

## Tire Models Exercises

Vehicle Dynamics, Planning and Control of Robotic Cars

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## Assignment

These simple exercises are meant to assess your proper understanding and basic use of Pacejka tire model and slip definitions. You are supposed to attach the solution of the following homework in the final report that you are going to deliver a few days before the oral exam.

### **Exercises**

### Exercise 1 - pure longitudinal slip

A fitting performed with Pacejka model on experimental tire test data provides the following results:

$$\begin{cases}
D_x = 2100 \\
B_x = 12.2 \\
C_x = 1.87 \\
E_x = 0.13 \\
S_{Hx} = 0.851 \cdot 10^{-3} \\
S_{Vx} = -77
\end{cases} \tag{1}$$

The convention for parameters names is the same as the one used in Eq. (20) of the file Tire\_Models\_doc.pdf.

It is required to complete the following tasks:

- Using Pacejka Magic Formula, plot the longitudinal tire force  $F_{x0}$  obtained in pure longitudinal slip conditions, as a function of slip  $\kappa \in [-1, 1]$ . You can use any program for plotting the curve (MATLAB, Maple, Wolfram Mathematica, Excel or others). Which comments are you able to make about the obtained graph? If you were supposed to design a traction control system for maximizing vehicle longitudinal acceleration, which would be the target value of longitudinal slip  $\kappa$  that you would try to achieve?
- Assuming that wheel rotational speed is  $\omega = 70 \text{ rad/s}$ , tire effective rolling radius is  $R_e = 0.2 \text{ m}$ , while the longitudinal component of tire contact point speed  $v_{Cx} = 13 \text{ m/s}$ , compute the longitudinal slip  $\kappa$ . In these conditions,

is the wheel accelerating, braking or is it in pure rolling? Compute also the corresponding longitudinal tire force  $F_{x0}$ .

• Compute the cornering stiffness  $C_{f\kappa}$ , that is the derivative for  $\kappa = 0$  of the  $F_{x0}$  function. Up to which value of  $\kappa$  is the linear approximation of Pacejka curve acceptable?

### Exercise 2 - combined slip

Using the same data of the previous exercise, suppose now that the vehicle starts steering, so that the side slip angle  $\alpha \neq 0$ . Again referring to Eq. (20) of the file Tire\_Models\_doc.pdf, the Pacejka fitted coefficients for the combined longitudinal-lateral case are:

$$\begin{cases}
B_{xa} = \frac{23.308}{\sqrt{(19.414 \cdot \kappa)^2 + 1}} \\
C_{xa} = 0.926 \\
S_{Hxa} = -0.1257 \cdot 10^{-2} \\
D_{xa} = \frac{1}{\cos(C_{xa} \arctan(B_{xa}S_{Hxa}))}
\end{cases} \tag{2}$$

It is required to complete the following tasks:

- Assume that the tire contact point velocity components along the tire x and y axes are  $v_{Cx} = 15$  m/s and  $v_{Cy} = -1.3$  m/s, respectively. Calculate the side slip angle  $\alpha$ . Moreover, compute the combined tire force  $F_x$  using this value of  $\alpha$ , for a longitudinal slip  $\kappa = 0.08$ .
- Plot the combined longitudinal tire force  $F_x$  as a function of  $\kappa \in [-1, 1]$ , for the following levels of side slip angle  $\alpha = \{0, 2, 4, 6, 8\}$  degrees (but remember that  $\alpha$  must be in radians in Pacejka formulas). Which comments can you make about the 5 curves obtained in this way? Finally, plot the weighting function  $G_{xa}$  as a function of  $\kappa \in [-1, 1]$  for each of the previously defined values of  $\alpha$ , and briefly comment also these 5 curves.