Problem Set 2

Due Feburary 1, 2018 PHY 215B

Problem 1. In the abstract Hilbert space $\mathcal{H}$, the energy eigenvectors of a particle in an infinite square well of width L are designated by $|E_1\rangle, |E_2\rangle, |E_3\rangle, \ldots$ for the ground state, the first excited state, the second excited state and so on. Suppose that at a given time a particle is in the state

$$|\Psi\rangle = \frac{1}{2}|E_1\rangle - \frac{\sqrt{3}}{2}|E_3\rangle.$$

(a) Express this state in the x–representation.

(b) In the Schrödinger picture, how does this state develop in time? What is the state in the Heisenberg picture?

(c) If an electromagnetic wave is incident onto the square well, find the equation of motion of this state in the interaction picture. Assume that the particle has charge q.

Problem 2. Consider a wave–function, $\Phi_a(\boldsymbol{r}) = Nxy$, and a rotation of $120°$ about the z–axis.

(a) Construct the operator R for the rotation.

(b) Find a way to apply R on $\Phi_a(\boldsymbol{r})$ using the active point of view.

(c) Express $\Phi_a(\boldsymbol{r})$ in terms of $Y_{\ell m}$'s.

(d) Do you apply the operator in (b) to the wave function in (c)? If yes, why? If no, why?

Problem 3. Given $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ as unit vectors for a square,

(a) Construct matrices for rotations of $90°$, $180°$ and $270°$ about the z–axis (these matrices are representations of the operators).

(b) Combine a unit two-dimensional matrix and show these operators form a group and work out the multiplication table for the group.

(c) To see the origin of the symmetries of the system, work out the Coulomb potentials from four ions at the corners of the square to a small $\boldsymbol{r}(x, y, z)$ from the center of the square, Taylor expand each potential so that the resultant potential is non-vanishing. Show that the potential is invariant under the symmetries in (a). (Show at least two operators.)