FOURIER ANALYSIS

Due Date: 3/20/2014 @ 11:59 pm

Fourier transforms are one of the most commonly used signal processing tools. In a nutshell, think of a time-domain function $h(t)$ and a corresponding frequency-domain function $H(f)$ as two representations of the same function, with the following mapping that lets you go back and forth between the two:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f t} dt,$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i f t} df.$$

For N discrete data points $h(t_k) \equiv h_k$, the discrete Fourier transform is given by:

$$H(f_n) \equiv H_n = \sum_{k=0}^{N-1} h_k e^{2\pi i k n / N},$$

and its corresponding inverse is given by:

$$h(t_k) \equiv h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi i k n / N}.$$

- Compute and plot the Fourier transform and the inverse Fourier transform for the following functions: sine, cosine, square pulse, triangle, delta. Compute power spectral density (one-sided) for all functions. This will give you an idea of what the Fourier transform does.

- Create a mixture of sines and cosines, i.e.

  $$h(t) = \sin(\pi t) + 3\sin(3\pi t) + 5\cos(5\pi t).$$

  Try the same with non-integer frequencies. Can you detect aliasing? Calculate transforms of some non-periodic functions as well.

- Analyze the sound of a guitar (the wav and the text files are on the course homepage). Can you detect harmonics? Assuming the guitar was tuned in standard tuning with $A_4 = 440$ Hz, can you determine which note was played?

- Listen to Bach's Partita in the mp3 format. Then listen to a 2.3 s excerpt that has been sampled at different rates: 882, 1378, 2756, 5512, 11025 and 44100 Hz. Once you hear what the effect of under-sampling is, quantify it by the Fourier transform. All data (in both mp3 and ascii) are available on the course web page.