Compound Poisson Processes

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Concept

Definition (Counting Processes)**.** *A random process* $\{N(t), t \in [0, \infty)\}$ *is a counting process if,*

1. $N(0) = 0$.
2. $N(t) \in \{0, 1, 2, 3, 4, ...\}$ *and is non-decreasing.*

Definition (Poisson Processes)**.** *A random process* $N(t)$ *is a poisson process with rate* λ *if,*

1. $N(t)$ *has independent increments. That is the set* $N(t_j + s_j) - N(t_j),\ j \in \{0, 1, 2, ..., n\}$ *is independent for each non-overlapping increment* $(t_j, t_j + s_j]$.
2. *For all* $t \geq 0$ *and* $h > 0$ *,* $N(t + h) - N(t) \sim POIS(\Lambda)$ *where* $\Lambda = \int_t^{t+h} \lambda(z)dz$.

Definition (Compound Poisson Process)**.** *A compound poisson process S(t) is defined as follows:*

1. *For* $t > 0, S(t) = \sum_{i=1}^{N(t)} X_i$*, where* $N(t)$ *is a poisson process with rate function* λ*,*
2. *All random variables* X_i *and* $\{N(t), t > 0\}$ *are independent and identically distributed,*
3. $N(t) = 0 \implies S(0) = 0$.

Examples and Exploration

Sources

https://www.probabilitycourse.com/chapter11/11_1_1_counting_processes.php

https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.734.750&rep=rep1&type=pdf