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Low cost inertial orientation tracking with Kalman filter

Markus Haid *, Jan Breitenbach

Fraunhofer Technology-Development Group (TEG), Nobekstrasse 12, 70569 Stuttgart, Germany

Abstract

In this paper the theory of estimation will be examined and applied to inertial measurement units (IMUs). IMUs sense the motion of an object without any contact. The sensors, that are used, are inertial sensors giving the IMU it's name. It will be considered, how the theory of estimation can compensate the problems with the sensor drift. The application of the theory of estimation is obvious, because the central aim of the theory of estimation is the extraction of information based on instable and deranged measured values.

Keywords: Indoor; Navigation; Inertial; Sensor; Bias; Drift; Kalman; Estimation; Orientation; Tracking; Gyroscope

1. Introduction

In the last few years the development of lower inertial sensors has been pushed forward. The dimensions of the newest sensors measure a few millimetres only. With the size decreases the price of the sensors caused in the abnormal increasing production figures and the application of low cost micromechanical silicon structures. This makes applications with low cost systems possible in the meantime.

The Fraunhofer Technology Development Group (TEG) in Stuttgart has shown the possibility of motion-tracking of an object with new low cost

* Corresponding author.
E-mail address: mah@teg.fraunhofer.de (M. Haid).

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Nomenclature

$\underline{\phi}(k)$	discrete transition marix
$\underline{C}(k)$	observation matrix
$\underline{I}$	unit matrix
$K(k)$	Kalman gain
M^{T}	transposed matrix
M^{-1}	inversed matrix
ω_D	turnrate
φ_D	rotation angle
$P(k)$	prediction matrix
$Q(k)$	covariance matrix caused by $\underline{w}(k)$
$R(k)$	covariance matrix caused by $\underline{v}(k)$
$\underline{v}(k)$	noise process
$\underline{w}(k)$	discreter noise process
$\underline{x}(k)$	state vector
$\underline{y}(k)$	measurement vector

miniaturised sensors. The main drawback of low cost IMUs is the sensor drift of the sensors, because with decreasing volume and price the accuracy of the sensors goes down. This disadvantage has obviated the advantage of the low cost sensors up to now [2].

To realise IMUs in spite of these boundary conditions innovative and complex algorithms are needed to compensate for the inaccuracies. On that occasion the calibration of the sensors is an important factor. The research focus includes first of all the handling and analysis of the recorded sensor signals. At this the needed algorithms transcend the algebraical coherence. Complex algorithms from statistical signal processing and the theory of probabilities come into operation. Furthermore the application specific algorithms analyse the custom-designed environment to extract information for improvement of the accuracy of our orientation tracking.

The motivation in scope of developing IMUs at Fraunhofer TEG is both the need of IMUs in hand-held-devices for engineering applications, and for low cost systems for the mass market. Marketable products like computer mice and electronic pens are only two of many applications of IMUs in the low cost segment [2].

2. Task

The sensor drift of the used low cost sensors is currently limiting broad application of the IMUs. This sensor drift is an offset. The reasons for the offset are fabrication inaccuracy, temperature change, environment ascendancies or ageing. The bias is the biggest problem to realise IMUs with low cost sensors, although the greatest part of the bias will be compensated, because there is a stochastical remaining error. This remaining error, called bias, comes into play from non-analysable sources [6].

Fraunhofer TEG has examined the consequences of the bias for inertial object tracking. The orientation is the result of the integration of the turnrate over the time. So it can be shown, that the desired specifications are unreachable for measurement times longer than 1 min with available sensors. Deviations of up to 150° after 1 min and over 300° after 2 min have been measured with the gyroscope ENC-03JA from Murata [3].

In the framework of this paper the theory of estimation and the improvement of the inertial object tracking with Kalman filtering has been investigated.

3. Theory of estimation

The Kalman filter is an optimal estimator, allowing to approximate system states in the state space by the recursive use of redundant data [4].

The special applicability of the Kalman filter for inertial object tracking is due to the accreditation of time invariant systems and transient processes by the time variable amplification matrix. Furthermore the Kalman filter considers the settling process, because an endless monitoring interval is not necessary [3]. Fig. 1 shows the structure of a Kalman filter algorithms as observer.

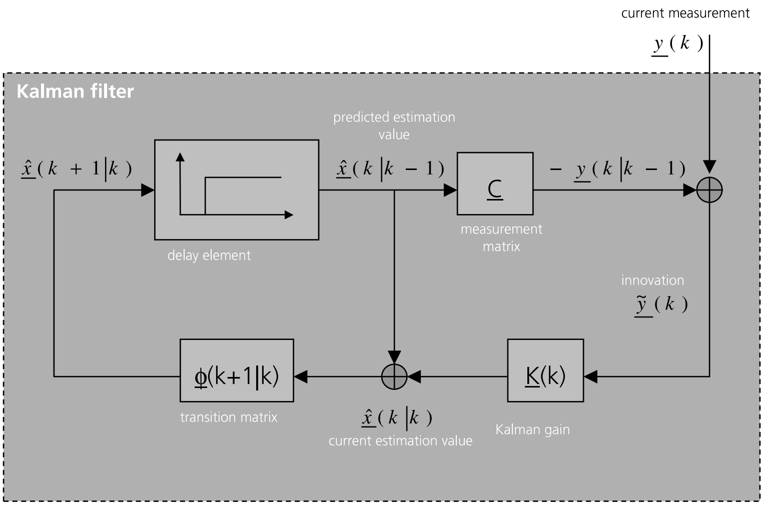

Fig. 1. Structure figure of a Kalman filter algorithm as observer [3].

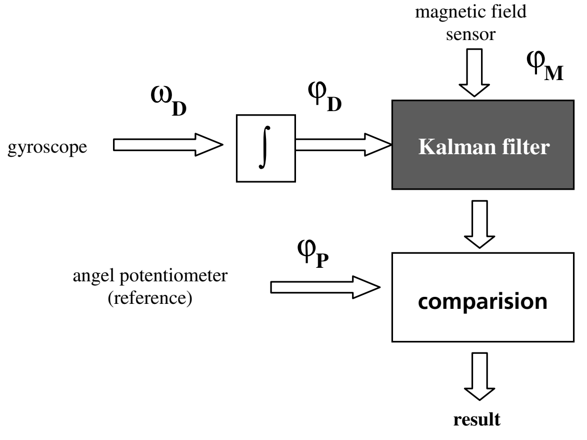

Fig. 2. Application of the Kalman filters in a one-dimensional model [3].

The attributes of the Kalman filter are profitable for many systems of inertial object tracking, although the filter theory can only realise suboptimal resolutions [1]. The advantages of the Kalman filter are the methodical determination of the filter coefficients, the good estimation of the system states during the settling process and the consideration of the stochastical process ascendancy of measurement and model errors [5].

Fig. 2 shows the basic algorithm of our one-dimensional model. The main measurement signal is coming from the gyroscope in form of the turnrate ω_D and by integration we find the rotation angle φ_D. The Kalman filter algorithm delivers an estimate of the rotation angle with the additional information in form of the sensor signal of a magnetic field sensor φ_M. In our test set-up the estimation of the rotation angle can be compared with the reference signal of an angle potentiometer φ_P. By comparing the experimental results against the reference signal from the potentiometer the improvements obtained with the Kalman filter algorithm can be quantified.

4. Modelling

In general, the Kalman filter describes a system with a state model as shown in Eq. (1) and a measurement model as in Eq. (2)

$$\underline{x}(k+1|k) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \varphi_D(k|k) \\ \omega_D(k|k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \underline{w}(k) \tag{1}$$

$$\underline{y}(k|k) = \underline{C} \cdot \begin{bmatrix} \varphi_D(k) \\ \omega_D(k) \end{bmatrix} + \underline{v}(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \varphi_D(k) \\ \omega_D(k) \end{bmatrix} + \underline{v}(k) \tag{2}$$

The advanced observation is realised in Eq. (3).

$$\underline{y}(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \underline{x}(k) + \underline{v}(k) \tag{3}$$

The prediction equations, (4)–(9), result from the state model and the measurement model.

At first the Kalman gain is computed in Eq. (4).

$$\underline{K}(k+1|) = \underline{P}(k+1|k) \cdot \underline{C}^{\mathrm{T}} \left[\underline{C} \cdot \underline{P}(k+1|k) \cdot \underline{C}^{\mathrm{T}} + \underline{R} \right]^{-1} \tag{4}$$

After that the system state is updated using Eq. (5).

$$\hat{\underline{x}}(k+1|k+1) = \hat{\underline{x}}(k+1|k) + \underline{K}(k+1) \cdot \tilde{\underline{y}}(k+1) \tag{5}$$

The innovation is calculated in Eq. (6).

$$\tilde{\underline{y}}(k+1) = \underline{y}(k+1) - \hat{\underline{y}}(k+1|k) = y(k+1) - \underline{C}(k) \cdot \hat{\underline{x}}(k+1|k) \tag{6}$$

Then follows the update of the error covariance in Eq. (7).

$$\begin{aligned} \underline{P}(k+1|k+1) = {} & (\underline{I} - \underline{K}(k+1) \cdot \underline{C}^{\mathrm{T}}) \cdot \underline{P}(k+1|k) \cdot (\underline{I} - \underline{K}(k+1) \cdot \underline{C}^{\mathrm{T}})^{\mathrm{T}} \\ & + \underline{K}(k+1) \cdot \underline{R} \cdot \underline{K}^{\mathrm{T}}(k+1) \end{aligned} \tag{7}$$

After that the next state is estimated in Eq. (8).

$$\hat{\underline{x}}(k+2|k+1) = \underline{\phi}(k+1) \cdot \hat{\underline{x}}(k+1|k+1) \tag{8}$$

And at last the prediction is updated in Eq. (9).

$$\underline{P}(k+2|k+1) = \underline{\phi}(k+1) \cdot \underline{P}(k+1|k+1) \cdot \underline{\phi}^{\mathrm{T}}(k+1) + \underline{Q} \tag{9}$$

The complete feedback cycle of Kalman filter is shown in Fig. 3.

5. Experimental description

The following assembly has been used for particular examinations of the Kalman filter. A gyroscope is arranged on a turntable with a magnetic field sensor (Fig. 4). The turntable allows a horizontal rotation of the total system around the vertical axis. A third sensor is an angle potentiometer arranged in the rotation axis and represents the reference value. The sensor data is recorded with a National Instruments PC card and is processed with the analysis software LabView. The sensor signal from the gyroscope is the measurement value and the sensor signal from the magnetic field sensor is an additional backup value. Both are available for the Kalman filter with a sampling rate of 500 Hz.

During the experiment the execution of arbitrary rotations has been carried out on the turntable. These rotations have been measured over several

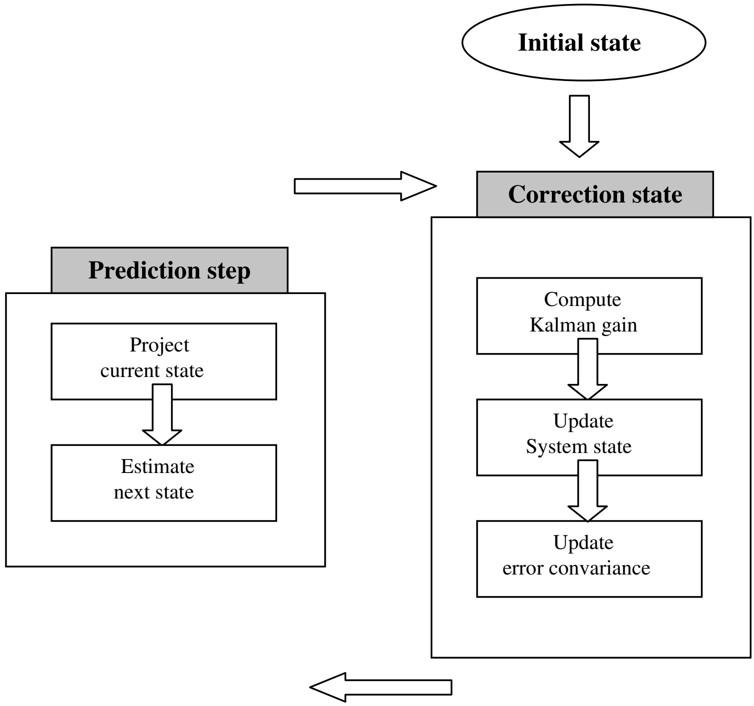

Fig. 3. Feedback cycle of the Kalman filter.

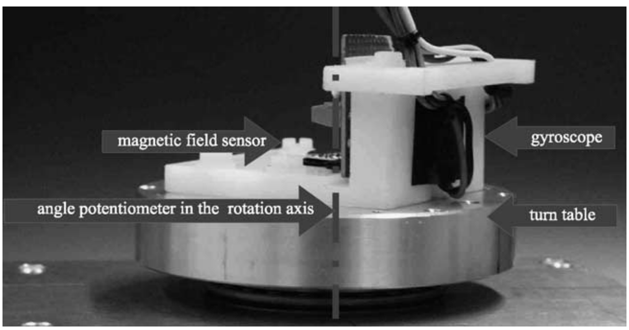

Fig. 4. Turntable for the one-dimensional sensor rotation with reference record [3].

durations. Thereby was recoded the integrated gyroscope signal as the real measurement value from the signal with the drift, the rotation angle of the angle potentiometer as the reference value of the real rotation and the estimation of the measurement value of the Kalman algorithms.

The low cost sensors used are the gyroscope ENC-03JA from Murata, the two-axis fluxgate-sensor-system FGS1/COB_07 from Fraunhofer IMS in Dresden and the angle potentiometer MCP 40 from Megatron. The gyroscope is based on the phenomenon of Coriolis-acceleration having an effect on a rotating and vibrating system. The sensor element is a piezo-electrical ceramic. Characteristics of the sensor are small dimensions, fast reaction time, low supply voltage and low power consumption. The system FGS1/COB_07 is a miniaturised two-axis magnetic sensor consisting of two planar fluxgate sensors manufactured in a CMOS process in an orthogonal arrangement and an ASIC for analysis. The fluxgate-sensor is a high sensitive sensor operating in a close loop mode using a harmonic readout method. The angle potentiometer MCP 40 is a conduct plastic element with a resolution of 0.001° and a resistance resolution from 1 to 500 kΩ. The reader is twice precision ball beared.

6. Results

In Fig. 5 are the estimated measurement value of the Kalman filter, the signal of the gyroscope afflicted with the drift and the real angle over a period of 120 s.

The graph shows explicitly the quantitative innovation by the Kalman filter. The estimated value of the Kalman filter diverges less than 5° from the reference signal after 120 s. In contrast the signal of the gyroscope afflicted with the drift diverges more than 322° from the reference signal. This corresponds approximately to an improvement of more than 90%.

Altogether nine different measurement times (20, 40, 60,. . .,180 s) have been executed with 10 measurements. After each measurement the divergences of the

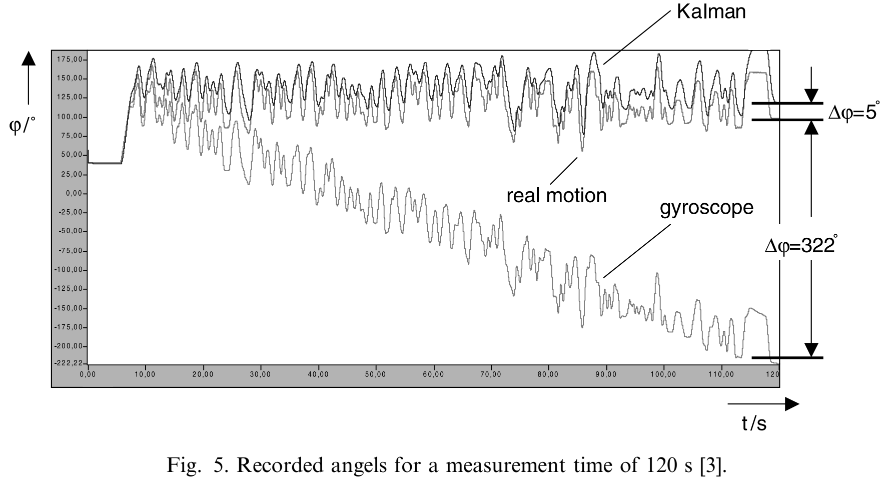

Fig. 5. Recorded angels for a measurement time of 120 s [3].

Table 1
Improvement of the Kalman filter for several measurement times [3]

Measurement time (s)	20	40	60	80	100	120	140	160	180
The median improvement for 10 measurements (%)	70	80	82	90	92	94	94	95	98

gyroscope signal has been compared with the estimation value of the Kalman filter. Out of this comparison the improvement has been calculated in per cent for each measurement. The results (average value) are shown in Table 1.

The results of the experiment prove that the estimated measurement value of the Kalman filter does not follow the gyroscope signal afflicted with the drift. While the integrated gyroscope signal does not represent the real rotation angle of the experimental system, the estimation value of the Kalman filter represents the real rotation angle with continuous measurement time. This fact will be more distinct for longer measurement times. At long measurement times the signal of the gyroscope afflicted with the drift is not suitable to obtain information of the absolute orientation of the experiment system. In contrast the estimation value of the Kalman filter delivers a realistic value of the rotation angle over longer measurement times and makes therefore a more exact prediction of the absolute orientation of the system.

7. Conclusion and outlook

As the sensor drift negatively influences the measurement signals, there are significant position errors concerning the integration over time. Therefore low cost systems do not achieve the requirements of industrial applications. For example the accuracy for an orientation detection system for a industrial application need to be better than 5° [2].

To get over these disadvantages of to date low cost sensor systems and to augment the analysis accuracy, this paper presents a simple estimation algorithm. It estimates the real state of a system as a result of measurement values afflicted with errors. The complete error is essentially lower than a measurement without the estimation algorithm.

The simulation and the practical tests have shown, that an significant improvement of the orientation tracking can be reached as a result of using a Kalman filter algorithm. Thereby improvements up to 98% have been obtained for the one-dimensional case. The claimed accuracy for industrial application from 5° has been comfortably achieved with our algorithm [3].

As a result of the capability of the developed Kalman filter Fraunhofer TEG decided to appoint this algorithm in form of a three-dimensional orientation detection system for an industrial application. Furthermore the Fraunhofer

TEG aims to improve the already developed prototypes and update the inertial object tracking with the Kalman filter algorithm to marketable products.

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