PS03

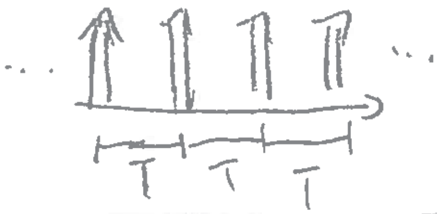
Mitchell Kwon

1.

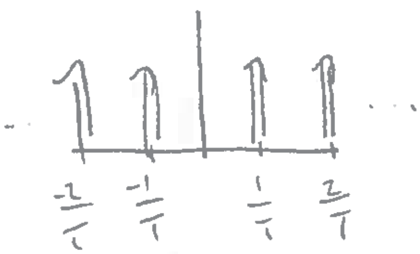
$x(t)$

$X(\omega)$

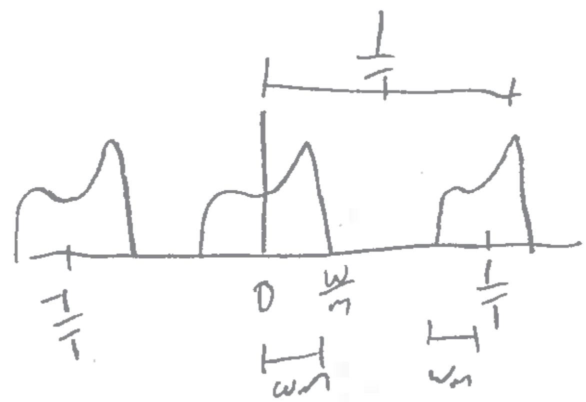
a) $p(t)$



b) $P(\omega)$



c) $X_p(\omega) = X(\omega) * P(\omega)$

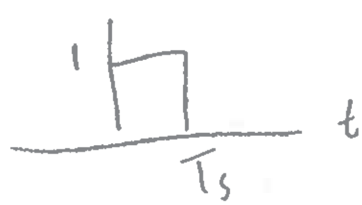


d) $\frac{1}{T} > 2 \cdot \omega_M$

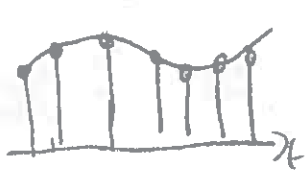
$$\boxed{2 \cdot T \cdot \omega_M < 1}$$

e) Apply LPF of $f_{cutoff} = \omega_M$

f) $z(t)$



g) $x_p(t) * z(t) = x_z(t)$



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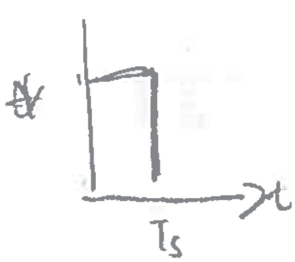

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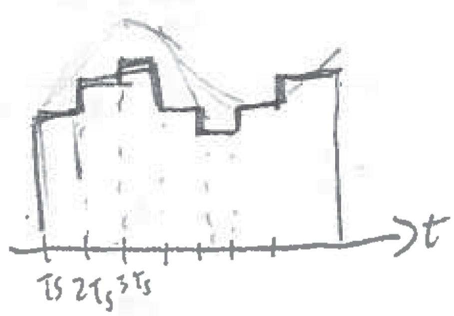
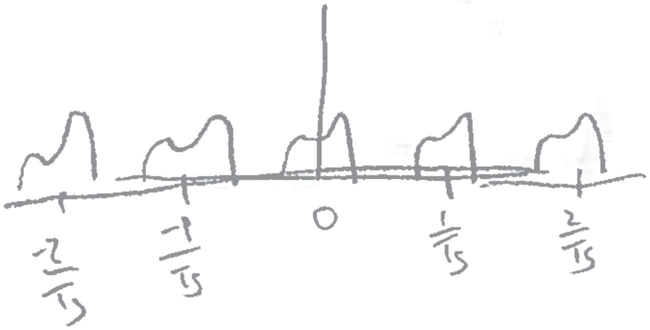


P. (b) $\bar{X}_Z(\omega) = \bar{X}_p(\omega) \cdot Z(\omega)$

$X_p(\omega)$



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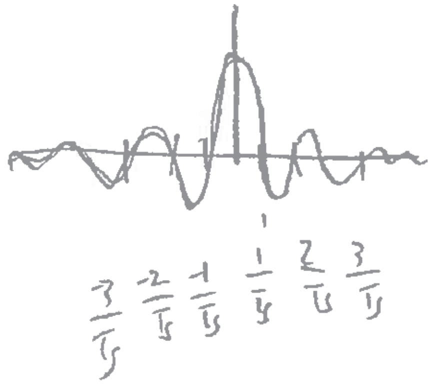

$X_Z(\omega)$

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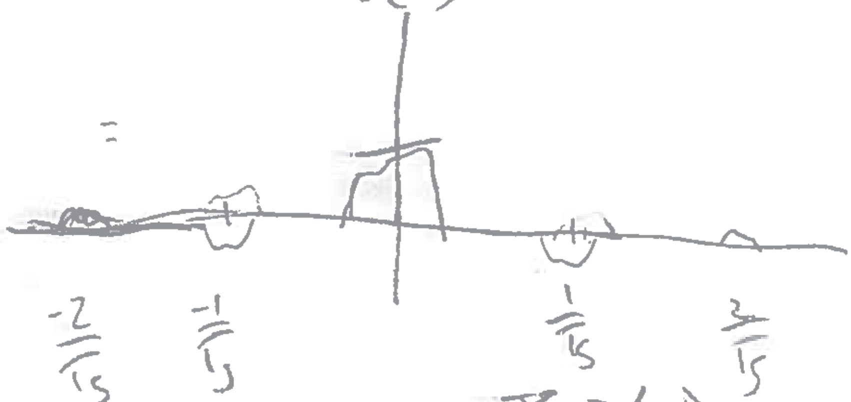
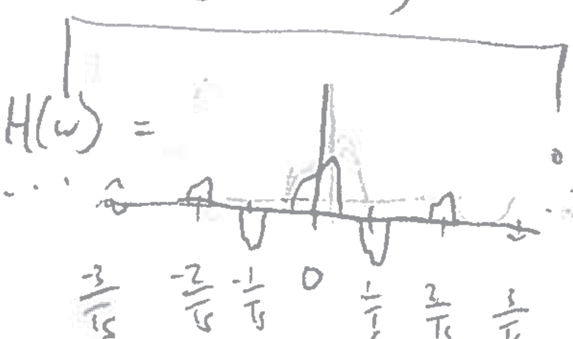
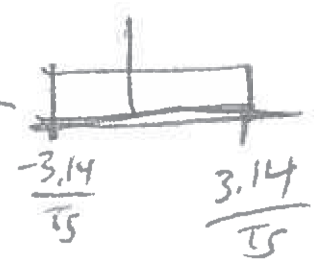


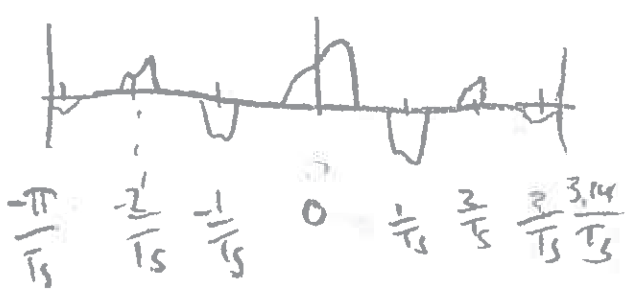
$\bar{X}_Z(\omega)$ $H(\omega)$

i) $\bar{X}(\omega) = X_Z(\omega) H(\omega) =$





$\bar{X}(\omega) =$



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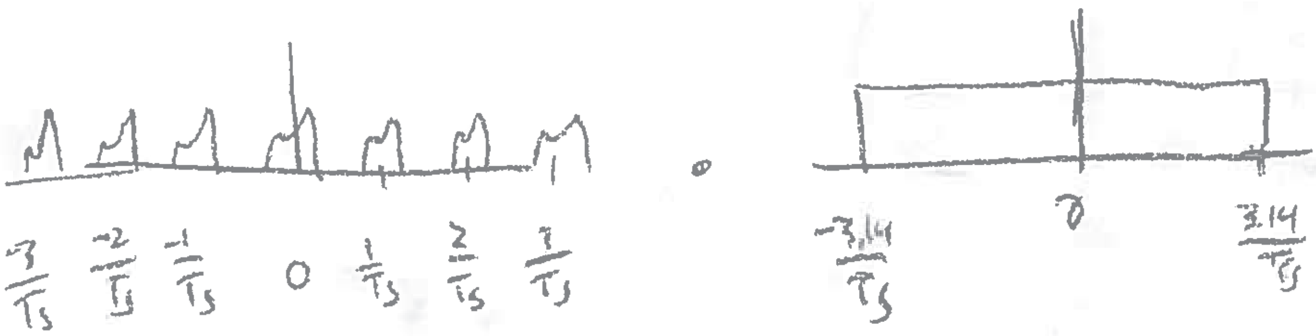
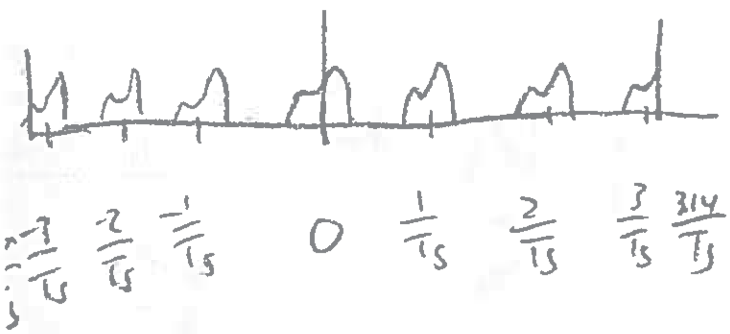
i) Continued

$\hat{X}(\omega) = X_p(\omega) H(\omega)$

$X_p(\omega)$



$\hat{X}(\omega)$



j) $\bar{X} = \hat{X}(\omega) \cdot \mathrm{sinc}(T_s \omega)$. Innermost is largest and outside harmonics are decreased by $\mathrm{sinc}(T_s \omega)$

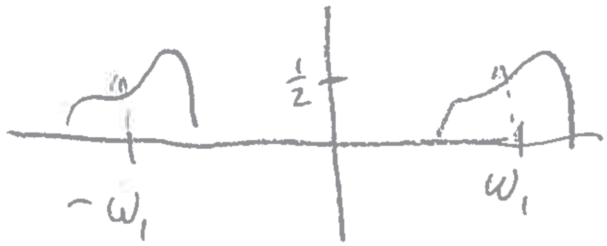
k. At $\omega = \frac{\pi}{T_s}$, $\bar{X}(\omega) = X_z(\omega) = X_p(\omega) \cdot Z(\omega) = X_p(\omega_n) \cdot Z(\omega_n)$

As $\omega = \frac{\pi}{T_s}$, $\hat{X}(\omega) = X_p(\omega) = X_p(\omega_n)$

$$\therefore \frac{\bar{X}(\omega_n)}{\hat{X}(\omega_n)} = \frac{X_p(\omega_n) Z(\omega_n)}{X_p(\omega_n)} = \boxed{Z(\omega_n)}$$

2. a) $Y(\omega) = X_1(\omega) * \boxed{\cos(\omega_1 t)} + X_2(\omega) * \boxed{\cos(\omega_2 t)}$

$\frac{1}{2}(\delta(t \pm \omega_1))$ $\frac{1}{2}(\delta(t \pm \omega_2))$



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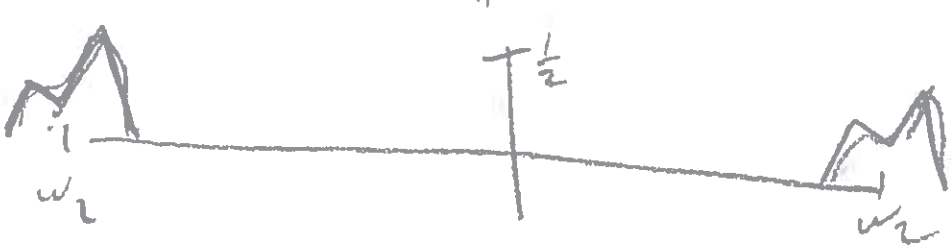

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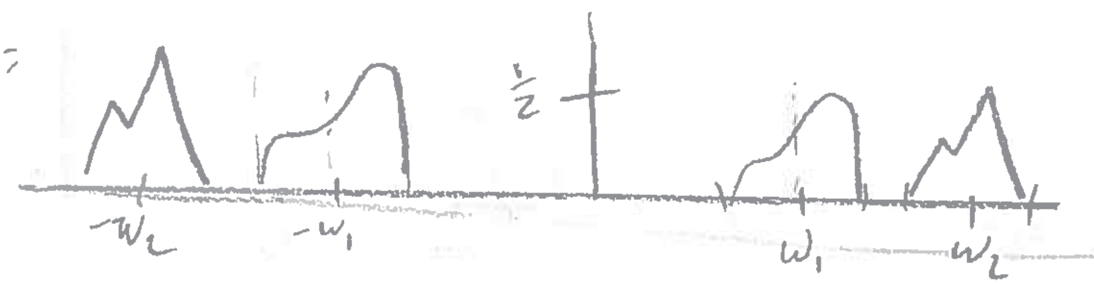
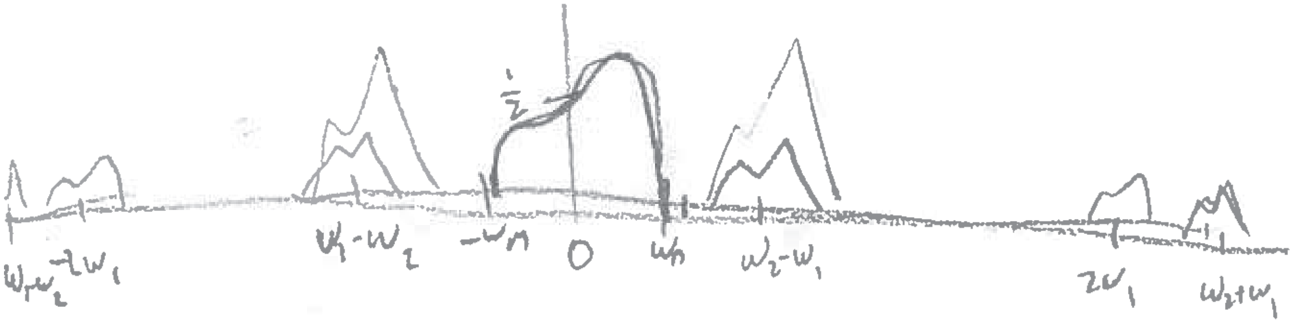
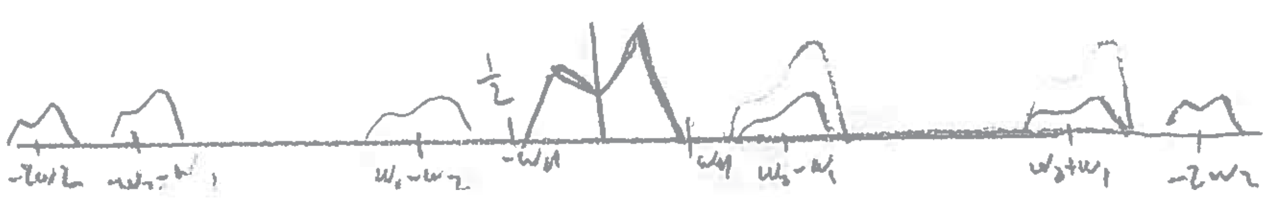


b) $y(t)\cos(\omega_1 t) \Rightarrow Y(\omega) * \left(\frac{1}{2}\delta(\omega \pm \omega_1)\right)$



$y(t)\cos(\omega_2 t) \Rightarrow Y(\omega) * \left(\frac{1}{2}\delta(\omega \pm \omega_2)\right)$



2. c) Recovering $x_1(t)$ from $y(t)$:

LPF (cutoff $= \omega_M$)

$= \frac{1}{2} x_1(t)$

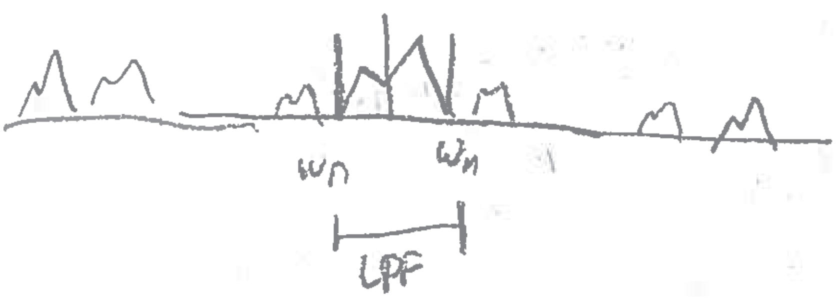
$$x_1(t) = \mathrm{LPF}(y(t)\cos(\omega_1 t),\ \omega_M) \cdot 2$$

$$= \underline{2 \cdot \mathrm{LPF}\left(y(t)\cos(\omega_1 t),\ \text{cutoff} = \omega_M\right)}$$

$x_2(t)$: Same process, but replace ω_1 with ω_2

$$\boxed{x_2 = 2 \cdot \mathrm{LPF}\left(y(t)\cos(\omega_2 t),\ \text{cutoff} = \omega_M\right)}$$

$y(t)\cos(\omega_2 t) \Rightarrow$



$= \frac{1}{2} x_2(t)$

3. a) $V_{out} + V_L + V_R(t) = V_{in}(t)$ $\qquad V_L(t) = L\frac{di}{dt}$

$$V_{out} + L\frac{di}{dt} + R\,i(t) = V_{in}(t)$$

$V_{out} = \frac{Q}{C}$ $\quad V_L = L\,Q''$ $\quad V_R = RQ'$

$V_L = LC(V_{out})''$ $\quad V_R = RC(V_{out})'$

a) $\boxed{V_{out} + LC(V_{out})'' + RC(V_{out})' = V_{in}}$

b) $F(V_{out}) + (j\omega)^2 LC[F(V_{out})] + (j\omega)RC[F(V_{out})] = F(V_{in})$

$F(V_{out})(1 + j\omega RC - \omega^2 LC) = F(V_{in})$

$F(V_{out}) = \dfrac{F(V_{in})}{1 + j\omega RC - \omega^2 LC}$

$H(\omega) = \dfrac{1}{1 + j\omega RC - \omega^2 LC}$

c) $|H(\omega)| = \sqrt{R(H(\omega))^2 + I(H(\omega))^2}$

$\dfrac{1}{(1-\omega^2 LC) + \omega RC[j]}$

$R(H(\omega)) = \dfrac{1-\omega^2 RC}{(1-\omega^2 LC)^2 - (\omega RC)^2}$

$\dfrac{(1-\omega^2 LC) - \omega RCj}{(1-\omega^2 LC)^2 - (\omega RC)^2}$

$I(H(\omega)) = \dfrac{-\omega RC}{(1-\omega^2 LC)^2 - (\omega RC)^2}$

$\therefore |H(\omega)| = \dfrac{\sqrt{(1-\omega^2 RC)^2 + (\omega RC)^2}}{(1-\omega^2 LC)^2 - (\omega RC)^2}$

$a = 1-\omega^2 RC$
$b = \omega RC$

$\dfrac{\sqrt{a^2+b^2}}{a^2-b^2}$

d) $\dfrac{d(H(\omega))}{d\omega} = \cancel{\tfrac{1}{2}\left((1-\omega^2 RC)^2 + \omega^2 R^2 C^2\right)^{-\frac{1}{2}}}($

$(a^2-b^2)\cdot \tfrac{1}{2}(a^2+b^2)^{-\frac{1}{2}}(2(1-\omega^2 RC)(2\omega RC) + 2R^2C^2\omega$

Max when $\frac{d|H(\omega)|}{d\omega} = 0$

$$\frac{d\,\frac{a}{b}}{d\omega} = \frac{b\,da - a\,db}{b^2}$$

$$\frac{d|H(\omega)|}{d\omega} =$$

$b\,da = a\,db$

$$(a^2-b^2)\left[\tfrac{1}{2}(a^2+b^2)^{-\frac{1}{2}}\boxed{\left(2(a)(2\omega RC)(RC)+2b(RC)\right)}\right]$$

$$= (a^2+b^2)^{\frac{1}{2}}\boxed{\left[2a(2\omega RC)(RC) + 2b(RC)\right]}$$

$$\frac{a^2-b^2}{2} = a^2+b^2 \qquad a = 1-\omega^2 LC \qquad b = \omega RC$$

$$a^2-b^2 = 2a^2+2b^2 \qquad a^2 = 1-2\omega^2 LC + \omega^4 L^2C^2$$

$$0 = a^2+3b^2 \qquad b^2 = \omega^2 R^2 C^2$$

$$1 - (2LC + 3R^2C^2)\omega^2 + (L^2C^2)\omega^4 = 0$$

$$u = \omega^2 \qquad u = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{C}{2}\left[2L+3R^2C \pm \sqrt{4L^2+6R^2CL-4L^2+9R^4C^2}\right] = \frac{2LC+3R^2C^2 \pm \sqrt{(2LC+3R^2C^2)^2-4L^2C^2}}{2}$$

$$\frac{C}{2}\left[2L+3R^2C \pm R\sqrt{6CL+9R^2C^2}\right] \qquad \frac{C}{2}\left[\frac{2L+3R^2C \pm \sqrt{(2L+3R^2C)^2-4L^2}}{2}\right]$$

$$\frac{d|H(\omega)|}{d\omega} = 0$$

$= \omega^2$

$$\boxed{\omega = \left[\frac{C}{2}\left[2L+3R^2C \pm R\sqrt{6CL+9R^2C^2}\right]\right]^{-\frac{1}{2}}}$$