POLYNOMIAL EXPANSION OF THE GEOID

Due Date: 3/13/2014 @ 11:59 pm

The usual way to treat Earth is as a point particle, $V(r) = GM/r$. This is adequate when we want to propagate the motion of a distant body, but not good enough for spy satellites. In that case we need to consider Earth as a geoid:

$$V(r, \phi, \lambda) = \frac{GM}{r}\left(1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n} a_{nm}\mathbf{Y}_{nm}(\phi, \lambda) + \frac{\omega_0^2 r^3}{2GM}\sin^2(\phi)\right)$$

Here a = 6378136.3 m is the semi-major axis of the reference ellipsoid, ω_0 = 7292115 $\times 10^{-11}/$ s is the angular velocity of rotation, and a_{nm} are the expansion coefficients. These have been tabulated by the EGM96 model found on the course web page.

- Plot the deformation of a massive body due to rotation for several different values of rotational velocity.
- Plot the geoid potential. Think of a good way (or look it up) to represent the 3D sphere as a map. Is it surprising that you do not see the continents?

★ Integrate the trajectories of three types of satellites: geostationary (r = 42, 000 km), TOPEX/POSEIDON (h = 1330 km) and spy satellites (h = 180 km). How well are the energy and the angular momentum conserved?

★ Determine the time cost and the numerical integration accuracy as a function of the number of expansion terms taken.