ELEC-E8103 - Modelling, Estimation and Dynamic Systems

Assignment 2

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1 Exercise 1: RLC circuit

1.1 Part a

Outputs: I_1 , q
Inputs: $E(t)$
Constants: R_1 , R_2, R_3, L, C
Internal time varying variables: I_2, I_3

1.2 Part b

By applying Kirkhoff's law we have the following

$$\begin{cases} I_1 - I_2 - I_3 = 0 \\ I_3 = \frac{dq}{dt} \end{cases} \tag{1}$$

Furthermore to derive the differential equations describing the system, starting from the left side of the circuit we have:

$$I_1R_2 - \frac{dq}{dt}R_2 + L\frac{dI_1}{dt} + I_1R_1 = 0 \tag{2}$$

while on the right side of the circuit we have

$$I_2R_2 - \frac{q}{C} + E - I_3R_3 = 0 \tag{3}$$

by knowing that:

$$\begin{cases} I_3 = \frac{dq}{dt} \\ I_2 = I_1 - \frac{dq}{dt} \end{cases}$$

and substituting it into Eq. 3, we obtain the following differential equation to describe the time varying charge of the capacitor:

$$\frac{dq}{dt} = \frac{E}{R_2 + R_3} - \frac{q}{c \cdot (R_3 + R_2)} + \frac{I_1R_2}{R_2 + R_3} \tag{4}$$

Therefore we can rearrange Eq. 2, to obtain the differential equation which describes the behavior of the current I_1 and obatin the following:

$$\frac{dI_1}{dt} = \frac{dq}{dt} \cdot \frac{R_2}{L} - \frac{I_1(R_2 + R_1)}{L} \tag{5}$$

Thus the system of differential equations which describes the RLC circuit is the following:

$$\begin{cases} \frac{dq}{dt} = \frac{E}{R_2+R_3} - \frac{q}{c \cdot (R_3+R_2)} + \frac{I_1R_2}{R_2+R_3} \\ \frac{dI_1}{dt} = \frac{dq}{dt} \cdot \frac{R_2}{L} - \frac{I_1(R_2+R_1)}{L} \end{cases} \tag{6}$$

1.3 Part c

When the input of the system is the following:

$$E(t) = sin(t) \tag{7}$$

The system response for a 20 seconds simulation time can be observed in Figure 1.

1.4 Part d

The simulink model of the RLC circuit is shown in the figure below

The results obtained through the simulation of the simulink model present a similar behavior of the results obtained in the simulation with Matlab, as it can be seen in the figure below:

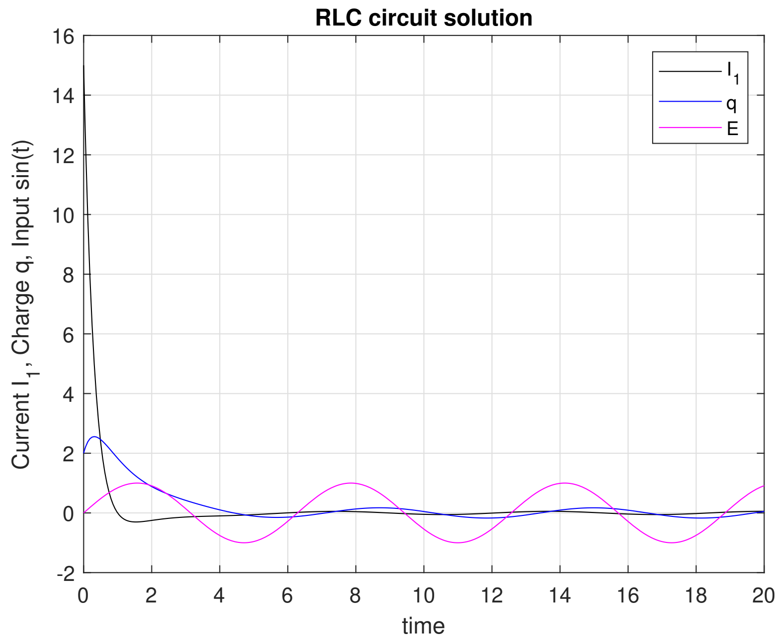

Figure 1: q and I_1 behaviors over a 20 seconds time Matlab solution

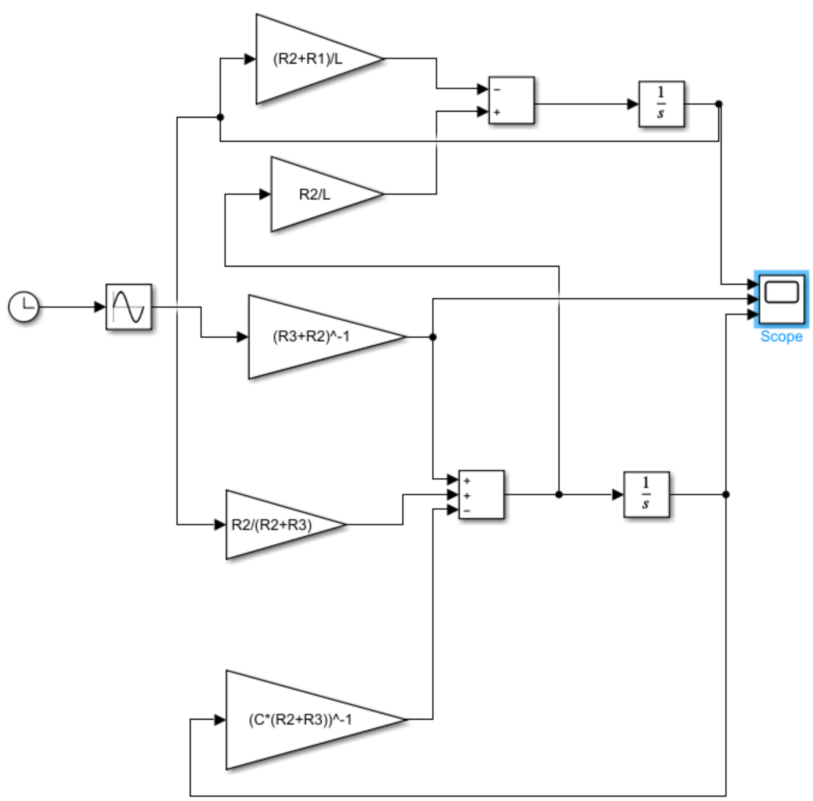

Figure 2: Simulink model of the RLC circuit

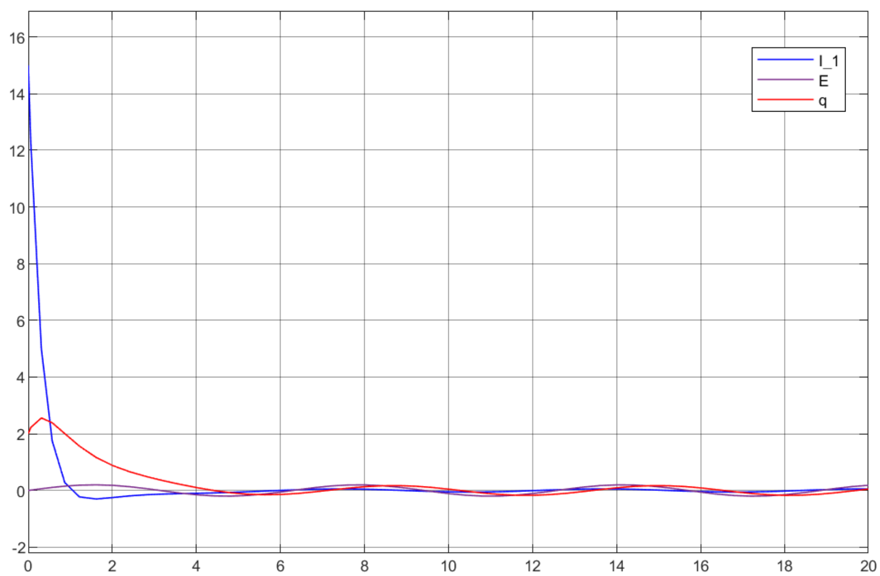

Figure 3: Simulink model simulation results

2 Exercise 2: Cruise Control modeling

2.1 Part a

Following the equations 3.3 and 3.2 of the cruise control file which was handed out, the following simulink model was constructed.

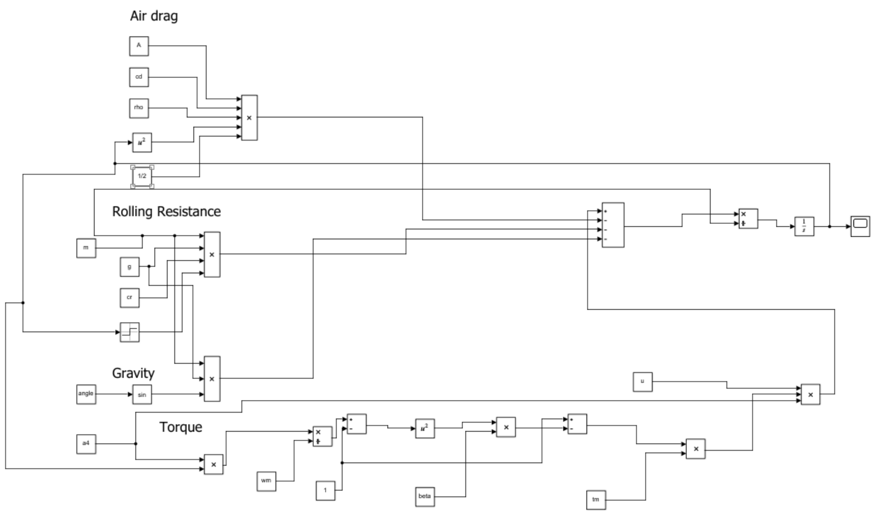

Figure 4: Simulink Cruise Control model

2.2 Part b

With an input u of 0.7 and a car mass of 800 kg and in the third gear, the steady state velocity of the car is 50.95 m/s as it is shown in the simulation results below.

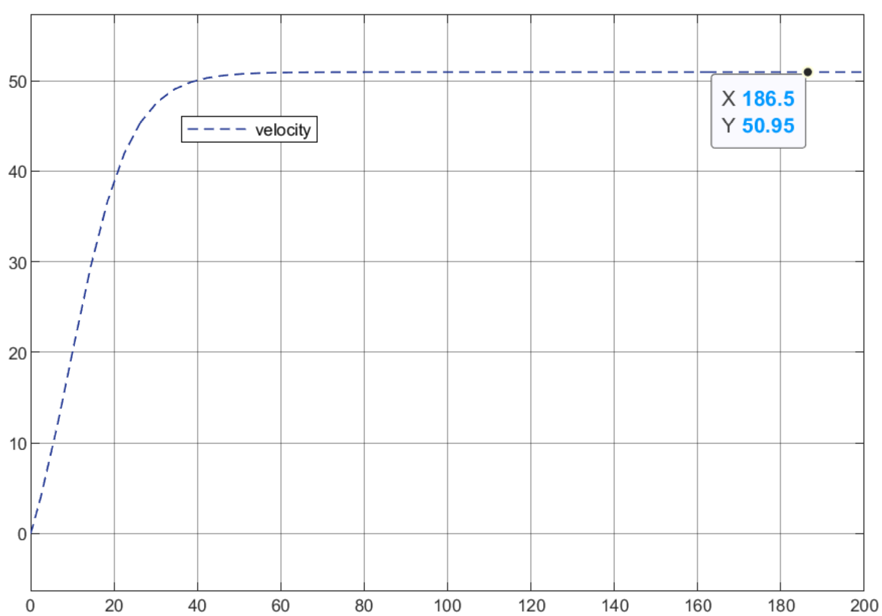

Figure 5: Simulink Cruise Control velocity on flat surface

2.3 Part c

Keeping the same input parameters as in part b, the speed decreases to 44.12 m/s. Through a trial and error approach it was found a value of u equal to 0.9844 when shifting to the fourth gear ratio in order to obtain the same velocity as obtained in part b.

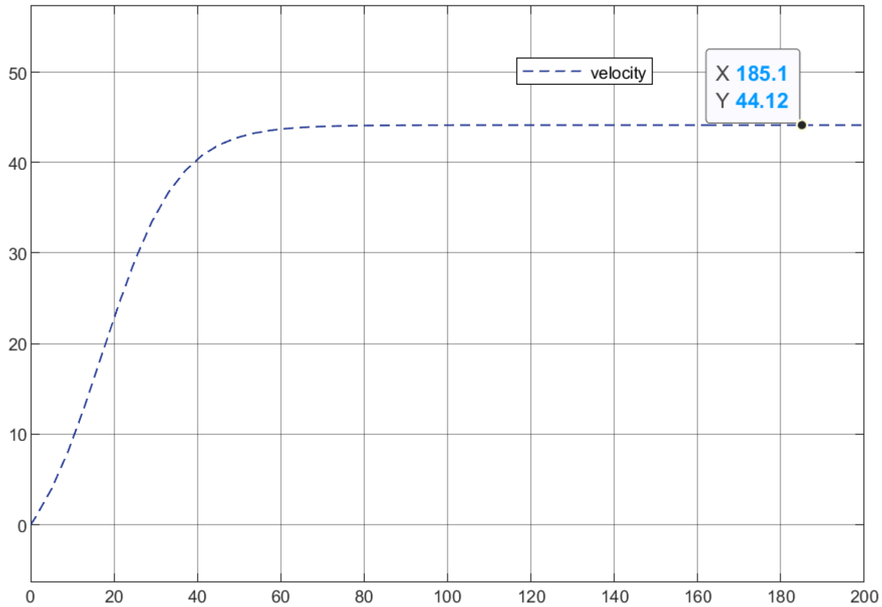

Figure 6: Simulink Cruise Control velocity on a 5 degree inclined surface