Poisson Processes

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Concept

Definition (Counting Processes)**.** *A random process* $\{N(t), t \in [0, \infty)\}$ *is a counting process if,*

1. $N(0) = 0$.
2. $N(t) \in \{0, 1, 2, 3, 4, ...\}$ *and is non-decreasing.*

Definition (Poisson Processes)**.** *A random process* $N(t)$ *is a poisson process with rate* λ *if,*

1. $N(t)$ *has independent increments. That is the set* $N(t_j + s_j) - N(t_j),\ j \in \{0, 1, 2, ..., n\}$ *is independent for each non-overlapping increment* $(t_j, t_j + s_j]$.
2. *For all* $t \geq 0$ *and* $h > 0$, $N(t + h) - N(t) \sim POIS(\Lambda)$ *where* $\Lambda = \int_t^{t+h} \lambda(z)dz$.

Examples and Exploration