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1. Własności dla X - przestrzeń, $A: I \to P(X)$, $B: I \to P(X)$,$C-$zbiór
   (a) Jeśli $i_0 \in I$, to $A_{i_0} \subseteq \bigcup_{i\in I} A_i$
   (b) Jeśli $i_0 \in I$, to $\bigcap_{i\in I} A_i \subseteq A_{i_0}$
   (c) Jeżeli $\forall_{i\in I} A_i \subseteq C$,to $\bigcup_{i\in I} A_i \subseteq C$
   (d) Jeżeli $\forall_{i\in I} C \subseteq A_i$,to $C \subseteq \bigcap_{i\in I} A_i$
      i. D: Niech $x \in C \implies \forall_{i\in I} x \in A_i \implies x \in \bigcap_{i\in I} A_i$
   (e) $\bigcup_{i\in I}(A_i \cup B_i) = (\bigcup_{i\in I} A_i) \cup (\bigcup_{i\in I} B_i)$
   (f) $\bigcap_{i\in I}(A_i \cap B_i) \subseteq (\bigcap_{i\in I} A_i) \cap (\bigcap_{i\in I} B_i)$
      i. D:$x \in \bigcap_{i\in I}(A_i \cap B_i) \iff \forall_{i\in I} x \in A_i \cap B_i \iff \forall_{i\in I}(x \in A \wedge x \in B) \iff (x \in \bigcap_{i\in I} A_i) \wedge (x \in \bigcap_{i\in I} B_i) \iff x \in (\bigcap_{i\in I} A_i \cap \bigcap_{i\in I} B_i)$
   (g) $\bigcup_{i\in I}(A_i \cap B_i) \subseteq (\bigcup_{i\in I} A_i) \cap (\bigcup_{i\in I} B_i)$
      i. D:$x \in \bigcup_{i\in I}(A_i \cap B_i) \implies \exists_{i\in I} x \in A_i \cap B_i \implies \exists_{i\in I}(x \in A_i \wedge x \in B_i) \implies (\exists_{i\in I} a \in A_i) \wedge (\exists_{i\in I} x \in B_i) \implies x \in \bigcup_{i\in I} A_i \wedge x \in \bigcup_{i\in I} B_i \implies x \in (\bigcup_{i\in I} A_i) \cap (\bigcup_{i\in I} B_i)$
   (h) $(\bigcap_{i\in I} A_i) \cup (\bigcap_{i\in I} B_I) \subseteq \bigcap_{i\in I}(A_i \cup B_i)$, Inkluzja przeciwna nie zachodzi
   (i) $-\bigcup_{i\in I} A_i = \bigcap_{i\in I} -A_i$
      i. D: $x \in -\bigcup_{i\in I} A_i \iff x \notin \bigcup_{i\in I} A_i \iff \neg(x \in \bigcup_{i\in I} A_i) \iff \neg(\exists_{i\in I} x \in A_i) \iff \forall_{i\in I} \neg(x \in A_i) \iff x \in \bigcap_{i\in I} -A_i$
   (j) $-\bigcap_{i\in I} A_i = \bigcup_{i\in I} -A$
   (k) Jeżeli $\forall_{i\in I} A_i \subseteq B_i$to $\bigcup_{i\in I} A_i \subseteq \bigcup_{i\in I} B_i$ oraz $\bigcap_{i\in I} A_i \subseteq \bigcap_{i\in I} B_i$
   (l) $\bigcup_{i\in I}(C \cap A_i) = C \cap \bigcup_{i\in I} A_i$
      i. D: $x \in \bigcup_{i\in I}(C \cap A_i) \iff \exists_{i\in I} x \in C \cap A_i \iff \exists_{i\in I}(x \in C \wedge x \in A_i) \iff x \in C \wedge \exists_{i\in I} x \in A_i \iff x \in C \wedge x \in \bigcup_{i\in I} A_i \iff x \in C \cap \bigcup_{i\in I} A_i$
   (m) $\bigcap_{i\in I}(C \cup A_i) = C \cup (\bigcap_{i\in I} A_i)$
   (n) Jeżeli $J \subseteq I$ to $\bigcup_{j\in J} A_j \subseteq \bigcup_{i\in I} A_i$ oraz $\bigcap_{i\in I} A_i \subseteq \bigcap_{j\in J} A_j$
      i. D: $x \in \bigcap_{i\in I} A_i \implies \forall_{i\in I} x \in A_i \implies \forall_{j\in J} x \in A_j \implies x \in \bigcap_{j\in J} A_j$
   (o) $\bigcup_{i\in\emptyset} A_i = \emptyset$
      i. D: Przypuśćmy, że istnieje $x \in \bigcup_{i\in\emptyset} A_i$, to $\exists_{i\in\emptyset} x \in A_i \iff \exists_i i \in \emptyset \wedge x \in A_i$, ale $i \in \emptyset$ to zdanie fałszywe stąd sprzeczność
   (p) $\bigcap_{i\in\emptyset} A_i = X$
   D: $x \in \bigcap_{i\in I} A_i \iff \forall_{i\in\emptyset} x \in A_i \iff \forall_i i \in \emptyset \implies x \in A_i \iff x \in X$

2. Indeksowanie dwoma indeksami I, J - zbiory indeksów$C: I \times J \to P(X), (i,j) \mapsto c_{ij} = C(i,j)$
   $x \in \bigcup_{j\in J} \bigcap_{i\in I} C_{ij} \iff \exists_{j\in J} x \in \bigcap_{i\in I} C_{ij} \iff \exists_{j\in J} \forall_{i\in I} x \in C_{ij}$
   Analogicznie definiujemy $\bigcap_{j\in J} \bigcup_{i\in I} C_{ij}$ oraz $\bigcup_{j\in J} \bigcup_{i\in I} C_{ij}$ oraz$\bigcap_{j\in J} \bigcap_{i\in I} C_{ij}$
   Własności: $C: I \times J \to P(X)$
   (a) $\bigcup_{j\in J} \bigcup_{i\in I} C_{ij} = \bigcup_{i\in I} \bigcup_{j\in J} C_{ij} \overset{\text{def.}}{=} \bigcup_{\substack{i\in I \\ j\in J}} C_{ij}$
   (b) $\bigcap_{j\in J} \bigcap_{i\in I} C_{ij} = \bigcap_{i\in I} \bigcap_{j\in J} C_{ij} \overset{\text{def.}}{=} \bigcap_{\substack{i\in I \\ j\in J}} C_{ij}$
   (c) $\bigcup_{i\in I} \bigcap_{j\in J} C_{ij} \subsetneqq \bigcap_{j\in J} \bigcup_{i\in I} C_{ij}$
      i. $x \in \bigcup_{i\in I} \bigcap_{j\in J} C_{ij} \iff \exists_{i\in I} \forall_{j\in J} x \in C_{ij} \implies \forall_{j\in J} \exists_{i\in I} x \in C_{ij} \iff x \in \bigcap_{j\in J} \bigcup_{i\in I} C_{ij}$

3. Nieskończone rodziny indeksowane. $I = J = \mathbb{R}$, $X = \mathbb{R}^2$
   (a) Dla każdego $a, b \in \mathbb{R}$, niech $C_{ab} = \{< x, y > \in \mathbb{R}^2 : y \leq ax + b\}$
   $\underline{\bigcup_{a\in\mathbb{R}} C_{ab} = \mathbb{R}^2 \setminus \{0\} \times (b, +\infty)}$
   $\bigcup_{b\in\mathbb{R}} C_{ab} = \mathbb{R}^2$
   $\bigcap_{a\in\mathbb{R}} C_{ab} = \{0\} \times (-\infty, b)$
   $\bigcap_{b\in\mathbb{R}} C_{ab} = \emptyset$
   $\bigcap_{b\in\mathbb{R}} \bigcup_{a\in\mathbb{R}} C_{ab} = \bigcap_{b\in\mathbb{R}}(\mathbb{R}^2 \setminus \{0\} \times (b, +\infty)) = \mathbb{R}^2 \setminus \{0\} \times \mathbb{R}$
   $\bigcup_{b\in\mathbb{R}} \bigcap_{a\in\mathbb{R}} C_{ab} = \bigcup_{b\in\mathbb{R}}(\{0\} \times (-\infty, b)) = \{0\} \times \mathbb{R}$