

Assignment_9.11476

Solution 1:

$$H_0: \mu = \text{Rs. } 52 \quad \text{vs} \quad H_1: \mu > \text{Rs. } 52$$

$$n = 100, \quad \bar{x} = \text{Rs. } 52.80, \quad \sigma = \text{Rs. } 4.50, \quad \alpha = 0.05$$

Calculate the Z score as follows

$$\begin{aligned} Z^* &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{52.8 - 52}{\frac{4.50}{\sqrt{100}}} = \frac{0.8}{0.45} = \mathbf{1.778} \end{aligned}$$

Now we need to compute the appropriate **p-value** based on our alternative hypothesis.

$$\begin{aligned} P(Z > Z^*) &= P(Z > 1.778) \\ &= 1 - P(Z < 1.778) \\ &= 1 - 0.9623 = \mathbf{0.0377} < 0.05 \end{aligned}$$

Since the calculated **p-value (0.0377)** is less than the level of significance (**0.05**), we reject the null hypothesis and conclude that the average cost of the bookstore textbook is higher than Rs. 52

Solution 2:

$$H_0: \mu = 34 \quad \text{vs} \quad H_1: \mu < 32.5$$

$$n = 50, \quad \bar{x} = 32.5, \quad \sigma = 8, \quad \alpha = 0.01$$

Calculate the Z score as follows

$$\begin{aligned} Z^* &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{32.5 - 34}{\frac{8}{\sqrt{50}}} = \frac{-1.5}{1.13} = \mathbf{-1.33} \end{aligned}$$

Now we need to compute the appropriate **p-value** based on our alternative hypothesis.

$$\begin{aligned} P(Z < Z^*) &= P(Z < -1.33) \\ &= 1 - P(Z < 1.33) \\ &= 1 - 0.9082 = \mathbf{0.0918} > 0.01 \end{aligned}$$

From the above calculations the test statistic lies in the Acceptance Region for H_0 Therefore we fail to reject H_0

Solution 3:

$$H_0: \mu = 1135 \quad vs \quad H_1: \mu \neq 1031.31$$

$$n = 22, \quad \bar{x} = 1031.31, \quad \sigma = 240.37, \quad \alpha = 0.05/2$$

Calculate the Z score as follows

$$\begin{aligned} Z^* &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{1031.31 - 1135}{\frac{240.37}{\sqrt{22}}} = \frac{103.69}{51.25} = -2.023 \end{aligned}$$

Now we need to compute the appropriate **p-value** based on our alternative hypothesis.

$$\begin{aligned} P(Z > Z^*) &= P(-2.02 < Z < 2.02) \\ &= 1 - P(Z < 2.02) \\ &= 1 - 0.9783 = 0.0217 < 0.025 \end{aligned}$$

From the above calculations the test statistic lies in the rejection Region for H_0
Therefore we reject H_0

Solution 4:

$$H_0: \mu = 48,432 \quad vs \quad H_1: \mu \neq 48,574$$

$$n = 400, \quad \bar{x} = 48,574, \quad \sigma = 2000, \quad \alpha = 0.05$$

Calculate the Z score as follows

$$\begin{aligned} Z^* &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{48,432 - 48,574}{\frac{2000}{\sqrt{400}}} = \frac{-142}{100} = -1.42 \end{aligned}$$

Now we need to compute the appropriate **p-value** based on our alternative hypothesis.

$$\begin{aligned} P(Z > Z^*) &= P(-2.02 < Z < 2.02) \\ &= 1 - P(Z < 2.02) \\ &= 1 - 0.9222 = 0.0778 > 0.025 \end{aligned}$$

From the above calculations the test statistic lies in the Acceptance Region for H_0
Therefore we fail to reject H_0

Solution 5:

$$H_0: \mu = 32.28 \quad \text{vs} \quad H_1: \mu \neq 31.67$$

$$n = 19, \quad \bar{x} = 31.67, \quad \sigma = 1.29, \quad \alpha = 0.05/2$$

Calculate the Z score as follows

$$\begin{aligned} Z^* &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{31.67 - 32.28}{\frac{1.29}{\sqrt{19}}} = \frac{-0.61}{0.3} = -2.03 \end{aligned}$$

Now we need to compute the appropriate **p-value** based on our alternative hypothesis.

$$\begin{aligned} P(Z > Z^*) &= P(-2.03 < Z < 2.03) \\ &= 1 - P(Z < 2.03) \\ &= 1 - 0.9788 = 0.0212 < 0.025 \end{aligned}$$

From the above calculations the test statistic lies in the Acceptance Region for H_0 . Therefore we fail to reject H_0

Solution 6

B at $\mu = 52$

$$\begin{aligned} &= \frac{48.81-52}{\frac{2.5}{\sqrt{16}}} = \frac{-3.19}{0.625} = -5.104 \\ &= \frac{51.19-52}{\frac{2.5}{\sqrt{16}}} = \frac{-0.81}{0.625} = -1.30 \\ &= P(-5.10 \leq Z \leq -1.30) \\ &= P(Z \leq -1.30) - P(Z \leq 5.10) \\ &= (1-0.9032) - 0 \\ &= 0.0968 \end{aligned}$$

B at $\mu = 50.5$

$$\begin{aligned} &= \frac{48.81-50.5}{\frac{2.5}{\sqrt{16}}} = \frac{-1.69}{0.625} = -2.7 \\ &= \frac{51.19-50.5}{\frac{2.5}{\sqrt{16}}} = \frac{-0.69}{0.625} = -1.1 \\ &= P(-2.7 \leq Z \leq -1.1) \\ &= P(Z \leq -2.7) - P(Z \leq 1.1) \\ &= (1-0.9965) - (1-0.8643) \\ &= 0.1322 \end{aligned}$$

☐ **at $\mu = 50$**

$$\begin{aligned} &= \frac{48.81-50}{\frac{2.5}{\sqrt{16}}} = \frac{-1.19}{0.625} = -1.904 \\ &= \frac{51.19-50}{\frac{2.5}{\sqrt{16}}} = \frac{1.19}{0.625} = 1.904 \\ &= P(Z < -1.904) + P(Z > 1.904) \\ &= (1-0.9713) + (1-0.9713) \\ &= 0.0574 \end{aligned}$$

☐ **at $\mu = 50$**

$$\begin{aligned} &= \frac{48.42-50}{\frac{2.5}{\sqrt{16}}} = \frac{-1.58}{0.625} = -2.528 \\ &= \frac{51.58-50}{\frac{2.5}{\sqrt{16}}} = \frac{1.58}{0.625} = 2.528 \\ &= P(Z < -2.528) + P(Z > 2.528) \\ &= (1-0.9943) + (1-0.9943) \\ &= 0.0114 \end{aligned}$$

Acceptance Region	Sample Size	α	B at $\mu = 52$	B at $\mu = 50.5$
$48.5 < x < 51.5$	10	0.0574	0.2643	0.8923
$48 < x < 52$	10	0.0114	0.5	0.9705
$48.81 < x < 51.19$	16	0.0574	0.0968	0.1322
$48.42 < x < 51.58$	16	0.0114	0.2514	0.95815

Solution 7:

$$\begin{aligned}
 t^* &= \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \\
 &= \frac{12 - 10}{\frac{1.5}{\sqrt{16}}} = \frac{2}{3.75} \\
 &= \mathbf{0.53}
 \end{aligned}$$

Solution 8:

$$\begin{aligned}
 1 - \alpha &= 0.99 \rightarrow \alpha = 1 - 0.99 = 0.01 \\
 df &= n - 1 = 16 - 1 = 15 \\
 t^*_{0.99}(15) &= -t^*_{0.01}(15) = -2.602
 \end{aligned}$$

Solution 9:

$$\begin{aligned}
 \bar{x} &= \frac{1}{16} \sum X_i = \frac{1}{16} (304 + 367 + \dots + 333) = \mathbf{320} \\
 s^2 &= \frac{1}{16-1} \sum (X_i - \bar{x})^2 = \frac{1}{15} [(304 - 320)^2 + \dots + (333 - 320)^2] = \mathbf{1702.93} \\
 s &= \sqrt{s^2} = \sqrt{1702.93} = \mathbf{41.27}
 \end{aligned}$$

Thus $n = 16$, $\bar{x} = 320$, $s = 41.27$, $\mu = 300$, $\alpha = 0.05$

$$H_0: \mu = 300 \quad \text{vs} \quad H_1: \mu > 300$$

We reject H_0 at 5% level of significance if $Z < -1.96$ or $Z > +1.96$

$$\begin{aligned}
 Z^* &= \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \\
 &= \frac{320 - 300}{\frac{41.27}{\sqrt{16}}} = \frac{20}{10.3175} = \mathbf{1.9384}
 \end{aligned}$$

The appropriate **p-value** is given by

$$\begin{aligned}
 P(Z > Z^*) &= P(Z > 1.9384) \\
 &= 1 - P(Z < 1.9384) \\
 &= 1 - 0.9737 = \mathbf{0.0263} < 0.05
 \end{aligned}$$

The **z-score** falls within the acceptance region and the **p-value** is less than the level of significance which makes us **fail to reject** the NULL hypothesis and conclude that the sales have not increased.