Problem Set 5

Due March 1, 2018 PHY 215B

---

1. Add $j_1 = 1$ and $j_2 = 1$ by constructing the table to account for all possible states. Use the table to find states $|i_1 i_2 J J-2\rangle$, $|i_1 i_2 J\, J-1\, J-2\rangle$, and $|i_1 i_2 J\, J-2\, J-2\rangle$ in terms of $|j_1 m_1, j_2 m_2\rangle$.

2. Given the state
$$\frac{\sqrt{3}}{5}\left|\frac{3}{2}-\frac{1}{2}\right\rangle|1\,-1\rangle+\frac{\sqrt{2}}{5}\left|\frac{3}{2}-\frac{3}{2}\right\rangle|1\,0\rangle\,,$$
   (a) What are the $|j_1 m_1, j_2 m_2\rangle$ for the two components and $|j_1 j_2 J M\rangle$?

   (b) What are the Clebsch-Gordan coefficients?

   (c) What are the expansion coefficients for the two components onto the state $|j_1 j_2 J-1 M\rangle$?

3. Let the deuteron spin be 1 and the electric quadrupole moment operator be $Q(2,0)$. Calculate the ratios of the expectation values of Q for the three states of the spin of the deuteron by considering Q as a tensor operator and using the Wigner-Eckart theorem.

4. Given that $T(k,q)$ is a tensor operator, and
$$\mathbf{J}^2\{A\} = [J_x,[J_x,A]] + [J_y,[J_y,A]] + [J_z,[J_z,A]],$$
   show that
$$\mathbf{J}^2\{T(k,q)\} = k(k+1)T(k,q).$$

5. Work out an operator for an infinitesimal translation. Examine the effect of the inversion symmetry on this operator.

Suggested readings: Messiah 560-577; Schiff 212-224.