PARTIAL DIFFERENTIAL EQUATIONS

Due Date: 4/2/2014 @ 11:59 pm

When solving partial differential equations constrained by some initial and boundary conditions we can employ two different classes of methods. Spectral methods involve expanding the solution over a set of functions (sines and cosines, for example), and then evolving the coefficients of the expansion in time. When using difference methods, we approximate partial time and space derivatives with finite differences. This translates the problem to solving algebraic equations.

You will be using a method from each class to find a solution of the heat equation,

$$\frac{\partial T}{\partial t} - D\frac{\partial^2 T}{\partial x^2} = 0,$$

where D is the heat diffusion constant, and the damped wave equation,

$$\frac{\partial^2 u}{\partial t^2} - c\frac{\partial^2 u}{\partial x^2} + 2k\frac{\partial u}{\partial t} = 0,$$

where c is the propagation speed of the wave, and k is the damping constant.

- Calculate the time evolution of a box temperature profile in a solid metal rod of length L. Compare the results you obtain with the spectral method to the results you get by convolving a box profile with a Gaussian function.
- Construct a non-symmetrical temperature profile and study its evolution in time. Observe how the temperature profile behaves at the edges of the rod. Reconstruct your profile using only 10-20 components and compare it to the fully reconstructed profile at different times.
- Model the motion of a plucked string with the finite differences method. Study the case where the string is plucked both at its center and off-center, with and without damping.
- How does a Gaussian pulse on a string propagate with time?