Pacejka Model - Fitting Exercises

Vehicle Dynamics, Planning and Control of
Robotic Cars

Gastone Pietro Papini Rosati
& Mattia Piccinini

M.Sc. in Mechatronics Engineering
University of Trento
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Assignment

These exercises will let you familiarize with real tire data, which you are going to fit with a Pacejka model. You are supposed to attach the solution of the following homework in the final report that you are going to deliver a few days before the oral exam.

Notes

For this homework, you need to install the Optimization MATLAB Toolbox.

Exercises

Exercise 1 - Understanding tire data

In this exercise, you are provided with a dataset resulting from a real tire testing campaign. The data were acquired by the Tire Test Consortium (TTC) using Formula SAE-style tires (tire compound: Hoosier 18x6-10 LCO).
The dataset is in the file `B1464run30.mat`. As you load it in MATLAB, you will find several variables in the workspace, but the most important for you are the following:

- `FX`,`FY`,`FZ`,`MZ` being respectively the longitudinal, lateral and vertical tire forces (in [N]) and the self-aligning torque (in the tire reference frame);
- `SA` and `SL` being the side slip angle α (in [deg]) and the longitudinal slip κ;
- `P` being the tire pressure (in [kPa]);
- `IA` being the camber angle γ (also called inclination angle by the Tire Test Consortium).

It is required to complete the following tasks:

- Plot the raw data in different graphs, specifically focusing on κ, α, γ, F_z and pressure P. Comment on what you see. What is, according to you, the main target of these tests?

Hint: think of the variable that changes most frequently in the tests.
Hint: use the `subplot` command in MATLAB to align vertically the 5 plots. This will be useful for the next exercises.

- Focus on the data with $\alpha = 0$ and $\gamma = 0$, and plot the curves F_x vs κ for each of the 4 vertical loads F_z used in the experiments. Plot the 4 curves on the same graph, with different colors. Comment on what you see.

- Focus on the data with $\gamma = 0$ and $F_z = 150$ lbf ≈ 670 N, and plot the curves F_x vs κ for each of the 3 side slip angles α used in the experiments. Plot the 3 curves on the same graph, with different colors. Comment on what you see.

Exercise 2 - Fitting tire data

In this exercise, you are going to fit the pure longitudinal force F_{x0} as a function of $\{\kappa, F_z, \gamma\}$, using the Pacejka Magic Formula (1996 version). Use the dataset of the file `B1464run30.mat`, and work in MATLAB.
Refer to the procedure explained in the document Tire_Models_doc.pdf, and use equation (20) of the same file.
Consider only the data in which the tire pressure is about 12 psi $\approx$ 82 kPa. Assume a nominal load $F_{z0} = 200$ lbf ≈ 890 N. This a pure longitudinal fitting exercise, so the side slip angle α will always be zero. You are required to fit 15 coefficients:

$$\begin{aligned}\boldsymbol{X} = &\{p_{Cx1}, p_{Dx1}, p_{Dx2}, p_{Dx3}, p_{Ex1}, p_{Ex2}, p_{Ex3}, p_{Ex4}, p_{Kx1}, p_{Kx2}, \dots \\ &p_{Kx3}, p_{Hx1}, p_{Hx2}, p_{Vx1}, p_{Vx2}\}\end{aligned}$$

Hint: Use a least-squares approach to find the optimal parameters $\boldsymbol{X}$ that minimize the squares of the residuals between the fitted function F_{x0} and the real data $\bar{F}_{x0}$:

$$\min_{\boldsymbol{X}} \quad \frac{\sum_{i=1}^{N} (F_{x0i}(\boldsymbol{X}) - \bar{F}_{x0i})^2}{\sum_{i=1}^{N} \bar{F}_{x0i}^2}$$

with N being the total number of samples. The problem above is an unconstrained minimization, which can be solved in MATLAB with `fmincon`.
Since the number of coefficients is quite high, subdivide the fitting in three phases:

- First consider the data with $F_z = F_{z0} = 890$ N, $\gamma = 0$ and (of course) $\alpha = 0$, and fit the coefficients $\boldsymbol{X}_1 = \{p_{Cx1}, p_{Dx1}, p_{Ex1}, p_{Ex4}, p_{Kx1}, p_{Hx1}, p_{Vx1}\}$. Build a function `resid_pure_Fx` that calculates the pure longitudinal force F_{x0} and the residuals. This function can look like the one provided as a template.

Then you can compute the optimal parameters $\boldsymbol{X}_{1,opt}$ as:
$\boldsymbol{X}_{1,opt} = \text{fmincon}(@(X)\text{resid\_pure\_Fx}(X, FX, SL, IA, FZ), X_0, [], [], [], [], [], [])$
with X_0 being an initial guess vector, and $\{FX, SL, IA, FZ\}$ being vectors of raw data suitably selected.
Plot the fitted curve F_{x0} vs κ that you obtained in these nominal conditions, together with the raw data.

- Now consider the data with the 4 different values of F_z, but still $\gamma = 0$ (and $\alpha = 0$). This enables the fitting of the parameters:
  $\boldsymbol{X}_2 = \{p_{Dx2}, p_{Ex2}, p_{Ex3}, p_{Hx2}, p_{Kx2}, p_{Kx3}, p_{Vx2}\}$.
  You can build another function:
  $\text{resid\_pure\_Fx\_varFz}(X, FX, SL, IA, FZ, X_{1,opt})$
  so as to use the optimal parameters $\boldsymbol{X}_{1,opt}$ found before for the coefficients already computed. Plot the fitted and raw curves F_{x0} vs κ for the 4 values of F_z and comment the results.

- Now consider the data with the 3 different values of γ, but with $F_z = F_{z0}$ (and $\alpha = 0$). This enables the fitting of the parameter:
  $\boldsymbol{X}_3 = \{p_{Dx3}\}$.
  You can build another function:
  $\text{resid\_pure\_Fx\_varCamber}(X, FX, SL, IA, FZ, X_{1,opt}, X_{2,opt})$
  so as to use the optimal parameters $\boldsymbol{X}_{1,opt}$ and $\boldsymbol{X}_{2,opt}$ found before for the coefficients already computed.
  Plot the fitted and raw curves F_{x0} vs κ for the 3 values of γ and comment the results.