POPULATION MODELS

Due Date: 2/19/2014 @ 11.59 pm

Population models are another nice example of dynamical systems that, in general, will not have an analytical solution. Their application is very widespread and they found their way in almost all branches of science. Studying them numerically gives us an insight into many of their properties.

- **Logistic model.** After the initial hype (and failure) of the basic (exponential) population growth model, we need to incorporate resource limitations into the model. These are described with the logistic equation (the Verhulst model):
$$\dot{N} = rN(1-\frac{N}{K})$$
where N is the population size, r is the increase rate, and K is the size constraint. Using the table of US population available from the course web page, compare the fit of the logistic model to the fit of the exponential growth model. What are suitable values of r and K ?

- **Predator-prey model.** Draw and carefully analyze the phase diagram of the Lotka-Volterra model. Determine the population equilibrium (the point in the phase diagram where the population levels of foxes and rabbits are not changing) and points of stability. Find period of your solutions and determine the phase shift between the prey and the predators. What happens if you add wolves into the system?

- **The epidemic model.** Divide the population into three groups: (1) healthy, (2) sick, and (3) immune. The disease spreads by contact between the sick and the healthy with some constant probability. The sick become immune (either healthy or dead) by another constant probability. During the epidemic, we are interested in: the time of peak in the number of sick people; the overall number of sick people; and the maximum number of sick people at one time. Determine these quantities for some realistic values of parameter(s). How does immunotherapy (shots) change these results?