# Homework 3, 601.465 - Natural Language Processing

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**1**

Using `switchboard-small`, the model's perplexity per word on each of the three sample files is as follows.

- sample1: 230.8033
- sample2: 316.5347
- sample3: 313.0219

Using `switchboard`, the model's perplexity per word on each of the three sample files is as follows.

- sample1: 280.4034
- sample2: 369.4896
- sample3: 456.3510

Log2-probabilities decrease and perplexities increase if I train on the larger `switchboard` corpus. It is because the larger corpus covers more vocabulary, reducing the number of instances of OOV while reading the sample texts. Since OOV is a placeholder for *all* words outside of the vocabulary, it results in having higher probability (and thus lower perplexity) than other words.

**3(a)** 69 out of 270 files were classified incorrectly. The total error rate is about 0.2556.

**3(b)** 20 out of 239 files were classified incorrectly. The total error rate is about 0.0837.

**3(c)** `textcat` classifies all the dev files as `spam` when the prior probability of `gen` is $10^{-324}$. When it is $10^{-323}$, some dev files are classified as `gen`.

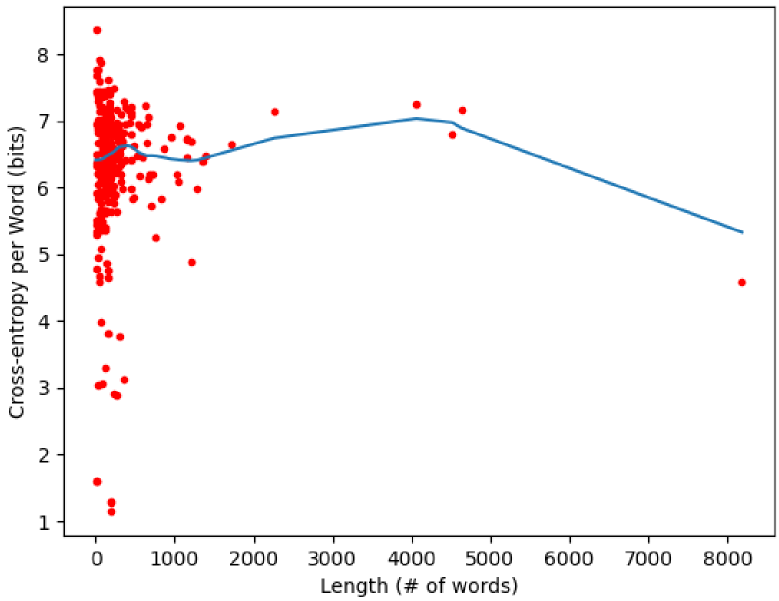
**3(d)** On the `gen` dev files, the minimum cross-entropy per token that I can achieve is 9.04616 bits, when $\lambda = 0.005$. On the `spam` dev files, the minimum cross-entropy per token that I can achieve is 9.09572 bits, when $\lambda = 0.005$.

**3(e)** The minimum cross-entropy per token that I can achieve on all development files together, if both models are smoothed with the same $\lambda$-value, is 9.06840 bits, when $\lambda^* = 0.005$.

**3(f)** As shown in the graph below, at $\lambda = 0.005$, there is no obvious relationship between the length of a file and its cross-entropy per token. Performance tends to be divergent for short files, and it converges to a high cross-entropy as the file gets longer.

**3(h)** I expect error rate to approach 0 as training size approaches $\infty$.

Figure 1: Performance of add-0.005 on dev data for different file lengths (lowess smoothing, frac=0.666..., it=3)




**4(a)** If I mistakenly took $V = 19999$ when there are 19999 different word types in training data, both the uniform estimate and the add-$\lambda$ estimate can face a problem. For both cases, the sum of probabilities of all vocabularies would exceed 1.

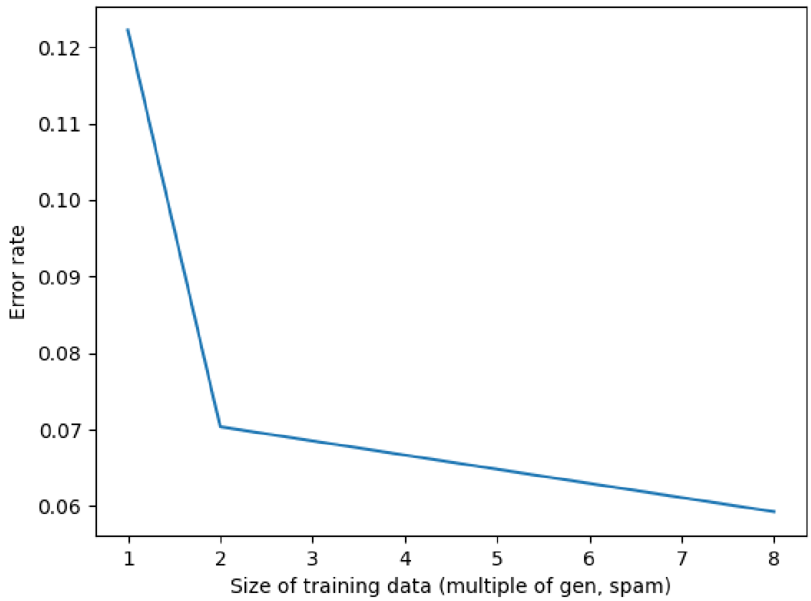
$$\text{Uniform:} \qquad \sum_{i=1}^{19999} \frac{1}{19999} + \frac{1}{19999}(\text{OOV}) = \frac{20000}{19999}$$

$$\text{add-}\lambda\text{:} \qquad \sum_{i=1}^{19999} \frac{c(xyv_i) + \lambda}{c(xy) + 19999\lambda} + \frac{\lambda}{c(xy) + 19999\lambda}(\text{OOV}) = \frac{c(xy) + 20000\lambda}{c(xy) + 19999\lambda}$$

**4(b)** If $\lambda = 0$, the add-$\lambda$ estimate would yield cross-entropy of $\infty$, since unseen words will yield the probability of 0, and their log-value will be $-\infty$.

**4(c)** Suppose that $c(xyz) = c(xyz') = 0$.

Figure 2: Performance of add-0.005 on dev data for different sizes of training data




$$
\begin{aligned}
\hat{p}(z|xy) &= \frac{c(xyz) + \lambda V \cdot \hat{p}(z|y)}{c(xy) + \lambda V} \\
&= \frac{c(xyz) + \lambda V \cdot \frac{c(yz)+\lambda V \cdot \hat{p}(z)}{c(y)+\lambda V}}{c(xy) + \lambda V} \\
&= \frac{c(xyz) + \lambda V \cdot \frac{c(yz)+\lambda V \cdot \frac{c(z)+\lambda}{\sum_v c(v)+\lambda V}}{c(y)+\lambda V}}{c(xy) + \lambda V} \\
\hat{p}(z'|xy) &= \frac{c(xyz') + \lambda V \cdot \frac{c(yz')+\lambda V \cdot \frac{c(z')+\lambda}{\sum_v c(v)+\lambda V}}{c(y)+\lambda V}}{c(xy) + \lambda V}
\end{aligned}
$$

It does not follow that $\hat{p}(z|xy) = \hat{p}(z'|xy)$. Even though the trigram counts are equal, the counts for the bigrams $yz/yz'$ and unigrams $z/z'$ may differ.

If, instead, $c(xyz) = c(xyz') = 1$, the answer is the same. Even though the trigrams are found in the training corpus, backoff still affects the overall estimated probability, so there is no guarantee that $\hat{p}(z|xy) = \hat{p}(z'|xy)$.

**4(d)** In add-$\lambda$ smoothing with backoff, increasing $\lambda$ increases the influence of the backoff probabilities to the probability estimates. Higher $\lambda$ results in the probability estimate being closer to the backoff probabilities based on the bigram and unigram counts.

**6** I generated sentences with two models: `gen_8_0.005.model` and `gen_5.model`.

`gen_8_0.005.model`:

- SUBJECT: Re : &NAME &NAME &NAME : The &NAME younger skills me moping Everybody blue revealed parts connector wondering filling
- SUBJECT: Re : &NAME &NAME Avenue hair heart NOTICE cunning lot practices retainer Repeat drinks dirty income r_a work-filled ONLY
- SUBJECT: Re : &NAME ] - editing challenge nearby parted teaching + THIS own non-returnable treats scheme he weighing Director
- SUBJECT: &NUM &NAME : &EMAIL ¡END_QUOTE¿ Me obviously shirts Everybody Of hundreds fee password overview was visiting top_docs hotmail single
- SUBJECT: Re : ? resources wear KNOW third People games perfect Below cuz drink Body AntoineIce present replying Incidentally including
- SUBJECT: Re : To issued folders believe hand In process sarcastic action electronic phones exercise Helpers volunteers occupation result Independence
- SUBJECT: OOV &NAME &CHAR any price House backup decided quite offer caused customer mind several huge captain affiliates Address sincerely
- SUBJECT: where alert replies involed reception visitors poster OVERALL Safety sick ever FOR Hop-on ie with indicate records sarcastic institutions
- SUBJECT: &NAME OOV OOV . &NAME &NAME : ( &NUM : &NUM ) &NUM &NAME &CHAR , &NAME and present
- SUBJECT: reminder traditional Independence finish AT relevant muscles You mortgage record chance ha took Event unintelligible wrong hard death For

`gen_5.model`:

- nearby matters old hour Science pm Okay continue ¡QUOTE¿ getting bill developed updates auld triples or French eval main offer
- year upon forwards convenient nothing modern credit pricing 26th &SMILEY factory assume Inc. rock include Event led fields task pre-varsity
- given between Evil respirators Make Are account months regret representation space audible advance promise awesome Dialogue supplies nicety miss identified

- appeal discourse economics permission break HILLSIDE conference last reasonable Hello realised chips photos anyway group booked HI call 'Stop choose
- time torture industry displayed awesome language If avoid job be Find lack Further huge hear quest just length x informatin
- clean November preference monsignor behavior FOR monsignor green approval feedback drive colleagues quality Acoustic never photograph candidate races beyond award
- waste Events Turn lesser tomorrow separate minerals psychologist +Do opt Dollars construct events elitist darn wo today Back yourself queries
- include Under # Inc. Unless aging lesser problem sleep nltools 1500m also imminent href' lasting event event differences afraid photo
- products mood External OF brief Smallest amusing tobacco expected relational finish Unless Upon disclose important saturday Citrate You Very slip
- wish trip applies forwarding masks President consist System younger base pictures purchased i.e. off Get check themselves writing watch door

One striking difference is that the former model generates sentences that start with "SUBJECT" whereas the latter model does not. It is because the latter model has a very high lambda value which gives less weight to the most probable word.

**8** Our goal is to maximize the probability that $\vec{w}_i$ was what the speaker was trying to say, given the input sound $u$. Since the language used here is English, we will assume that $p(\vec{w}) = p(\vec{w}|\text{language} = \text{English})$.

$$\begin{aligned}
\arg\ max_i p(\vec{w}_i|u) &= \arg\ max_i \frac{p(\vec{w}_i) \cdot p(u|\vec{w}_i)}{p(u)} \\
&= \arg\ max_i p(\vec{w}_i) \cdot p(u|\vec{w}_i) \\
&= \arg\ max_i \log_2 \left(p(\vec{w}_i) \cdot p(u|\vec{w}_i)\right) \\
&= \arg\ max_i \log_2 \left(p(\vec{w}_i)\right) + \log_2 \left(p(u|\vec{w}_i)\right)
\end{aligned}$$

Among the 9 candidates, we should choose the sentence that has the highest value of $\log_2 (p(\vec{w}_i)) + \log_2 (p(u|\vec{w}_i))$. The first term is the negative cross-entropy of the sentence given an English language model, and it can be computed with the English language model trained with `data/english_spanish/train/en.*`. The second term is explicitly stated in the speech file.