

POLYNOMIAL EXPANSION OF THE GEOID

Due Date: 3/13/2014 @ 11:59 pm

The usual way to treat Earth is as a point particle, so the gravitational potential can be written as $V(r) = GM/r$. This is adequate when we want to propagate the motion of a distant body, but not good enough for spy satellites. In that case we need to consider Earth as a geoid:

$$V(r, \phi, \lambda) = \frac{GM}{r} \left(1 + \sum_{n=2}^{\infty} \left(\frac{a}{r} \right)^n \sum_{m=0}^n a_{nm} Y_{nm}(\phi, \lambda) + \frac{\omega_0^2 r^3}{2GM} \sin^2(\phi) \right)$$

Here $a = 6378136.3$ m is the semi-major axis of the reference ellipsoid, $\omega_0 = 7292115 \times 10^{-11}$ / s is the angular velocity of rotation, and a_{nm} are the expansion coefficients. These have been tabulated by the EGM96 model found on the course web page.

- Plot the deformation of a massive body due to rotation for several different ratios of gravitational to rotational acceleration.
- Plot the geoid potential. Think of a good way (or look it up) to represent the 3D sphere as a map. Is it surprising that you do not see the continents?
- ★ Integrate the trajectories of three types of satellites: geostationary ($r = 42,000$ km), TOPEX/POSEIDON ($h = 1330$ km) and spy satellites ($h = 180$ km). How well are the energy and the angular momentum conserved?
- ★ Determine the time cost and the numerical integration accuracy as a function of the number of expansion terms taken.