Max Tjen
CSDS 313
Homework 3

6) a) Prove H(X, Y) = H(X) + H(Y | X)

$$H(X,\ Y) = -\Sigma_i\Sigma_j p_{i,j} log_2(p_{i,j})$$
$$H(X,\ Y) = -\Sigma_i\Sigma_j p_{i,j} log_2 p_i p_{j|i}$$
$$H(X,\ Y) = -\Sigma_i\Sigma_j p_{i,j} log_2 p_i - \Sigma_i\Sigma_j p_{i,j} log_2 p_{j|i}$$
$$H(X,\ Y) = -\Sigma_i p_i log_2 p_i - \Sigma_i\Sigma_j p_{i,j} log_2 p_{j|i}$$
$$H(X,\ Y) = H(X) + H(Y \mid X)$$

b) Prove I(X, Y) = H(Y) - H(Y | X)

$$I(X,\ Y) = H(X) + H(Y) - H(X,\ Y)$$
$$I(X,\ Y) = H(Y) - (H(X,\ Y) - H(X))$$
$$I(X,\ Y) = H(Y) - H(Y \mid X)$$