Compound Poisson Processes

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1 Concept

Definition 1.1 (Counting Processes)**.** *A random process* $\{N(t), t \in [0, \infty)\}$ *is a counting process if,*

1. $N(0) = 0$.
2. $N(t) \in \{0, 1, 2, 3, 4, ...\}$ *and is non-decreasing.*

Definition 1.2 (Poisson Processes)**.** *A counting process* $N(t)$ *is a poisson process with rate* λ *if,*

1. $N(t)$ *has independent increments. That is the set* $N(t_j + s_j) - N(t_j),\ j \in \{0, 1, 2, ..., n\}$ *is independent for each non-overlapping increment* $(t_j, t_j + s_j]$.
2. *For all* $t \geq 0$ *and* $h > 0$, $N(t + h) - N(t) \sim POIS(\Lambda)$ *where* $\Lambda = \int_t^{t+h} \lambda(z)dz$.

Definition 1.3 (Compound Poisson Process)**.** *A compound poisson process S(t) is defined as follows:*

1. *For* $t > 0, S(t) = \sum_{i=1}^{N(t)} X_i$, *where* $N(t)$ *is a poisson process with rate function* λ,
2. *All random variables* X_i *and* $\{N(t), t > 0\}$ *are independent and identically distributed,*
3. $N(t) = 0 \implies S(0) = 0$.

2 Examples and Exploration

References

Daniel, James W. 2008. "Poisson Processes (and Mixture Distributions)." http://www.casact.org/library/studynotes/3/_Poisson/_2008.pdf.

Pishro-Nik, Hossein. 2004. *Introduction to Probability.* Kappa Research LLC. https://www.probabilitycourse.com.