Homework 4

Due February 16, 2018 PHY 204B

11.6.2 Show that the function

$$w(z) = \left(z^2 - 1\right)^{1/2}$$

is single-valued if we make branch cuts on the real axis for $x > 1$ and for $x < -1$.

11.6.7 Show that negative numbers have logarithms in the complex plane. In particular, find $\ln(-1)$.

ANS. $\ln(-1) = i\pi$.

11.7.1 Determine the nature of the singularities of each of the following functions and evaluate the residues ($a > 0$).

(a) $\dfrac{1}{z^2 + a^2}$.

(b) $\dfrac{1}{(z^2 + a^2)^2}$.

(c) $\dfrac{z^2}{(z^2 + a^2)^2}$.

(d) $\dfrac{\sin 1/z}{z^2 + a^2}$.

(e) $\dfrac{ze^{+iz}}{z^2 + a^2}$.

(f) $\dfrac{ze^{+iz}}{z^2 - a^2}$.

(g) $\dfrac{e^{+iz}}{z^2 - a^2}$.

(h) $\dfrac{z^{-k}}{z+1}, \quad 0 < k < 1.$

Hint. For the point at infinity, use the transformation $w = 1/z$ for $|z| \to 0$. For the residue, transform $f(z)\,dz$ into $g(w)\,dw$ and look at the behavior of $g(w)$.

11.7.2 Evaluate the residues at $z = 0$ and $z = -1$ of $\pi \cot \pi z / z(z+1)$.

11.7.10 The statement that the integral halfway around a singular point is equal to one-half the integral all the way around was limited to simple poles. Show, by a specific example, that

$$\int_{\text{Semicircle}} f(z)\,dz = \frac{1}{2} \oint_{\text{Circle}} f(z)\,dz$$

does not necessarily hold if the integral encircles a pole of higher order.

Hint. Try $f(z) = z^{-2}$.

11.8.4 Evaluate $\displaystyle\int_0^{2\pi} \frac{\cos 3\theta\,d\theta}{5 - 4\cos\theta}$.

ANS. $\pi/12$.