1. $\dot{y} + y = x$

$\mathcal{L}\{\ \}$

$\rightarrow sY(s) + Y(s) = x$

$H(s) = \frac{X(s)}{Y(s)} = \frac{1}{s+1}$

Step response : $\frac{1}{s} \cdot \frac{1}{s+1} = \frac{A}{s} + \frac{B}{s+1}$

$A + B = 0 \qquad A = 1$

$B = -1$

$y(t) = Ae^{0t}u(t) + Be^{-1t}u(t)$

$\therefore y(t) = (1 + (-1)e^{-t})u(t)$

2. a) Black's Formula

$\frac{Y}{Y_{sp}} = \frac{KH}{1+KH}, \quad K = \frac{K_1}{s}$

$= \frac{K_1 H}{s + K_1 H}$ DC Gain: Final Value Theorem

$\lim_{s \to 0} sX(s) = x(0)$

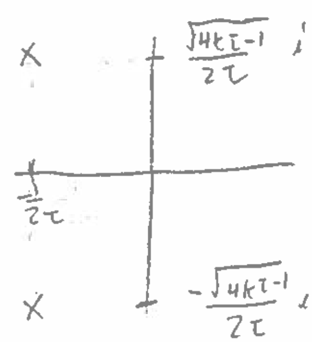
$\lim_{s \to \infty} \left(\frac{sK_1H}{s + K_1H} \right) = \boxed{K_1H}$, Yes, depends on K_1

b) $H(s) = \frac{1/\tau}{s + 1/\tau}$ $\quad \frac{Y}{Y_{sp}} = \frac{K_1 \frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}}{s + K_1 \frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}} = \frac{K_1/\tau}{s(s+\frac{1}{\tau}) + K_1/\tau} = \frac{K_1}{s^2\tau + s + K_1}$

Zeros : none

imaginary

poles : $s = \frac{-1 \pm \sqrt{1 - 4k\tau}}{2\tau} \qquad s = \frac{-1 \pm i\sqrt{4k\tau - 1}}{2\tau}$



3. a) Very simple high pass filter with no oscillations one pole at -1, and one zero at 0. Step response is ever decreasing.

b) Appears to make a band-pass filter that shifts frequency outside of its pass. ~~poles~~ One pole is exceptionally close to the ~~zero~~ zero as well.

c) A sharper band pass with two imaginary poles and one zero. The step response has a bit o oscillation.

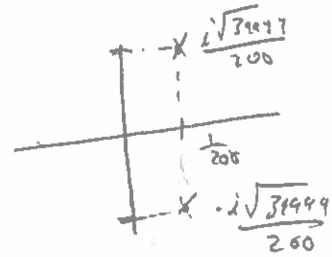
d) Two poles that are bothe nearly only imaginary. Very sharp band-pass filter and an extremely oscillatory step-response.

e) 2 Poles and zeros that are extremely close and mostly imaginary. Bode is constant and the step response is pretty close to a constant 1.

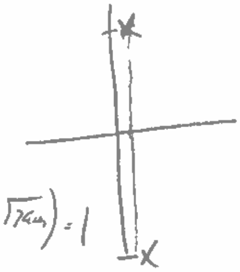
f) Very similar to e), But works as a band-stop filter. Step response and pole-zero are still similar.

4) A) Zeroes: None

Poles: $\dfrac{\frac{1}{100} \pm \sqrt{\frac{1}{10^4} - 4}}{2} = \frac{1}{200}\left(1 \pm i\sqrt{39999}\right)$



B Step response: $\frac{1}{s}\left(\frac{1}{s-r_1}\right)\left(\frac{1}{s-r_2}\right) = 1$



$\frac{A}{s} + \frac{B}{s-r_1} + \frac{C}{s-r_2} = 1$

$A\left(s^2 - \frac{1}{100}s + 1\right) + B(s)\left(s - \frac{1}{200} + \frac{1}{200}i\sqrt{39999}\right) + C(s)\left(s - \frac{1}{200} - \frac{1}{200}i\sqrt{39999}\right) = 1$

$A = 1, \quad A + B + C = 0, \quad B + C = -1$ $\qquad 1As^2 + Bs^2 + Cs^2 = 0s^2, \quad A + B + C = 0$

$1A + 0B + 0C = 1, \quad A = 1$

$2A + (-1 + i\sqrt{39999})B + (-1 - i\sqrt{39999})(-1 - B) = 0$

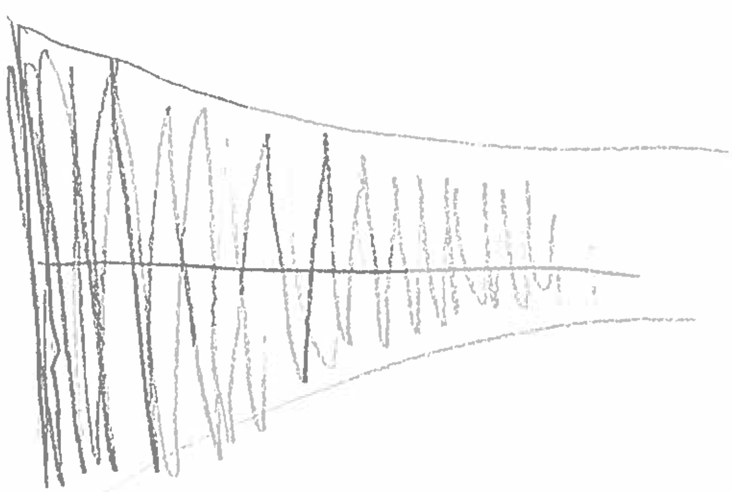
$2 - (-1 - i\sqrt{39999}) + \left[(-1 + i\sqrt{39999}) - (-1 - i\sqrt{39999})\right]B = 0$

$(2i\sqrt{39999})B = -3 - i\sqrt{39999}$

$B = \frac{3i}{2\sqrt{39999}} - \frac{1}{2} = -\frac{1}{2} + \frac{3}{2\sqrt{39999}}i$

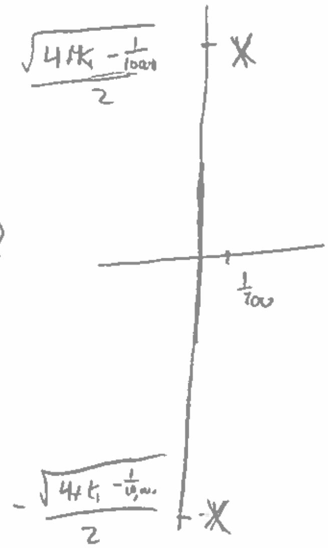
$C = -1 - B = -\frac{1}{2} - \frac{3i}{2\sqrt{39999}}$

Step response: $y(t) = \left[1 + \left(-\frac{1}{2} + \frac{3}{2\sqrt{39999}}i\right)e^{-\left(\frac{1}{200}\right)\left(1 + i\sqrt{39999}\right)t} + \left(-\frac{1}{2} - \frac{3i}{2\sqrt{39999}}\right)e^{-\left(\frac{1}{200}\right)\left(1 - i\sqrt{39999}\right)t}\right]u(t)$



B) Proportional Control: $\dfrac{K_1\left(\frac{1}{s^2-0.01s+1}\right)}{1+K_1\left(\frac{1}{s^2-0.01s+1}\right)}$

$= \dfrac{K_1}{s^2+0.01s+1+K_1}$ Poles: $\dfrac{\frac{1}{100} \pm \sqrt{\frac{1}{10^4}-4(1+K_1)}}{2}$



Stabilization: DC gain: $\dfrac{K_1}{1+K_1}$ Const: $\dfrac{1}{K_1+1}$

$\dfrac{K_1 e^{-st_0}}{s^2-0.01s+1+K_1 e^{-st_0}}$

$1-st_0$

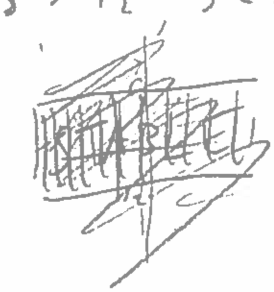
$s^2-0.01s+(1+K_1-K_1 st_0)$

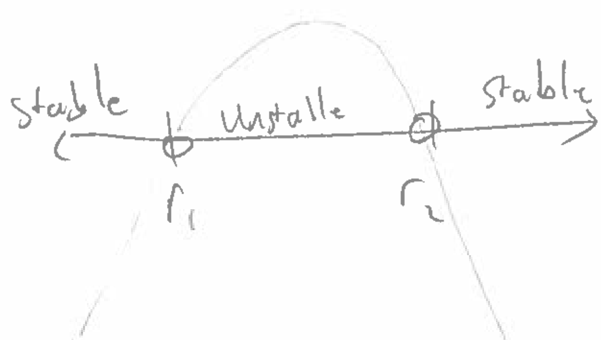
Roots $r_1, r_2 \to \mathbb{C}$

$\dfrac{1}{(s-r_1)(s-r_2)}$

Unstable if $\dfrac{K_1(1-st_0)}{s^2-0.01s+1+K_1(1-st_0)} > 6$ unstable if $(s-r_1)(s-r_2)>0$

or $s > r_2$, $s < r_1$



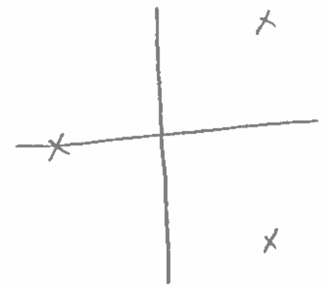


roots of $s^2+(-0.01-K_1t_0)s+(1+K)$

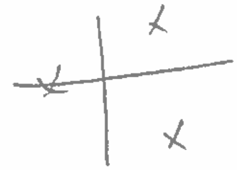
c) Integral $K = \frac{K_1}{s}$

$$\frac{\frac{K_1}{s}\left(\frac{1}{s^2-0.01s+1}\right)}{1+\frac{K_1}{s}\left(\frac{1}{s^2-0.01s+1}\right)} = \frac{K_1}{s^3-0.01s^2+s+K_1}$$

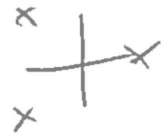
Poles:



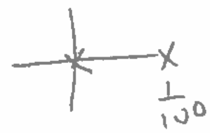
$K_1 > 0$



$K_1 < 0$



$K_1 = 0$,

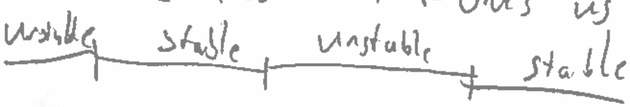


$$\frac{K_1}{s^3-0.01s^2+s+K_1}$$

DC Gain = 1

~~Unstable is $s^3-0.01s^2+s+K_1>0$ which is at all points~~

$$\frac{K_1 - K_1 s t_0}{s^3-0.01s^2+(1-K_1t_0)s+K_1}$$ ← Gives us r_1, r_2, r_3



$r_1 < s < r_2$ or $s > r_3$

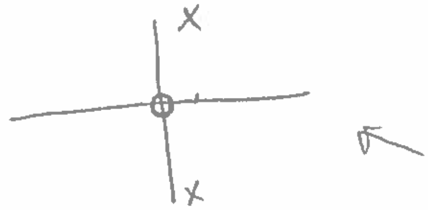
D) Derivative $K_{D} = sK_1$

$$\frac{sK_1 \frac{1}{\sim}}{1+sK_1\frac{1}{\sim}} = \frac{sK_1}{\sim + sK_1}$$

$b^2-4ac>0$

$b^2-4ac=0$

$b^2-4ac<0$



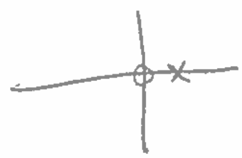
$$\frac{sK_1}{s^2+(-0.01+K_1)s+1}$$

$$\frac{0.01 \pm \sqrt{\frac{1}{10^4} - \frac{2K_1}{10^2} + K_1^2 - 4}}{2}$$

$b^2-4ac>0$

DC Gain: $s=0 \to 0$

$b^2-4ac=0$



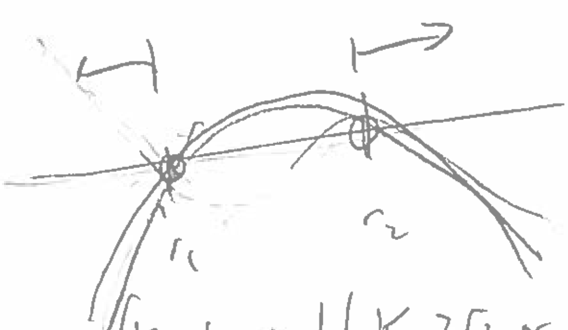
$$\frac{sK_1}{s^2+(-0.01+K_1)s+1}$$

$K_1 \to K_1(1-st_0)$

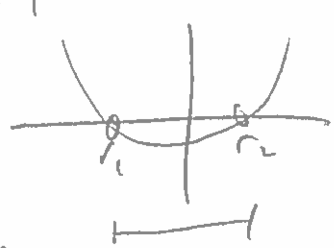
$$\frac{sK_1(1-st_0)}{(1-K_{t0})s^2+(-0.01+K_1)s+1}$$

Roots at $\dfrac{0.01-K_1 \pm \sqrt{K_1^2 - \frac{2K_1}{100} + \frac{1}{10^4} - 4 + 4K_{T0}}}{2-2K_{T0}}$

$K_{T0} \geq 1$



$K_{F0} > 1$



$(K_{T0}<1$ and $(K_1>r_2$ or $K_1<r_1))$ or $((r_1<K<r_2)$ and $K_{T0}>1)$