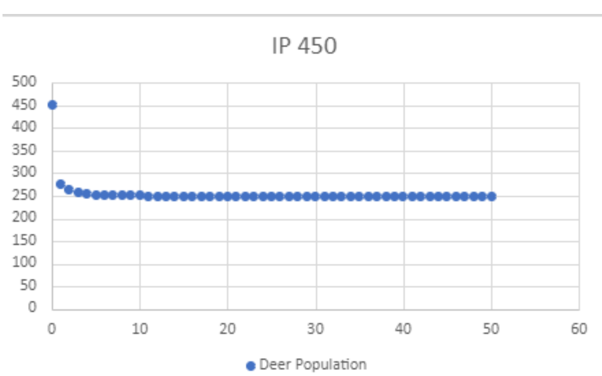
We found the percent harvest amount to get to half the carrying capacity (250 deer) was 48% of the deer population every year.

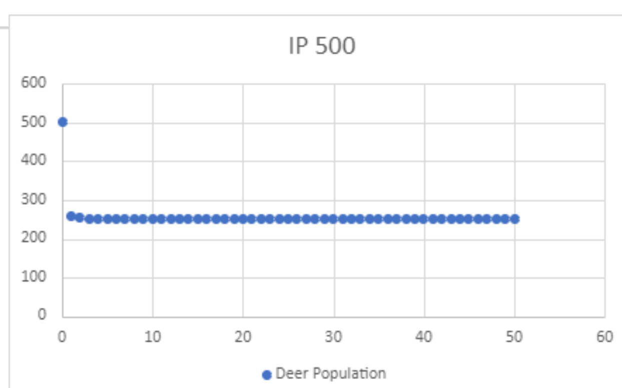


We found that the deer population is not extremely sensitive to the initial population. Changing the initial deer population by any amount will not significantly impact it. The deer population will return to 250.

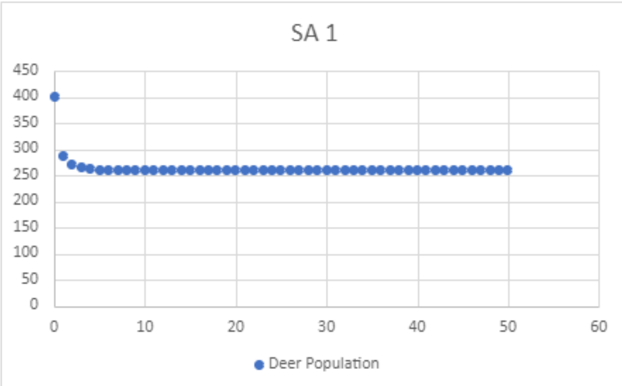


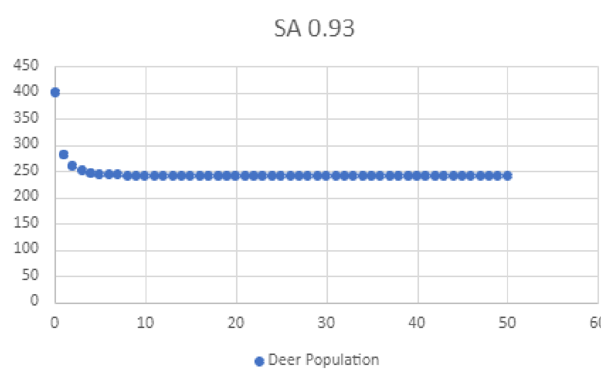


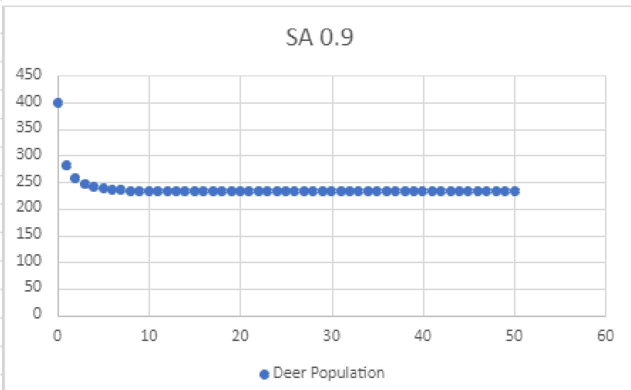




The deer population was not as sensitive to the growth rate as the last model. The deer population did not die off at 0.93 or 0.9. Instead, it changed the resulting population to 242 for a growth rate of 0.93 and 233 for a growth rate of 0.9.







Our Hunting Advice

If the Department of Natural Resources (DNR) were to seek guidance on establishing a sustainable framework for the annual deer hunt, our recommendation would be to implement a percentage-based harvest strategy. This approach involves setting a target harvest rate as a percentage of the estimated deer population, rather than a fixed number of deer. Such a method offers several advantages that align with conservation objectives and population management goals.

A percent harvest amount is inherently adaptive. By pegging the harvest quota to a proportion of the current population, the strategy automatically adjusts to fluctuations in deer numbers. This is particularly beneficial in years when the population may be lower due to factors such as disease, severe weather, or food scarcity. A fixed number harvest could inadvertently lead to overharvesting under these conditions, potentially destabilizing the population.

By adopting the percent harvesting approach, the DNR can ensure that the yearly deer hunt contributes positively to the ecological balance, provides recreational opportunities, and upholds the conservation of the species for future generations.

Two Species Model

Our next model represents the interaction between deer and wolf populations. We modeled the interaction between the species by using a predator-prey model. For more information, see spreadsheet "Question5".

We were given the following information to construct a model for two species:

- In the absence of deer, the wolf population would decrease by 25% each year
- The presence of 10 wolves decreases the deer population by 10%
- The presence of 100 deer increases the wolf population by 12.5%

We assumed the following information from the previous models:

- Deer growth rate of 0.96
- Initial deer population of 400
- Deer population carrying capacity of 500

We had to make the following assumption:

- Initial wolf population of 8
    - This information was obtained from the National Wildlife Federation
    - Link: https://www.nwf.org/Educational-Resources/Wildlife-Guide/Mammals/Gray-Wolf

Given the above information, we were able to construct our model. Below, we calculated the equilibrium points and obtained their eigenvalues to discern their type.

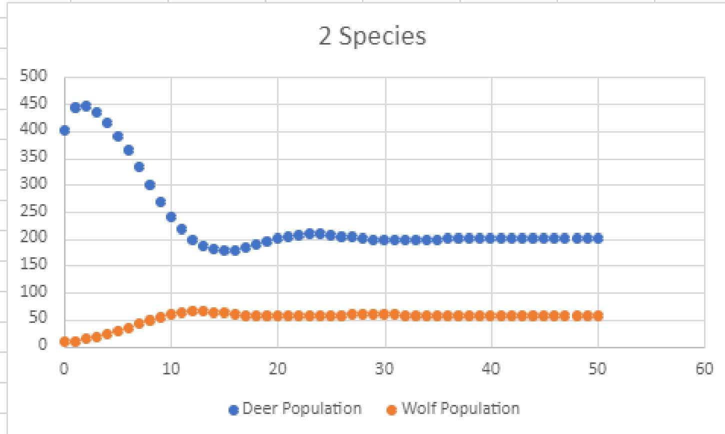
```
X deer, Y wolves

In[∘]:= growth = 0.96;

In[∘]:= Deer[x_, y_] = x + (x * growth * (1 - x / 500)) - (y / 10) * 0.1 * x;
        Wolves[x_, y_] = y - 0.25 y + (x / 100 * 0.125 * y);

In[∘]:= N[Solve[{Deer[x, y] == x, Wolves[x, y] == y}]]

  Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained
      by solving a corresponding exact system and numericizing the result.

Out[∘]= {{x → 200., y → 57.6}, {x → 500., y → 0.}, {x → 0., y → 0.}}

In[∘]:= J = ( ∂x Deer[x, y]     ∂y Wolves[x, y] )
            ( ∂x Wolves[x, y]   ∂y Deer[x, y]   )

Out[∘]= {{1 + 0.96 (1 - x/500) - 0.00192 x - 0.01 y, 0.75 + 0.00125 x}, {0.00125 y, -0.01 x}}

In[∘]:= N[Eigenvalues[J /. {x → 200.`, y → 57.6`}]]

Out[∘]= {-2.02724, 0.643239}

In[∘]:= N[Eigenvalues[J /. {x → 500.`, y → 0.`}]]

Out[∘]= {-5., 0.04}

In[∘]:= N[Eigenvalues[J /. {x → 0.`, y → 0.`}]]

Out[∘]= {1.96, 0.}
```

Equilibrium Points:

- (200, 57.6)
    - Saddle; the magnitude of one eigenvalue is greater than 1 (2.02724) while the other is less than 1 (0.643239)
- (500, 0)
    - Saddle; the magnitude of one eigenvalue is greater than 1 (5) while the other is less than 1 (0.04)
- (0, 0)
    - Unknown; an eigenvalue containing 0 gives us no indication of what happens around this eigenvalue and the other eigenvalue has a magnitude greater than 1 (1.96)

The long-term behavior is dependent on the initial conditions of the model. However, with the assumptions and constraints described above, the deer population will hover around 200 and the wolf population will hover around 57.6.

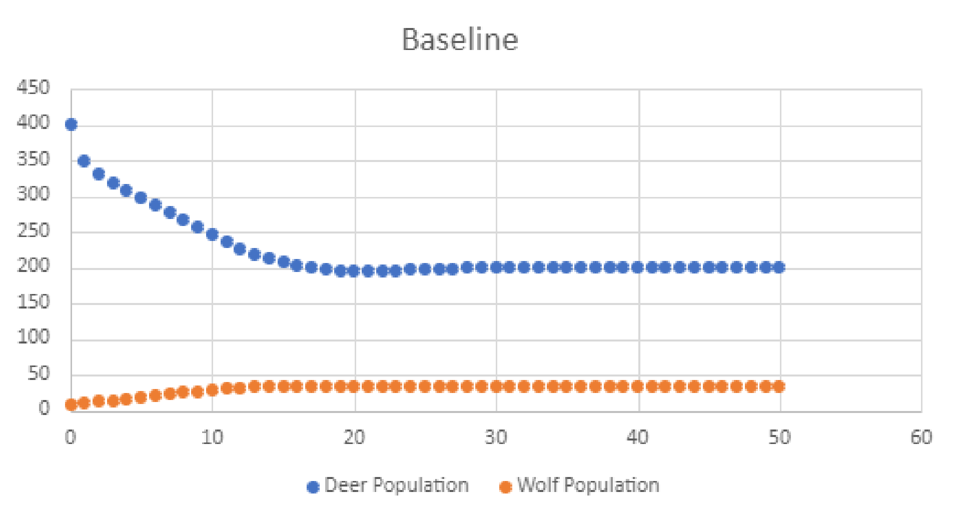
2 Species				
n	deer pop	wolf pop	deer growth	deer capacity
0	400	8	0.96	500
1	444.8	10		
2	447.4617	13.06		
3	434.1602	17.09981		
4	414.8029	22.10493		
5	390.9638	28.04019		
6	363.1849	34.73351		
7	332.4413	41.81849		
8	300.3699	48.74161		
9	269.0935	54.85685		
10	240.7773	59.59466		
11	217.1236	62.6323		
12	199.0588	63.97291		
13	186.7327	63.89765		
14	179.7296	62.83797		
15	177.3104	61.24578		



Two Species Model with Hunting Deer

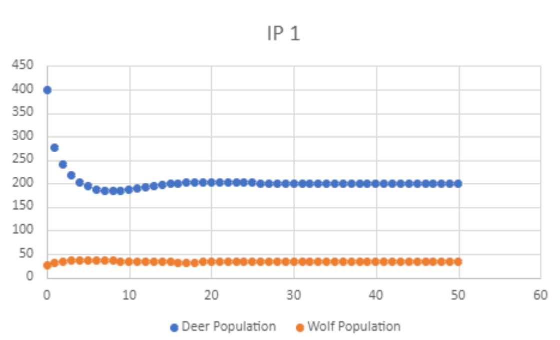
Our next models represent the interaction between deer and wolf populations considering deer hunting. We modeled the interaction between the species by using a predator-prey model as in the model above. We decided to use the percentage harvest to model hunting due to our reasoning from Our Hunting Advice. For more information, see spreadsheet “Question6”.

To create this model, we reused many of the constraints from our last model (Two Species Model). We decided to use a decreased hunting rate of 0.25 because the deer population fell below half the carrying capacity (250) when wolves were introduced.



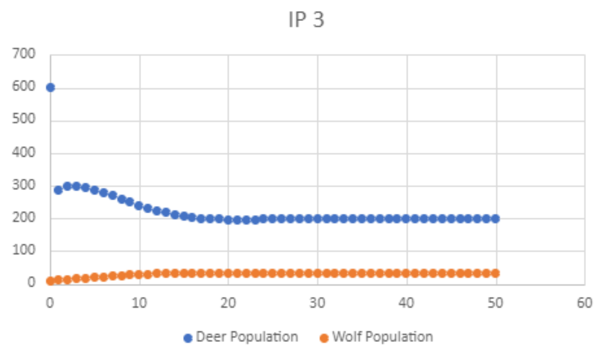
We then experimented with different initial populations of deer and wolves to see how the populations would evolve over time. We found that the populations were not sensitive to the initial conditions. The deer population made its way to 200 and the wolf population made its way to 57.6

Deer: 400
Wolves: 25

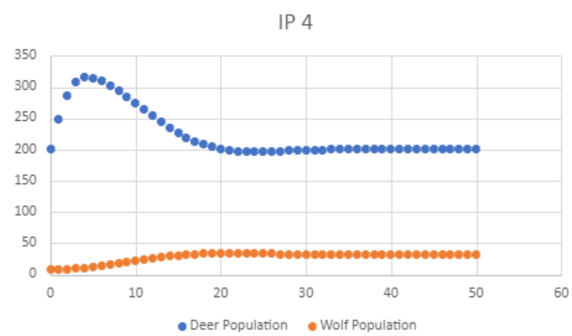


Deer: 400
Wolves: 1

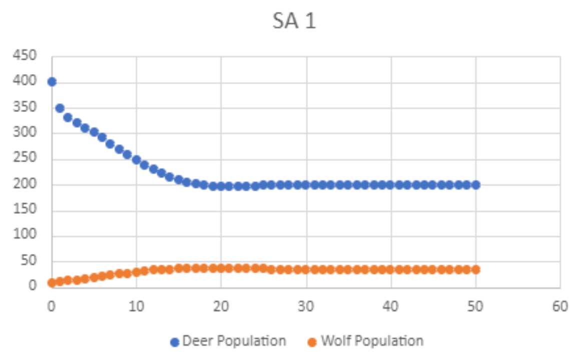
Deer: 600
Wolves: 8

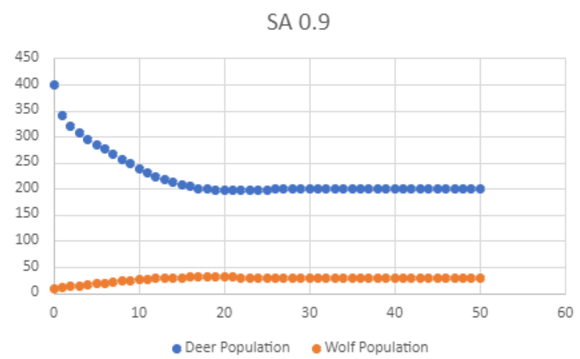


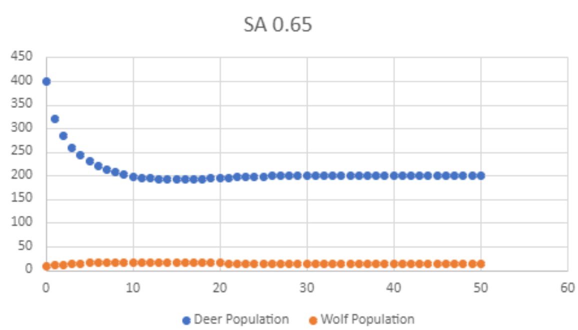
Deer 200
Wolves: 8



The deer and wolf populations were not sensitive to the growth rate. Neither population died off until the growth rate was below 0.65.





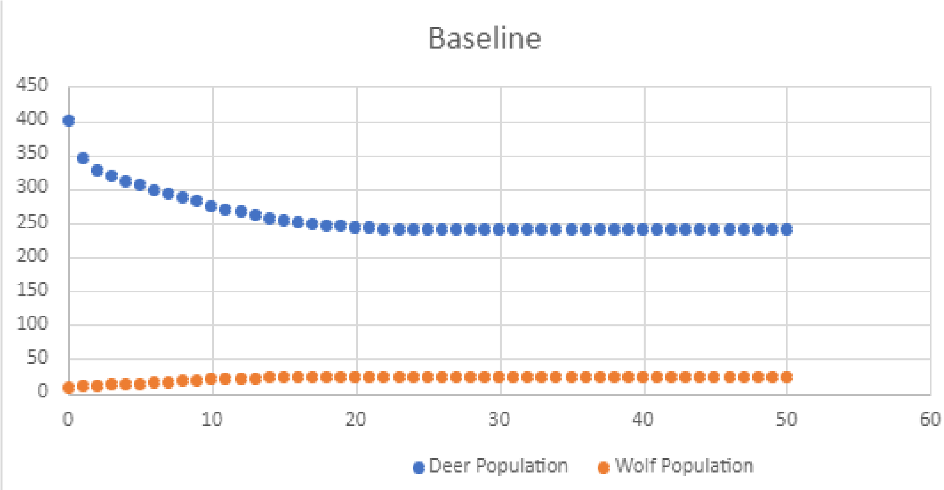


Since both the deer and wolf populations are stable given different initial conditions and growth rates, this method is sustainable.

Two Species Model with Hunting Deer and Wolves

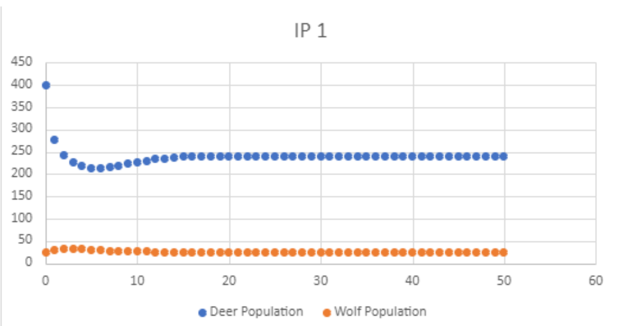
Our next models represent the interaction between deer and wolf populations considering deer and wolf hunting. We modeled the interaction between the species by using a predator-prey model as in the model above. We decided to use the percentage harvest to model hunting both populations due to our reasoning from Our Hunting Advice. For more information, see spreadsheet "Question7".

To create this model, we reused many of the constraints from our last model (Two Species Model with Hunting Deer). We included a hunting rate of 0.05 for wolves.

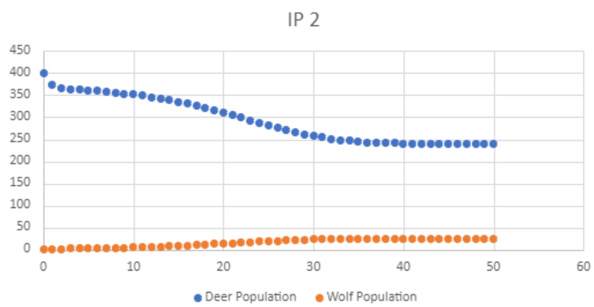


We then experimented with different initial populations of deer and wolves to see how the populations would evolve over time. We found that the populations were not sensitive to the initial conditions. The deer population made its way to 240 and the wolf population made its way to 25.

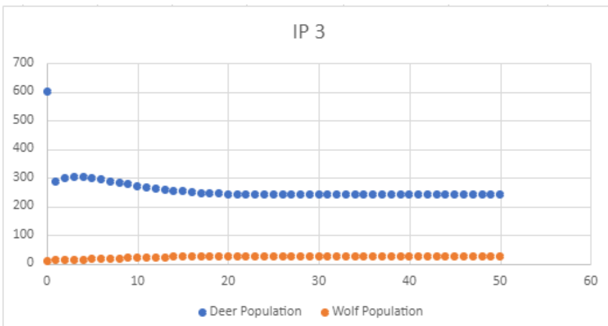
Deer: 400
Wolves: 25



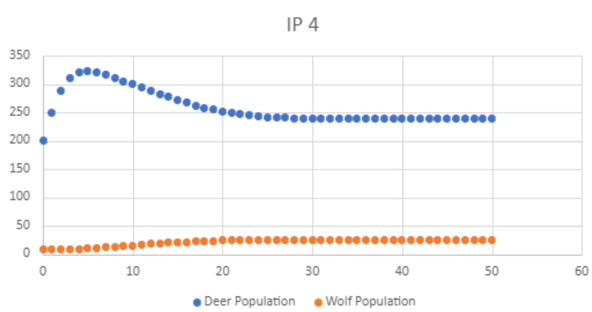
Deer: 400
Wolves: 1



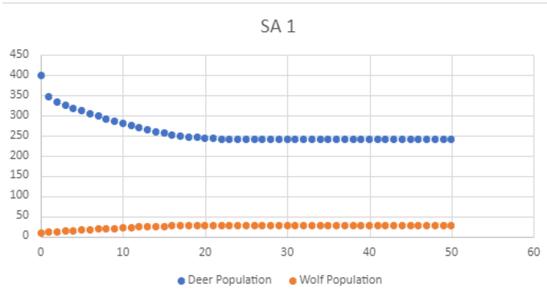
Deer: 600
Wolves: 8

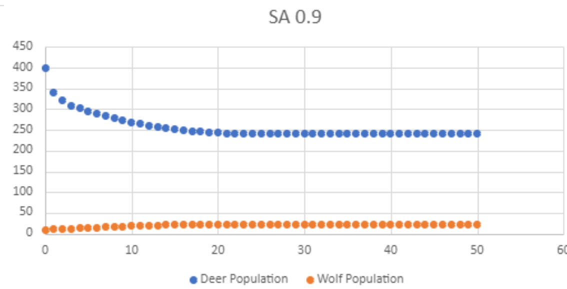


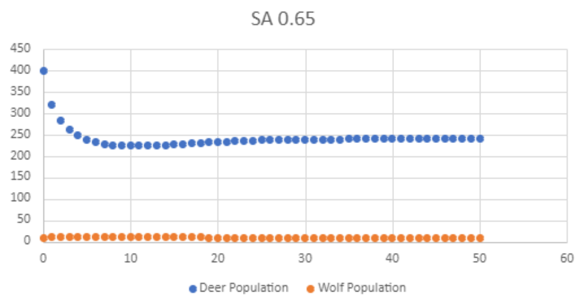
Deer 200
Wolves: 8



The deer and wolf populations were not sensitive to the growth rate. Neither population died off until the growth rate was below 0.65.







Since both the deer and wolf populations are stable given different initial conditions and growth rates, this method is sustainable.

Conclusion

Based on our models, we conclude that both deer and wolves can coexist. Not only can the species coexist, but they can coexist while being hunted. Given our models, both populations are resilient to changes in populations and the growth rate of the deer. Granted, for both populations to survive and coexist, the hunting must be managed appropriately. We suggest a proportionate approach to hunting as it evaluates the population of the hunted based on their current population. This accounts for bad years for a population. There could be a bad winter or other atypical events and a proportional approach accounts for this. Conversely, a single fixed rate of harvest could lead to an overharvest in a hard year.