FLOW MODELS FOR TURBOMACHINES

By J. H. Horlock* and H. Marsh†

The equations for the flow through cascades of blades are averaged across the pitch and then compared with those for the flow through hypothetical models of blade rows; a closely spaced blade row, an axisymmetric flow with body forces and the flow on a mean stream surface. It is shown that these models can provide an accurate representation for the overall flow changes across a blade row, but they cannot give an exact local matching with the averaged properties of the flow in the real cascade. The nature of the blade forces in the real flow and the body forces in the hypothetical flow is discussed.

INTRODUCTION

IN THE ANALYSIS of the flow through turbomachines, it is often necessary to represent a blade row by a simple mathematical model. For example, a real cascade, which has a finite number of blades, may be represented by a cascade which has infinitely many blades, or by an axisymmetric flow with a distributed body force; or the flow on a mean stream surface.

The problem is to relate the mathematical model to the original cascade flow. In the first model, it is necessary to define the geometry of the cascade with infinitely many blades which can represent the flow through the real cascade. In the second (axisymmetric) flow model, there are no gradients of the fluid properties or velocity in the tangential, or cascade, direction, but a hypothetical body force (comparable to the body force that appears in magnetohydrodynamics) changes the tangential momentum of the fluid. In the third model, in which the through-flow methods of analysis of Smith (1)‡ and Marsh (2) are used, it is necessary to specify the mean stream surface. Some method is therefore required for determining the shape of this hypothetical stream surface.

Ruden (3) discussed the problem of representing the flow through a cascade by the fluid properties and velocity components averaged across the blade passage. He showed that if the variations from the mean values are small, then the equations governing the mean fluid properties and the mean velocity components are the same as those for the flow through a cascade having infinitely many blades. Later, Smith (1) considered the problem in more detail, and by assuming a linear variation of the fluid properties and velocity components across the blade passage, showed that if these variations are small, then the equations governing the mean properties and velocities are the same as those for an axisymmetric flow. Smith also suggested that the blade loading might be the correct criterion for the validity of the models with infinitely many blades or an axisymmetric flow.

The basic concepts of an infinitely many bladed cascade, an axisymmetric flow with a distributed body force and flow on a mean stream surface have all been developed to describe the mainstream flow, the flow outside of the wall boundary layers. However, some of these concepts are now being applied in attempts to predict the development of the wall boundary layer, for example, Horlock (4) and Gregory-Smith (5). The purpose of this paper is to re-examine the basis for these simple models of the flow through cascades and to discuss their use in predicting the flow in turbomachines.

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‡ *References are given in Appendix 3.*

Notation

$\left.\begin{array}{l} a \\ b \end{array}\right\}$ Points on the blade surface.

B Passage width parameter in the model flow.

C_L Lift coefficient.

C_y Tangential force coefficient.

c Chord.

c^x Axial chord.

F_y $= \dfrac{p_a - p_b}{s}$.

f_y $= \dfrac{\rho}{s}(v'_b - v'_a)(\bar{u}\tan\beta - \bar{v})$.

g_y $= -\rho \dfrac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'})$.

k Ratio of specific heats.

P Pressure in the model flow.

p Pressure.
q Property.
R Density in the model flow.
s Pitch.
T Temperature.
$\left.\begin{matrix} u \\ v \end{matrix}\right\}$ Velocity components.
$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ Co-ordinate system.
α Flow angle.
β Blade angle.
β_c Angle of camber line.
γ Angle defined by equation (10).
ϵ Pitch in the many bladed cascade.

Superscripts

$'$ Perturbation from the passage averaged value.
$-$ Passage averaged value.

DEFINITIONS

Before considering the analysis, it is necessary to define a number of terms that arise in the present discussion.

Actual flow

The actual flow is the real flow through any given blade row, as shown in Fig. 1. It is, however, assumed that the actual flow has some properties of an ideal flow, that it is steady and that far upstream and far downstream the real flow is uniform. The fluid is also taken as inviscid. The analysis is based on a three-dimensional rectangular co-ordinate system (x, y, z) but can be extended to the cylindrical co-ordinate system, as used by Ruden. The three velocity components u, v and w and the fluid properties p, ρ and T are all functions of x, y and z.

Passage averaged flow

The passage averaged flow is obtained by averaging the velocity components and fluid properties in the y direction across the blade passage. Such velocities and properties are denoted by a superscript and are related to the actual flow by passage averages,

$$\bar{q} = \frac{1}{(b-a)} \int_a^b q \, dy$$

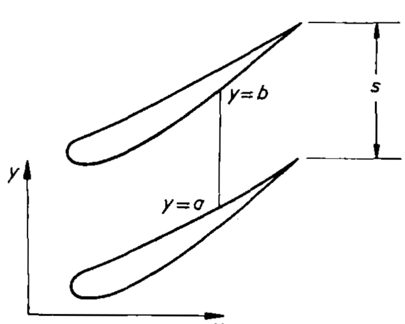

Fig. 1. *Cascade geometry*

The governing equations are then presented in terms of the averaged properties $\bar{q}$ and do not include averaged products.

Continuity and momentum averaged flow

The continuity and momentum averaged flow is obtained by averaging all terms in the continuity and momentum equations across the blade passage to provide averaged flow equations. Averaged products are retained, so that in the continuity equation,

$$\overline{\rho u} = \frac{1}{(b-a)} \int_a^b \rho u \, dy$$

and in the momentum equation

$$\overline{\rho u^2} = \frac{1}{(b-a)} \int_a^b \rho u^2 \, dy$$

The governing equations are therefore averaged across the blade passage.

S1 family of surfaces

Wu (**6**) has shown that the actual flow in a turbomachine can be obtained by calculating the flow on two intersecting familes of stream surfaces (Fig. 2). The S1 family of surfaces is essentially a set of blade-to-blade surfaces, whereas the S2 stream surfaces lie between the blades. If the local thickness and geometry of the S2 stream surfaces are known from the flow on the S1 surfaces, then the flow through the cascade can be calculated using the methods of Smith (**1**) or Marsh (**2**).

Hypothetical flows

The three mathematical models, or hypothetical flows, are now defined.

Many bladed cascade flow

The many bladed cascade flow is the flow through a

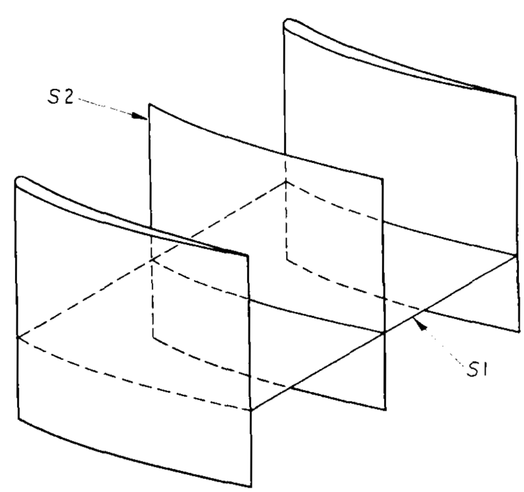

Fig. 2. *S1 and S2 stream surfaces*

cascade in which the pitch of the blades has been reduced to an infinitesimally small distance. The geometry of the many bladed cascade is chosen such that the flow is representative of the actual flow in the original blade row. The change of momentum in the y direction is produced by the blades, which still exist in the many bladed cascade flow. By definition, the thickness of the blades in this hypothetical flow must be zero, but the ratio of thickness to pitch can remain non-zero and may, if necessary, differ from that of the real cascade. An actuator disc can be regarded as a limiting case of a many bladed cascade in which the axial chord length is reduced to zero.

Axisymmetric flow

The axisymmetric flow is a flow in which there is no variation of the fluid properties or velocity components in the cascade, or y, direction. There are no blades in the axisymmetric flow and the change of momentum in the y direction is produced by a distributed body force $F(x, z)$.

Mean stream surface

The mean stream surface is a hypothetical S2 stream surface and it is chosen so that the solution for the flow on this surface is representative of some averaged flow through the real cascade. This is a hypothetical flow; it is not one in which the properties of the flow or the governing equations have been averaged across the blade passage. The correspondence between the flow on the mean stream surface and an axisymmetric flow has been discussed by Marsh (**2**).

Averaging

The following relationship is frequently used when averaging across the blade passage

$$\frac{\partial}{\partial x}\int_a^b q\,\mathrm{d}y = \int_a^b \frac{\partial q}{\partial x}\,\mathrm{d}y + q_b\frac{\partial b}{\partial x} - q_a\frac{\partial a}{\partial x} \quad . \quad (1)$$

On the blade surface

$$\frac{\partial a}{\partial x} = \tan\beta_a$$

$$\frac{\partial b}{\partial x} = \tan\beta_b$$

and defining

$$\bar{q} = \frac{1}{(b-a)}\int_a^b q\,\mathrm{d}y$$

it follows that

$$\int_a^b \frac{\partial q}{\partial x}\,\mathrm{d}y = \frac{\partial}{\partial x}[\bar{q}(b-a)] + q_a\tan\beta_a - q_b\tan\beta_b$$

OBJECTIVES

The objective of the designer or research worker is to establish equivalence between the averaged flow and a hypothetical flow, such as that for a many bladed cascade. The equivalence may be established locally between particular properties of the flow, or for overall changes in properties across the blade row.

These concepts are difficult and the problem is therefore approached here in three stages.

(1) The first step is to consider the simplest actual flow, an incompressible inviscid flow through a cascade of constant section blades of negligible thickness (see section entitled 'Incompressible, inviscid flow through a two-dimensional cascade of thin blades'). The conditions for equivalence of the averaged and hypothetical flows are established.

(2) A more general cascade flow is then considered in which the density may vary and blade thickness effects are important (see section entitled 'Compressible flow through a cascade of blades with thickness'). Again, equivalence of the averaged and hypothetical flows is discussed.

(3) Finally, the nature of the blade or body force in the hypothetical flow is discussed in relation to the blade force of the actual flow (see section entitled 'Discussion of the blade force and the body force').

INCOMPRESSIBLE, INVISCID FLOW THROUGH A TWO-DIMENSIONAL CASCADE OF THIN BLADES

The actual flow through the cascade is described by the equations of continuity and momentum,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad . \quad . \quad . \quad (3a)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \quad . \quad . \quad . \quad (3b)$$

or, alternatively, for the momentum equations

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) \quad . \quad . \quad (4a)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) \quad . \quad . \quad (4b)$$

The flow angle α at any point is $\tan^{-1}(v/u)$ and in general varies from β_a to β_b across the blade passage about a mean value (Fig. 3). We define $\bar{\alpha}$ as $\tan^{-1}(\bar{u}/\bar{v})$, and $\overline{\tan\alpha}$ as $\overline{(v/u)}$. For zero thickness blades $\beta_a = \beta_b = \beta$.

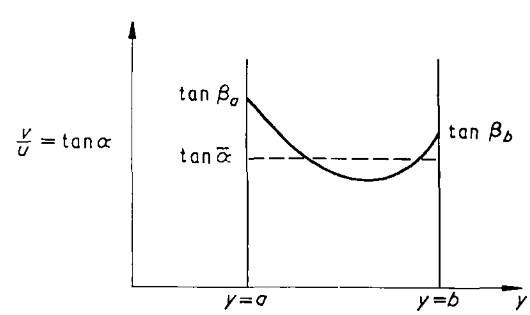

Fig. 3. *Variation of flow angle*

Passage averaged flow

The passage averaged flow equations are obtained by integrating the actual flow equations (2) and (3) across the blade passage and then expressing these in terms of the averaged properties $\bar{u}$, $\bar{v}$ and $\bar{p}$. Since the real cascade has blades of zero thickness, the blade passage $(b-a)$ is equal to the pitch s and is independent of x. It is assumed that the local properties q can be expressed as the passage mean $\bar{q}$, plus a small perturbation q', where $\int_a^b q'\,\mathrm{d}y = 0$. The passage averaged equations are then

$$\left.\begin{aligned} &\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} = 0 \\ &-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{(p_a-p_b)}{\rho s}\tan\beta = \bar{u}\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} \\ &\qquad -\frac{(u'_b-u'_a)}{s}\bar{u}\tan\beta+\frac{(u'_b-u'_a)}{s}\bar{v} \\ &\frac{(p_a-p_b)}{\rho s} = \bar{u}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x}-\frac{(v'_b-v'_a)}{s}\bar{u}\tan\beta+\frac{(v'_b-v'_a)}{s}\bar{v} \end{aligned}\right\} \quad (5)$$

in which averaged products of small quantities, such as $u'v'$, have been neglected.

If we define

$$F_y = \frac{(p_a-p_b)}{s}$$

and
$$f_y = \frac{\rho}{s}(v'_b-v'_a)(\bar{u}\tan\beta-\bar{v})$$

then equations (5) may be written as

$$\left.\begin{aligned} &\bar{u} = \text{constant} \\ &-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{F_y}{\rho}\tan\beta+\frac{f_y}{\rho\tan\beta} = 0 \\ &\frac{1}{\rho}(F_y+f_y) = \bar{u}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \end{aligned}\right\} \quad . \quad (6)$$

The blade force term F_y is the pressure difference across the blade, divided by the pitch s. The term f_y is of order $q'_i q'_j$, and if such terms are assumed small, it may be neglected. In Appendix 2 it is shown that quantities such as $q'_i q'_j$ are of the second order of smallness if the blades have low lift coefficient. The neglect of these second order terms is equivalent to neglecting the G functions defined by Smith (1) and also implies that $\bar{u}^2 = \overline{u^2}$. Equations (6) then become

$$\left.\begin{aligned} &\bar{u} = \text{constant} \\ &-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{F_y}{\rho}\tan\beta = 0 \\ &\frac{F_y}{\rho} = \bar{u}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \end{aligned}\right\} \quad . \quad . \quad (7)$$

These equations are then the same as those for three hypothetical flows.

(1) The flow through a many bladed cascade which has a camber line $\beta(x)$, a pitch ϵ and a blade thickness to pitch ratio of zero. There is an elementary pressure force acting normal to each part of the blade where

$$F_y = \frac{(p_a-p_b)}{s} = \frac{\Delta p}{\epsilon}$$

in which Δp is the pressure difference across each blade in the hypothetical flow. For this many bladed cascade there is little variation in the physical quantities across the blade passage, but the y derivatives remain finite.

(2) An axisymmetric flow in which a *body* force $F_y \sec\beta$ acts normal to the streamlines, but the y derivatives are zero. Thus, for these conditions, the blade force in the many bladed cascade and the body force in the axisymmetric flow are the same.

(3) The flow on a mean stream surface of constant thickness and with a local shape given by the angle β. The *body* force which is necessary for the flow to follow this mean stream surface is $F_y \sec\beta$ and it acts normal to the surface.

The three hypothetical flows can each represent the passage averaged flow of the real cascade provided that the blade loading is not high. The only difference between the three hypothetical flow models is whether the force term $F_y \sec\beta$ is regarded as a *blade* force or as a *body* force. The conclusion that the three models are satisfactory for lightly loaded blades is in agreement with Smith's analysis (1).

Continuity and momentum averaged flow

An alternative approach to this problem is directly to average the continuity and momentum equations (equations (2) and (4)), across the blade passage

$$\left.\begin{aligned} &\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} = 0 \\ &-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{(p_a-p_b)}{\rho}\tan\beta = \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}) \\ &\qquad -\frac{(u_b^2-u_a^2)}{s}\tan\beta+\frac{(u_b v_b-u_a v_a)}{s} \\ &\frac{(p_a-p_b)}{\rho s} = \frac{\mathrm{d}}{\mathrm{d}x}(\overline{uv})-\frac{(u_b v_b-u_a v_a)}{s}\tan\beta \\ &\qquad +\frac{(v_b^2-v_a^2)}{s} \end{aligned}\right\} \quad (8)$$

Since $v = u\tan\beta$ at the blade surface, these equations can be written as

$$\left.\begin{aligned} &\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} = 0 \\ &-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{F_y}{\rho}\tan\beta = \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}) \\ &\frac{F_y}{\rho} = \frac{\mathrm{d}}{\mathrm{d}x}(\overline{uv}) \end{aligned}\right\} \quad . \quad . \quad (9)$$

No assumptions about neglecting cross products of small quantities or the order of magnitude of the lift coefficient have been made.

We now attempt to identify these averaged equations with those for one of the hypothetical flows, e.g. the many bladed cascade. If we define an angle γ so that

$$\tan\gamma \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}\tan\gamma) = \tan\beta \frac{\mathrm{d}}{\mathrm{d}x}(\overline{uv}) \quad . \quad (10)$$

then
$$F_y \tan\beta = \tan\beta \frac{\mathrm{d}}{\mathrm{d}x}(\overline{uv})$$
$$= \tan\gamma \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}\tan\gamma)$$

Further, if $F_y{}^*$ is defined as

$$F_y{}^* = \frac{\tan\beta}{\tan\gamma}\cdot F_y \quad . \quad . \quad . \quad . \quad (11)$$

then equations (9) become

$$\left.\begin{aligned} \frac{\mathrm{d}\bar{u}}{\mathrm{d}x} &= 0 \\ -\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} - \frac{F_y{}^*}{\rho}\tan\gamma &= \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}) \\ \frac{F_y{}^*}{\rho} &= \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}\tan\gamma) \end{aligned}\right\} \quad . \quad (12)$$

It should be noted that the angle γ is not necessarily the same as the mean flow angle $\bar{\alpha}$, but is defined by equation (10).

In order to identify equations (12) with any of the hypothetical flows, further assumptions must be made about the relationship between $\overline{u^2}$ and $(\bar{u})^2$. To assume that they are the same is tantamount to assuming that products of small quantities can be neglected, $(u')^2 \simeq 0$ in the section entitled 'Objectives'. Equations (12) can then be written as

$$\left.\begin{aligned} \frac{\mathrm{d}\bar{u}}{\mathrm{d}x} &= 0 \\ -\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} - \frac{F_y{}^*}{\rho}\tan\gamma &= 0 \\ \frac{F_y{}^*}{\rho} &= \bar{u}\frac{\mathrm{d}}{\mathrm{d}x}(\bar{u}\tan\gamma) \end{aligned}\right\} \quad . \quad . \quad (13)$$

At first sight, it appears that we have identified the continuity and momentum averaged equations with a hypothetical flow; for example, a many bladed cascade flow with a camber line $\gamma(x)$ and a *blade* force $F_y{}^* \sec\gamma$ acting perpendicular to the blades.

However, the assumption that $\overline{u^2} = (\bar{u})^2$ at all axial positions implies that $\bar{\alpha}$, β and γ are little different and that $F_y{}^*$ is the same as F_y for the passage averaged flow. The analysis has therefore returned to the same results and conclusions as in the section entitled 'Passage averaged flow'. The three hypothetical flows can represent the continuity and momentum averaged flow of the real cascade, provided that the blade loading is low.

In this attempt to match the averaged flow through a real cascade of thin blades with a hypothetical flow which also has no blade blockage, only one free parameter was available, the blade angle of the many bladed cascade. It is evidently not possible to match the averaged (incompressible) flow with the hypothetical flow at all axial positions unless some simplifying assumptions can be made, such as the neglect of products of small quantities. This then implies a low lift coefficient in the actual flow. These problems are discussed in more detail in the next section, where the actual flow is assumed to be compressible, and the mathematical model may include a blade blockage effect as an additional free parameter.

COMPRESSIBLE FLOW THROUGH A CASCADE OF BLADES WITH THICKNESS

A more general problem is now considered, the flow of a compressible fluid through a cascade in which the blades have a non-zero thickness, so that $s > (b-a)$ and the blade angle $\beta_b = \tan^{-1}(\mathrm{d}b/\mathrm{d}x)$ is not equal to $\beta_a = \tan^{-1}\left(\frac{\mathrm{d}a}{\mathrm{d}x}\right)$.

Continuity and momentum averaged flow

The equations of continuity and momentum are now

$$\left.\begin{aligned} \frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v) &= 0 \\ -\frac{\partial p}{\partial x} &= \frac{\partial}{\partial x}(\rho u^2)+\frac{\partial}{\partial y}(\rho uv) \\ -\frac{\partial p}{\partial y} &= \frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho v^2) \end{aligned}\right\} \quad . \quad (14)$$

It is assumed that the fluid is a perfect gas and that the flow is reversible and adiabatic, so that

$$p \propto \rho^k \quad . \quad . \quad . \quad . \quad . \quad (15)$$

These equations may be averaged across the blade passage to obtain the continuity and momentum averaged equations

$$\frac{1}{(b-a)}\frac{\mathrm{d}}{\mathrm{d}x}[(b-a)\overline{\rho u}] = 0 \quad . \quad . \quad (16)$$

$$-\frac{1}{(b-a)}\frac{\mathrm{d}}{\mathrm{d}x}[(b-a)\bar{p}] - \left[\frac{p_a\tan\beta_a - p_b\tan\beta_b}{b-a}\right] = \frac{1}{(b-a)}\frac{\mathrm{d}}{\mathrm{d}x}[(b-a)\overline{\rho u^2}] \quad (17)$$

$$F_y = \left[\frac{p_a-p_b}{b-a}\right] = \frac{1}{(b-a)}\frac{\mathrm{d}}{\mathrm{d}x}[(b-a)\overline{\rho u^2\tan\alpha}] \quad (18)$$

$$\bar{p} = C(\overline{\rho^k}) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Hypothetical flows

Now consider the three hypothetical flows in which blade blockage is introduced as a new free parameter. The governing equations for the three models are the same; the only difference is in the interpretation of certain terms in these equations, e.g. whether the force is a blade or a body force.

For the analysis, the following quantities are defined in the hypothetical flows to *correspond* to properties of the actual flow.

- B corresponds to the ratio of the width of the blade passage to the pitch;
- R corresponds to the density;
- U corresponds to the axial velocity;
- P corresponds to the pressure;
- γ corresponds to the blade or stream surface angle; and
- F_y^* corresponds to the y component of the blade or body force.

The hypothetical flows are presented in terms of these quantities and it is not assumed that they are equal to the mean values for the actual flow. The governing equations for the hypothetical flows are as follows.

Continuity

$$\frac{1}{B}\frac{\mathrm{d}}{\mathrm{d}x}(BRU) = 0 \quad . \quad . \quad . \quad (20)$$

Far upstream, subscript 1, where the actual flow is uniform, the hypothetical and real flows must correspond, so that

$$BRU = R_1U_1 = \rho_1u_1 \quad . \quad . \quad . \quad (21)$$

Momentum (x direction)

$$-\frac{\mathrm{d}P}{\mathrm{d}x}-F_y^*\tan\gamma = \frac{1}{B}\frac{\mathrm{d}}{\mathrm{d}x}(BRU^2) \quad . \quad (22a)$$

or

$$-\frac{1}{B}\frac{\mathrm{d}}{\mathrm{d}x}(BP)-F_y^*\tan\gamma+\frac{P}{B}\frac{\mathrm{d}B}{\mathrm{d}x} = RU\frac{\mathrm{d}U}{\mathrm{d}x} \quad (22b)$$

Momentum (y direction)

$$F_y^* = \frac{1}{B}\frac{\mathrm{d}}{\mathrm{d}x}[BRU^2\tan\gamma]$$

$$= RU\frac{\mathrm{d}}{\mathrm{d}x}[U\tan\gamma] \quad . \quad . \quad (23)$$

State

$$\frac{P}{R^k} = C = \frac{p_1}{\rho_1{}^k} \quad . \quad . \quad . \quad . \quad (24)$$

The use of C and k in the equation of state implies that the working fluid is the same as that for the actual flow and that both the actual and hypothetical flows are isentropic with the same entry state (p_1, ρ_1).

Local matching of the averaged and hypothetical flows

We consider first the relation between the blade force in the averaged flow and the blade or body force in the hypothetical flow.

In the averaged and in the actual flow, the axial component of the blade force F_x is given by the second term in equation (17)

$$F_x = \frac{p_a\tan\beta_a-p_b\tan\beta_b}{b-a}$$

$$= \left(\frac{p_a-p_b}{b-a}\right)\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{a+b}{2}\right]-\left(\frac{p_a+p_b}{b-a}\right)\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{b-a}{2}\right]$$

Introducing β_c as the angle of the blade camber line,

$$\tan\beta_c = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{a+b}{2}\right]$$

it follows that

$$F_x = F_y\tan\beta_c-\left[\frac{p_a+p_b}{2}\right]\frac{1}{(b-a)}\frac{\mathrm{d}}{\mathrm{d}x}(b-a) \quad (25)$$

The axial component of the blade force in the hypothetical flow is, from equation (22b)

$$F_x^* = F_y^*\tan\gamma-\frac{P}{B}\frac{\mathrm{d}B}{\mathrm{d}x} \quad . \quad . \quad (26)$$

which may be compared with equation (25) for the actual flow. It is clear that if the passage width parameter B varies, then the axial component of the blade force in the hypothetical flow is not equal to $F_y^*\tan\gamma$.

More generally, in matching the hypothetical flows (equations (21)–(24)) to the continuity and momentum averaged flows (equations (16)–(19)) there are only two physical cascade parameters which can be varied, the camber line $\gamma(x)$ and passage width $B(x)$ in the many bladed cascade model. In general, the camber line $\gamma(x)$ is not equal to $\bar{\alpha}$, β_a, β_b or β_c and the passage width parameter is not equal to $(b-a)/s$. There is one more free variable than in the simple problem of the section entitled 'Continuity and momentum averaged flow', where $\gamma(x)$ was the only free parameter and B was assumed to be unity throughout. However, the problem is further complicated by the introduction of compressibility, although the entry states and working fluid of the actual and hypothetical flows have been matched.

To illustrate the difficulties involved, consider again the problem of the section under the heading 'Incompressible inviscid flow through a two-dimensional cascade of thin blades', that of matching the averaged flow of an incompressible fluid through a cascade of thin blades with a hypothetical flow, but with two free parameters (γ and B) in the latter. For the actual flow, $(b-a)/s = 1$ and ρ is constant. For the hypothetical flow, $R = \rho =$ constant and $\gamma(x)$ and $B(x)$ can be chosen to match the actual flow in certain respects. With constant density and constant blade passage width, the continuity and momentum averaged equations (16)–(18) reduce to equations (9). In the hypothetical flow, the parameter $B(x)$ can be determined by matching the rate of change of axial momentum, the right-hand side of equation (22), with that of the averaged flow

$$U\frac{\mathrm{d}U}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\overline{(u^2)}$$

so that

$$U^2 = \overline{2u^2} - u_1^2 \quad . \quad . \quad . \quad . \quad (27)$$

From the continuity equation

$$BU = u_1$$

so that B is then given by

$$B = \frac{u_1}{(\overline{2u^2} - u_1^2)^{1/2}} \quad . \quad . \quad . \quad (28)$$

Thus, as far as continuity and axial momentum are concerned, the real cascade with thin blades is therefore modelled by a many bladed cascade having a blade thickness effect. Further, the blade angle of the model can be obtained by matching the rate of change of tangential momentum, right-hand side of equation (23), with that of the averaged flow,

$$U \frac{\mathrm{d}}{\mathrm{d}x}[U \tan \gamma] = \frac{\mathrm{d}}{\mathrm{d}x}[\overline{u^2 \tan \alpha}] \quad . \quad (29)$$

It then follows from equations (18) and (23) that the blade, or body, force component $F_y{}^*$ is matched to the blade force component $(p_a - p_b)/s$.

It would appear at first sight that the hypothetical and averaged flows are now identical. Continuity is satisfied and both flows have the same rates of change of axial and tangential momentum. However, the second terms in equations (17) and (22) are

$$F_y \tan \beta = \left(\frac{p_a - p_b}{s}\right) \tan \beta$$

and

$$F_y{}^* \tan \gamma = \left(\frac{p_a - p_b}{s}\right) \tan \gamma$$

and these two terms are not matched since γ is not equal to β. It then follows that $\mathrm{d}\bar{p}/\mathrm{d}x$ and $\mathrm{d}P/\mathrm{d}x$ are not the same, even for this simple example,

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} = \frac{\mathrm{d}P}{\mathrm{d}x}\left(\frac{p_a - p_b}{s}\right)(\tan \gamma - \tan \beta) \quad . \quad (30)$$

The variation of the averaged pressure $\bar{p}$ in the actual flow is not paralleled by the variation of P in the hypothetical flow.

Thus, in this example, if the parameters $B(x)$ and $\gamma(x)$ are chosen to obtain matching of continuity and rates of change of axial and tangential momentum, the local pressure gradients are, as a result, unmatched. It would be possible to choose $B(x)$ and $\gamma(x)$ to match other quantities, but the rates of change of momentum would then be unmatched.

We must conclude, with reluctance, that it is not possible to obtain complete local identity of the averaged actual flow and the hypothetical flow in order to obtain the same rates of change of the mean and hypothetical properties. This inability of the mathematical model to match precisely the averaged actual flow has been shown only for an incompressible flow; however, for a compressible flow, it is clear that if the same working fluid is used, then again it will be impossible to match the two flows. In general, the hypothetical flows of the many bladed cascade, the axisymmetric flow and the flow on the mean stream surface are models which cannot fully represent the averaged flow of the real cascade.

Overall changes across the cascade

Although it has not been possible to obtain complete local identity between the actual and hypothetical flows, equivalence may be established for the overall changes across the real cascade and the model. If the mathematical model has the same working fluid and the same flow angle far downstream as in the actual flow, then, in the model, the fluid state and velocity far downstream are the same as in the real flow. The axial and tangential momentum changes are the same for the two flows and it follows that in the mathematical model, the total axial and tangential blade forces are the same as those for the real cascade. The overall behaviour of the cascade can therefore be correctly represented by the model, even though the local variations of the flow properties are not fully matched in the averaged and hypothetical flows.

Discussion of the mean stream surface

In Wu's general method for calculating the flow in turbomachines (6) the flow is calculated on two families of intersecting stream surfaces, the S1 and S2 stream surfaces shown in Fig. 2. For each S2 stream surface the governing equations are solved using information from the flow on the S1 stream surfaces. The analysis presented in this paper shows that, in general, it is not possible to define a mean S2 stream surface on which the variations of the flow properties are the same as the variations of the averaged flow properties in the actual flow, i.e. it is not possible completely to represent the flow through the real cascade by the flow on a mean stream surface, except for a cascade of blades of low lift coefficient.

In through-flow calculations (7) which use the method of streamline curvature (1) or the matrix method (2) to predict the flow in turbomachines, the flow within the blade rows is calculated on a mean stream surface, which must be specified as data for the calculation, and which is in some way representative of the whole family of S2 stream surfaces. The analysis of this paper shows that this flow model may predict the correct overall changes of the flow across a blade row, but the calculated flow variations may not provide a good local representation of the averaged actual flow. The numerical results obtained from flow calculations for turbomachines should therefore be interpreted with care, particularly if they are to be used for predicting the development of the wall boundary layer (4) (5).

DISCUSSION OF THE BLADE FORCE AND THE BODY FORCE

In this section, we shall examine the averaged equations in more detail, and shall discuss the nature of the blade and body forces. In Appendix 1 we give without derivation a summary of the various ways of stating the averaged equations for the incompressible flow through a cascade of thin blades.

The averaged continuity equation (41) is

$$\frac{d\bar{u}}{dx} = 0$$

and requires little further comment. It results from the assumption of incompressibility, that once the flow has entered the channel between the blades, the infinite speed at which pressure waves are transmitted ensures that $\bar{u}$ must remain invariant with x through the channel.

We have already demonstrated that it is not possible to simulate exactly the averaged flow through the real cascade with one of the hypothetical flows. Reference to the y and x momentum equations (42)–(51) of Appendix 1 will emphasize this, but we discuss here a number of points that arise in the interpretation of these equations.

(1) Equations (42) and (47) are the exact equations derived from the momentum averaged equations, as in the section entitled 'Continuity and momentum averaged flow'.

(2) The equations for a hypothetical flow through closely spaced thin blades of angle $\beta(x)$ (equations (7)) are obtained only if the terms $d/dx(\overline{u'v'})$ and $d/dx(\overline{u'^2})$ are neglected in equations (43) and (48). Note that this is not a question of neglecting, say, $u'v'$ compared with $\bar{u}\bar{v}$, which would, from Appendix 2, imply that $(C_L/4)^2$ should be small. It is the neglect of $d/dx(\overline{u'v'})$ compared with, say, $\bar{u}\,(d\bar{v}/dx)$ that is involved. This is discussed in Appendix 2.

(3) One attempt that we have made to improve the accuracy of the hypothetical model involves the use of equations (44) and (49). Neglect of the last term of equation (44) would appear to be justified since it not only involves terms $q'_i{}^2$, but also the difference in such squares, across the blade pitch. Neglect of the term on the right-hand side of equation (49) (possibly less justified) leads to the equations

$$\left.\begin{aligned} -\frac{1}{\rho}\frac{d\bar{p}}{dx}-\frac{F_y}{\rho}\,[\tan\beta-\tan(\beta-\bar{\alpha})] &= 0 \\ \frac{F_y}{\rho}\,[1+\tan\beta\tan(\beta-\bar{\alpha})] &= \bar{u}\,\frac{d\bar{v}}{dx} \end{aligned}\right\} \quad (31)$$

It should be noted that the ratio of the two components of the body force in these equations is

$$\frac{\tan\beta-\tan(\beta-\bar{\alpha})}{1+\tan\beta\tan(\beta-\bar{\alpha})} = \tan\bar{\alpha}$$

This leads to an interesting interpretation of these equations, which is illustrated in Fig. 4*a*.

We now have a hypothetical flow (say, in a many bladed cascade) in which the fluid follows a path described by $\bar{\alpha}(x)$, the mean flow angle of the real cascade. A force $[F_y/\cos\beta\cos(\beta-\bar{\alpha})]$ acts normal to the streamlines, and this force has components $F_y/\cos\beta$ and $f_y/\sin\beta$ normal to and along the camber line of the actual cascade.

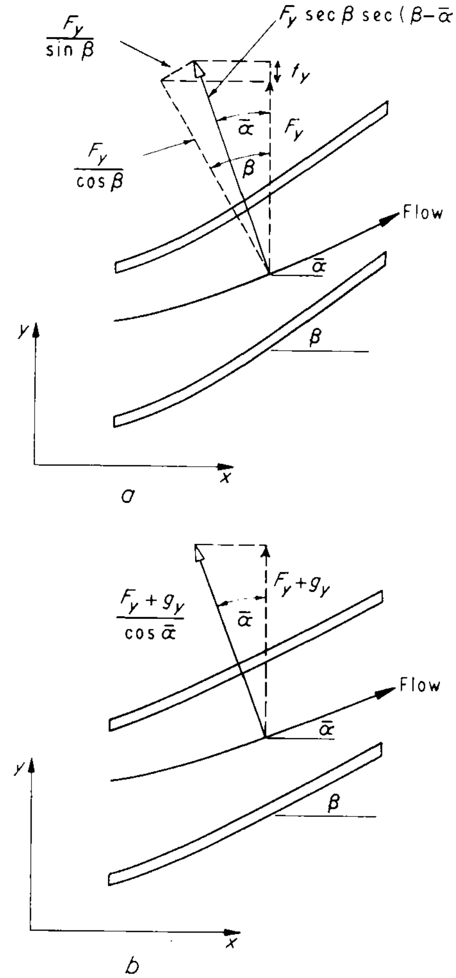

Fig. 4. *Flow models*

Once again, it would appear at first sight that an accurate hypothetical model has been found, but it must be appreciated that equations (31) are not exact, as the terms have been omitted from equations (44) and (49).

(4) Another hypothetical model is suggested by equations (45) and (50). Neglect of the term $d/dx[(\overline{u'^2+v'^2})/2]$ on the right-hand side of equation (50) enables the two equations to be written

$$\left.\begin{aligned} \frac{F_y+g_y}{\rho} &= \bar{u}\,\frac{d\bar{v}}{dx} \\ -\frac{1}{\rho}\frac{d\bar{p}}{dx}-\frac{(F_y+g_y)}{\rho}\tan\bar{\alpha} &= 0 \end{aligned}\right\} \quad . \quad (32)$$

These represent the flow through a hypothetical cascade in which the fluid again follows the mean angle $\bar{\alpha}(x)$, and a body force

$$\left(\frac{F_y+g_y}{\cos\bar{\alpha}}\right) = \left[\frac{p_a-p_b}{s}-\rho\,\frac{d}{dx}\,(\overline{u'v'})\right]\sec\bar{\alpha}$$

acts normal to the mean streamlines (Fig. 4*b*).

But again we are defeated in our search for an exact hypothetical model because of the neglect of the term on the right-hand side of equation (50).

(5) Finally, various other exact forms of the equations are listed in equations (46) and (51)–(54) but none leads to the definition of completely satisfactory hypothetical models.

CONCLUSIONS

Averaged equations for the flow through a real cascade have been compared with the corresponding equations for three hypothetical flows, the many bladed cascade, an axisymmetric flow and flow on a mean stream surface. It has been shown that it is only for a cascade of thin blades of low lift coefficient that the equations for the hypothetical flow become identical to the passage averaged equations, the hypothetical flow following the direction of the camber line of the real cascade, $\beta(x)$. In general, when the lift coefficient is not low, then it is not possible to specify a mathematical model in which the local flow properties are the same as the averaged properties of the actual flow, although the overall changes across the real cascade can be represented accurately in a hypothetical flow.

A number of hypothetical flows have been considered which give some increase in the accuracy of representing locally the averaged flow. In particular, hypothetical flows with streamlines following an angle $\bar{\alpha}(x)$ given by the mean flow direction of the averaged flow, $\bar{\alpha} = \tan^{-1}(\bar{u}/\bar{v})$, show promise. The body force acting normal to the hypothetical streamlines may be written

$$F_y \sec\beta \sec(\beta-\bar{\alpha}) = \left(\frac{p_a-p_b}{s}\right) \sec\beta \sec(\beta-\bar{\alpha}) \quad (33)$$

or, alternatively, as

$$(F_y+g_y)\sec\bar{\alpha} = \left[\frac{p_a-p_b}{s} - \rho\frac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'})\right]\sec\bar{\alpha} \quad (34)$$

but both representations involve a degree of approximation in the neglect of terms that arise in the exact equations listed in Appendix 1.

APPENDIX 1

EXACT FORMS OF THE AVERAGED EQUATIONS

We summarize here, without derivation, various ways of stating the exact averaged equations for the incompressible flow through a cascade of thin blades. The local velocities may be written as $u = \bar{u}+u'$, $v = \bar{v}+v'$, and on the surfaces of the blades at a and b

$$v_a = \bar{v}+v'_a = (\bar{u}+u'_a)\tan\beta$$

$$v_b = \bar{v}+v'_b = (\bar{u}+u'_b)\tan\beta$$

It may be noted that if the flow between the blades is irrotational then

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and averaging of this equation across the pitch yields

$$u'_b - u'_a = s\cos^2\beta\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \quad . \quad . \quad . \quad (36)$$

(see Appendix 2).

Further, we define the following

$$F_y = \frac{p_a-p_b}{s} \quad . \quad . \quad . \quad . \quad (37)$$

$$\bar{\alpha} = \tan^{-1}\left(\frac{\bar{v}}{\bar{u}}\right) \quad . \quad . \quad . \quad . \quad (38)$$

$$f_y = \rho\frac{(v'_b-v'_a)(\bar{u}\tan\beta-\bar{v})}{s}$$

$$= F_y\tan\beta\tan(\beta-\bar{\alpha}) \quad . \quad . \quad . \quad (39)$$

$$\frac{g_y}{\rho} = -\frac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'}) \quad . \quad . \quad . \quad . \quad (40)$$

The averaged equations may then be written, *exactly*,

Continuity

$$\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The y momentum equation

$$\frac{F_y}{\rho} = \frac{\mathrm{d}}{\mathrm{d}x}(\overline{uv}) \quad . \quad . \quad . \quad . \quad . \quad (42)$$

$$= \bar{u}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'}) \quad . \quad . \quad . \quad (43)$$

If the flow in the blade passage is irrotational, then

$$\frac{F_y+f_y}{\rho} = \bar{u}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} + \left(\frac{u'^2_b+v'^2_b-u'^2_a-v'^2_a}{2s}\right) \quad (44)$$

$$\frac{F_y+g_y}{\rho} = \bar{u}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \quad . \quad . \quad . \quad . \quad (45)$$

$$\frac{F_y}{\rho} = \left(\frac{u_a+u_b}{2}\right)\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \quad . \quad . \quad . \quad (46)$$

The x momentum equation

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} - \frac{F_y}{\rho}\tan\beta = \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u^2}) \quad . \quad . \quad (47)$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}(\overline{u'^2}) \quad . \quad . \quad (48)$$

and for an irrotational flow in the blade passage,

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} - \frac{F_y}{\rho}\tan\beta + \frac{f_y}{\rho\tan\beta} = \frac{1}{s}\int_a^b \frac{\partial}{\partial x}\left[\frac{u'^2+v'^2}{2}\right]\mathrm{d}y \quad . \quad . \quad . \quad (49)$$

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} - \frac{(F_y+g_y)}{\rho}\tan\bar{\alpha} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\overline{\frac{u'^2+v'^2}{2}}\right] \quad (50)$$

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x} - \bar{v}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\overline{\frac{u'^2+v'^2}{2}}\right] \quad . \quad . \quad (51)$$

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{F_y}{\rho}\tan\beta+\frac{(v'_a+v'_b)}{2}\cdot\frac{\mathrm{d}\bar{v}}{\mathrm{d}x}=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\overline{u'^2+v'^2}}{2}\right] \quad . \quad . \quad . \quad (52)$$

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\frac{F_y}{\rho}\tan\beta+\frac{(v'_a+v'_b)(u'_b-u'_a)}{2s\cos^2\beta}=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\overline{u'^2+v'^2}}{2}\right] \quad . \quad . \quad . \quad (53)$$

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}x}-\left[\frac{F_y}{\rho}-\left(\frac{u'_a+u'_b}{2}\right)\frac{\mathrm{d}\bar{v}}{\mathrm{d}x}\right]\tan\bar{\alpha}=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\overline{u'^2+v'^2}}{2}\right] \quad . \quad . \quad . \quad (54)$$

APPENDIX 2

NEGLECT OF PRODUCTS OF SMALL QUANTITIES

In considering the averaged equations for the incompressible flow through a cascade of thin blades, terms involving the products of small quantities arise (see Appendix 1). It is necessary to establish the condition under which these terms may be neglected, the requirement being that such terms are small compared with the terms retained in the analysis.

The flow in the blade passage is assumed to have zero vorticity, so that

$$\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

If the axial and tangential velocities are written as $\bar{u}+u'$ and $\bar{v}+v'$ and if equation (55) is integrated across the passage width, then with $s = b-a =$ constant,

$$s\frac{\mathrm{d}\bar{v}}{\mathrm{d}x}+(v_a-v_b)\tan\beta=u_b-u_a \quad . \quad . \quad (56)$$

where β is the blade angle. At the blade surface the two velocity components are related by

$$v = u\tan\beta$$

so that equation (56) can be written as

$$u'_b-u'_a=s\cos^2\beta\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \quad . \quad . \quad . \quad (57)$$

If it now assumed that $u'_a \simeq -u'_b$, then a typical value for u'_b is given by

$$u'_b \simeq \frac{s\cos^2\beta}{2}\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \quad . \quad . \quad . \quad (58)$$

The value for $\mathrm{d}\bar{v}/\mathrm{d}x$ can be based on the overall change of tangential velocity across the cascade,

$$\frac{\mathrm{d}\bar{v}}{\mathrm{d}x}\simeq\frac{\bar{u}}{c^x}(\tan\alpha_2-\tan\alpha_1) \quad . \quad . \quad . \quad (59)$$

where c^x is the axial chord length. From equations (58) and (59)

$$\left(\frac{u'}{\bar{u}}\right)_{\max}\simeq\frac{s\cos^2\beta}{2c^x}(\tan\alpha_2-\tan\alpha_1) \quad . \quad (60)$$

But the blade lift coefficient, for zero total pressure loss, is given by

$$C_L=\frac{2s}{c}\cos\alpha_m(\tan\alpha_2-\tan\alpha_1) \quad . \quad . \quad (61)$$

where α_m is the mean flow angle through the cascade. The ratio $(u'/\bar{u})_{\max}$ can then be expressed in terms of the lift coefficient,

$$\left(\frac{u'}{\bar{u}}\right)_{\max}\simeq\frac{C_L}{4}\frac{c\cos^2\beta}{c^x\cos\alpha_m} \quad . \quad . \quad (62)$$

The term $(c\cos^2\beta/c^x\cos\alpha_m)$ is of order unity, so that

$$\left(\frac{u'}{\bar{u}}\right)_{\max}\simeq\frac{C_L}{4} \quad . \quad . \quad . \quad . \quad (63)$$

Similarly, it follows that $(v'/\bar{v})_{\max}\simeq C_L/4$ and the ratio $[(u'v')/(\bar{u}\bar{v})]_{\max}$ is therefore of order $(C_L/4)^2$. However, it must be emphasized that it is the rate of change of $(\overline{u'v'})$ with x [e.g. $g_y/\rho = -\mathrm{d}/\mathrm{d}x(\overline{u'v'})$] in comparison with, say, F_y/ρ or $\bar{u}(\mathrm{d}\bar{v}/\mathrm{d}x)$, that is critical in equations such as equation (43).

From equations (43) and (44) the term $\mathrm{d}/\mathrm{d}x(\overline{u'v'})$ can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'})=\left[\frac{u'^2_b+v'^2_b-u'^2_a-v'^2_a}{2s}\right]-\frac{f_y}{\rho}$$

and substituting equation (39) for f_y,

$$\frac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'})=\left(\frac{u'_a+u'_b}{2}\right)\frac{\mathrm{d}\bar{v}}{\mathrm{d}x} \quad . \quad . \quad (64)$$

Thus,

$$\frac{\dfrac{\mathrm{d}}{\mathrm{d}x}(\overline{u'v'})}{\bar{u}\dfrac{\mathrm{d}\bar{v}}{\mathrm{d}x}}=\left(\frac{u'_a+u'_b}{2\bar{u}}\right) \quad . \quad . \quad . \quad (65)$$

and if $u'_a \simeq -u'_b$, then $(u'_a+u'_b)$ is very small and this suggests that the neglect of $\mathrm{d}/\mathrm{d}x(\overline{u'v'})$, cf. $\bar{u}(\mathrm{d}\bar{v}/\mathrm{d}x)$, is justified.

The lift coefficient of an aerofoil in a compressor cascade operating at its design point may be of the order of unity or slightly less, so that $C_L/4 \simeq 0{\cdot}2$. The justification for the neglect of products $q'_iq'_j$ in the averaged equations is therefore marginal. For turbine blades, it is customary to express the blade loading in terms of a tangential force coefficient,

$$C_y=\frac{2s}{c^x}\cos^2\alpha_2(\tan\alpha_2-\tan\alpha_1)$$

$$=\frac{c}{c^x}\cdot C_L\frac{\cos^2\alpha_2}{\cos\alpha_m} \quad . \quad . \quad . \quad . \quad . \quad (66)$$

$$\simeq C_L\frac{\cos^2\alpha_2}{\cos\alpha_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

For an impulse blade with $\alpha_1 = \alpha_2 = 60°$ and $\alpha_m = 0$, C_y is of the order 0·8 and $(C_L/4)$ is also of order 0·8. It is clearly not justifiable to neglect the products $q'_iq'_j$ in heavily loaded turbine cascades.

APPENDIX 3

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