Problem Set 1

Due January 25, 2018 PHY 215B

Problem 1. If operators A and B do not commute, show that

$$e^{A+B} = e^A e^B e^{(-1/2)[A,B]},$$

where $[A, B]$ is a commutator. Discuss this result in your own terms.

Problem 2. Show that if A and B are Hermitian operators, then all of the following operators are Hermitian:

(a) $A + B$

(b) $AB + BA$

(c) $i(AB - BA)$

(d) $A^n B^m + B^m A^n$, where m and n are integers

(e) $(AB)^\dagger = B^\dagger A^\dagger$

Problem 3. Show that $AA^\dagger$ is Hermitian even if A is not.

Problem 4. Given a truncated harmonic oscillator centered at the origin:

$$H = \begin{cases} \dfrac{p^2}{2m} + \dfrac{kx^2}{2} & |x| \leq a, \\ \dfrac{p^2}{2m} & a \leq |x| \leq \ell,\ a < \ell, \end{cases}$$

and a wave function of the form:

$$\Psi = b_1 \exp\left(\frac{i\pi x}{\ell}\right) + b_2 \exp\left(\frac{2i\pi x}{\ell}\right) + b_3 \exp\left(\frac{3i\pi x}{\ell}\right),$$

set up a 3×3 matrix of H using the functions given in Ψ. b_i are not normalization constants. If the integrations are too hard, you should just set up the matrix correctly. If Ψ is expanded onto three lowest energy basis functions of the un-truncated harmonic oscillator, set up the matrix for the transformation between the two sets of functions. Is the matrix unitary? If it is not, form a unitary matrix.

Problem 5. For the one-dimensional harmonic oscillator problem, show that:

(i) its energy is positive definite.

(ii) the difference between neighboring states is $\hbar\sqrt{k/m}$ using Heisenberg's approach of quantum mechanics.