

Problem Statement 1: [50 marks]

- a) $X \sim \text{Bin}(6, 0.3)$, therefore the probability is given by

$$\begin{aligned} P(X = 2) &= \binom{n}{x} * p^x * (1 - p)^{n-x} \\ &= \binom{6}{2} * 0.3^2 * (1 - 0.3)^{6-2} \\ &= 15 * 0.09 * 0.2401 = \mathbf{0.324135} \end{aligned}$$

- b) The average value is given by $E(X) = n p = 6 * 0.3 = \mathbf{1.8}$

- c) The standard deviation associated with it is given by $s = \sqrt{\text{Var}(X)} = n p(1 - p)$
 $= \sqrt{6 * 0.3 * (1 - 0.3)}$
 $= \sqrt{1.8 * 0.7} = \mathbf{1.122}$ (to 3dp)

Problem Statement 2: [50 marks]

a) $G \sim \text{Bin}(8, 0.75)$, therefore the probability is given by

$$\begin{aligned} P(G \text{ will get 5 correct}) &= \binom{8}{5} * p^x * (1 - p)^{n-x} \\ &= \binom{8}{5} * 0.75^5 * (1 - 0.3)^{8-5} \\ &= \mathbf{0.2076} \end{aligned}$$

a) $B \sim \text{Bin}(12, 0.45)$, therefore the probability is given by

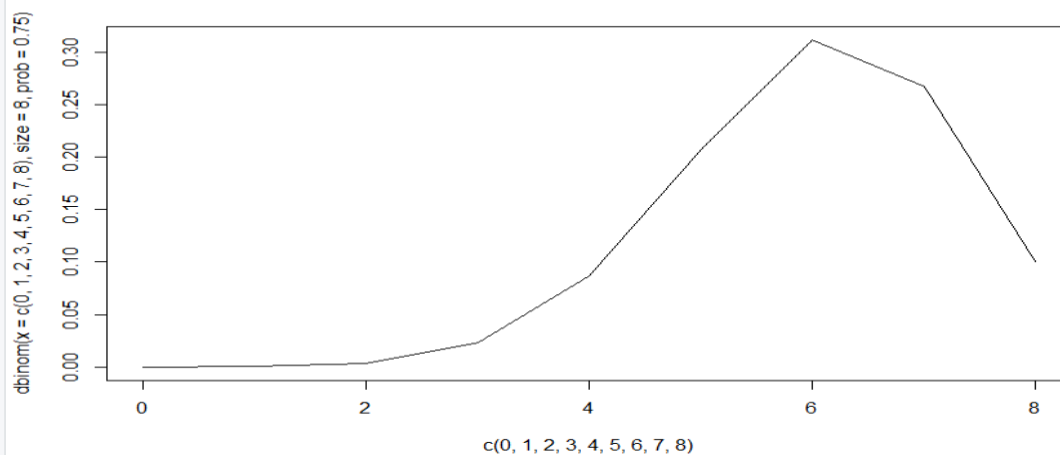
$$\begin{aligned} P(B \text{ will get 5 correct}) &= \binom{12}{5} * p^x * (1 - p)^{n-x} \\ &= \binom{12}{5} * 0.45^5 * (1 - 0.45)^{12-5} \\ &= \mathbf{0.2224} \end{aligned}$$

In cases of 4 and 6 probabilities of getting 6 correct is much higher than that of getting 4 correct

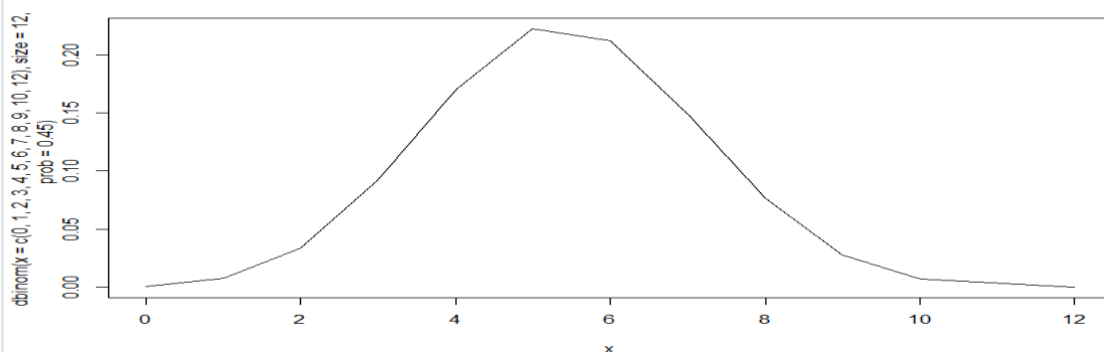
- Number of trials
- Probability of correction

Pictorial View

$$G \sim \text{Bin}(8, 0.75)$$



$$B \sim \text{Bin}(12, 0.45)$$



Problem Statement 3: [100 marks]

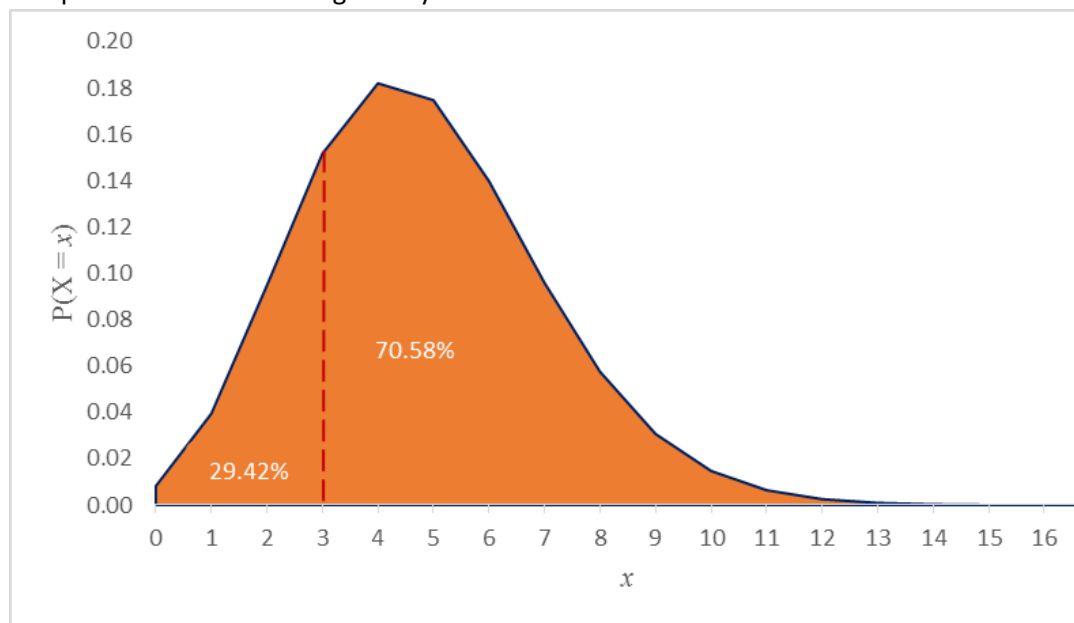
$$\mu = \left(\frac{72}{60}\right) * 4 = 4.8, \text{ thus } X \sim Po(4.8)$$

$$\text{a) } P(X = 5) = \frac{e^{-\mu} \mu^x}{x!} = \frac{e^{-4.8} * 4.8^5}{5!} = \mathbf{0.1747}$$

$$\begin{aligned} \text{b) } P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\ &= \frac{e^{-4.8} * 4.8^0}{0!} + \frac{e^{-4.8} * 4.8^1}{1!} + \frac{e^{-4.8} * 4.8^2}{2!} + \frac{e^{-4.8} * 4.8^3}{3!} = \mathbf{0.2942} \end{aligned}$$

$$\text{c) } P(X > 3) = 1 - P(X \leq 3) = 1 - 0.2942 = \mathbf{0.7058}$$

d) The pictorial view of this is given by



Problem Statement 4: [100 marks]

$X \sim Po(6)$

- a) For a 455 word document, the error rate becomes lower i.e. $\mu = \frac{455}{4260} * 6 = 0.591$, thus $X \sim Po(0.591)$

$$P(X = 2) = \frac{e^{-0.591} * 0.591^2}{2!} = \mathbf{0.0964}$$

- b) For a 1000 word document, the error rate becomes $\mu = \frac{1000}{4260} * 6 = \frac{100}{77} = 1.299$, thus $X \sim Po(1.299)$

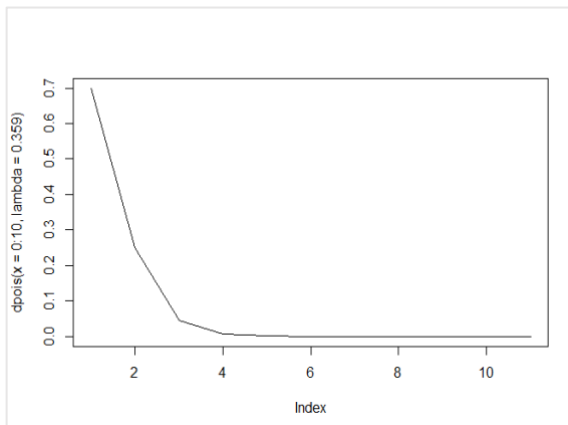
$$P(X = 2) = \frac{e^{-1.299} * 1.299^2}{2!} = \mathbf{0.2301}$$

- c) For a 255 word document, the error rate becomes $\mu = \frac{255}{4260} * 6 = 0.359$, thus $X \sim Po(0.359)$

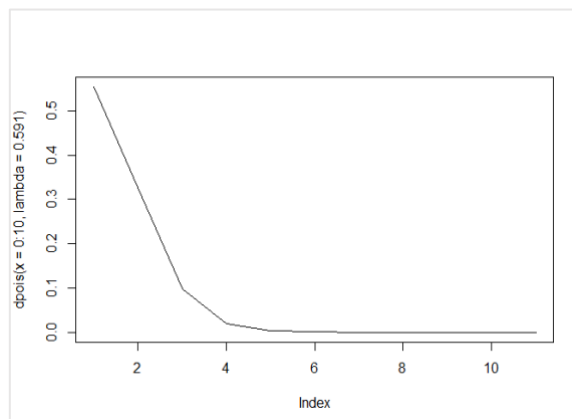
$$P(X = 2) = \frac{e^{-0.359} * 0.359^2}{2!} = \mathbf{0.045}$$

The likelihood of making 2 errors increases as the number of words increases and decreases as the number of words in a document decreases.

255 Words



455 Words



1000 Words

