PDE Homework

Burgers' equation with dissipation is commonly used to test methods.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \nu\frac{\partial^2 u}{\partial x^2} = 0$$

Use the initial condition $u_0(x) = \{ \begin{matrix} 1, 0 \le x \le 1 \\ 0, 1 < x \le 2 \end{matrix}$

And boundary conditions $u(0,t) = 1, \quad u(2,t) = 0$

Solve the equation using ν=.01
Integrate from t=0 to t=1.0
Plot the initial timestep, the final timestep, and two intermediate times as for Problem 1.

I suggest that you write a basic structure for your codes (grid setup, initial condition, boundary condition, etc.) then write subroutines/functions to perform the update for each timestep using the appropriate method. Trying out different methods will then be mostly a matter of swapping in subroutine calls. This will minimize duplication of effort and make it easier to code. Keep in mind that this is why we have stressed modular programming so much during this course.

1. Solve this system using leapfrog (CTCS) differencing. Use Euler's method for the first timestep. Since you have a viscosity present you should not need to add an artificial viscosity.

2. Solve the strong form (primitive variables) using Lax-Wendroff for the advection and Crank-Nicolson for the diffusion. First-order (in time) operator splitting is sufficient.