

Homework 5

Due February 23, 2018

PHY 204B

11.8.8 Evaluate $\int_{-\infty}^{\infty} \frac{\cos bx - \cos ax}{x^2} dx$, $a > b > 0$.

ANS. $\pi(a - b)$.

11.8.10 Show that $\int_0^{\infty} \frac{x \sin x}{x^2 + 1} dx = \frac{\pi}{2e}$.

11.8.12 Show that ($a > 0$):

(a) $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2} dx = \frac{\pi}{a} e^{-a}.$

How is the right side modified if $\cos x$ is replaced by $\cos kx$?

(b) $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2} dx = \pi e^{-a}.$

How is the right side modified if $\sin x$ is replaced by $\sin kx$?

11.8.14 In the quantum theory of atomic collisions, we encounter the integral

$$I = \int_{-\infty}^{\infty} \frac{\sin t}{t} e^{ipt} dt,$$

in which p is real. Show that

$$I = 0, \quad |p| > 1$$

$$I = \pi, \quad |p| < 1.$$

What happens if $p = \pm 1$?

11.8.22 Show that

$$\int_0^{\infty} \frac{dx}{1+x^n} = \frac{\pi/n}{\sin(\pi/n)}.$$

Hint. Try the contour shown in Fig. 11.30, with $\theta = 2\pi/n$.

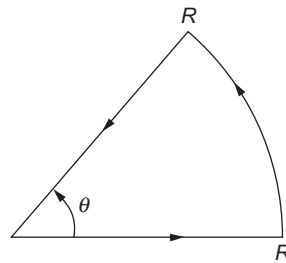


FIGURE 11.30 Sector contour.

11.8.24 Show that

$$\int_0^{\infty} \frac{x^{-a}}{x+1} dx = \frac{\pi}{\sin a\pi},$$

where $0 < a < 1$.

Hint. You have a branch point and you will need a cut line. Try the contour shown in Fig. 11.26.

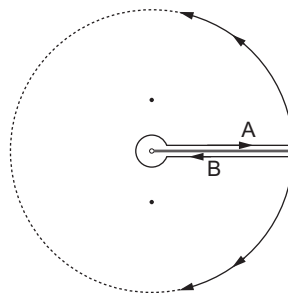


FIGURE 11.26 Contour for [Example 11.8.8](#).

11.8.27 Show that $\int_0^1 \frac{1}{(x^2 - x^3)^{1/3}} dx = \frac{2\pi}{\sqrt{3}}.$

Hint. Try the contour shown in Fig. 11.31.

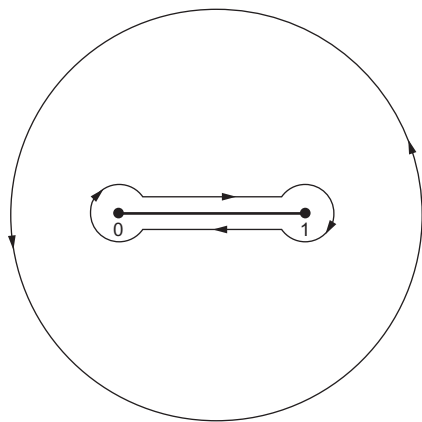


FIGURE 11.31 Contour for [Exercise 11.8.27](#).

11.9.4 Evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}.$