Problem Set 4

Due February 22, 2018 PHY 215B

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1. If $\mathbf{A}$ and $\mathbf{B}$ are two vectors that commute with $\boldsymbol{\sigma}$, prove that

$$(\boldsymbol{\sigma} \cdot \mathbf{A})(\boldsymbol{\sigma} \cdot \mathbf{B}) = \mathbf{A} \cdot \mathbf{B} + i\boldsymbol{\sigma} \cdot (\mathbf{A} \times \mathbf{B}),$$

   where $\boldsymbol{\sigma}$ is the Pauli matrix.

2. In two successive Stern-Gerlach experiments, the first one has the magnetic field pointing in the $\hat{\mathbf{z}}$–direction. The beam of atoms of this first experiment is fed into the second experiment with the magnetic field oriented in the $\hat{\mathbf{x}}$–direction. What is the state occupied by the beam of atoms as the output of the second experiment? What is the probability of the beam of atoms occupying the up-spinor state, the 2×1 column matrix with '1' at the top row, with respect to the $\hat{\mathbf{z}}$–axis? (Hint: construct a unitary matrix.)

3. Express a rotational operator about ϕ acting on spinors in terms of S, the spin operator, and the Pauli matrices. For the Pauli matrices, give an expression of the 'rotation' operator in the 2×2 matrix form.

4. Show that $\det\left(e^{A}\right) = e^{\mathrm{tr}(A)}$ for an Hermitian operator.

5. Explain that $\langle \chi' | \sigma | \chi' \rangle = \langle \chi | U^{\dagger} \sigma U | \chi \rangle$ is consistent with the active point of view of rotations.

Suggested readings:

Messiah, vol. 2, pages 508-518; 540-554; 643-646.

Schiff: pages 194-199; 203-210.