Notes from a reading of Kalicharan's Advanced C

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1 Sorting, Searching, and Merging

Note: Only two simple sorting algorithms, selection sort and insertion sort, are covered in this section.

1.1. Sorting an Array: Selection Sort

Sorting involves arranging a set of values in ascending/descending order. We often find this desirable because many operations, like searching, can be done much more efficiently on sorted data.

We first consider selection sort. With this sorting method, we maintain separate sorted and unsorted portions of the list. In each iteration, we iterate over the unsorted part, fine the smallest number, and add it to the end of the sorted part. Example:

```
57 48 79 65 15 33 52
15 48 79 65 57 33 52 // 15 swaps with 57
15 33 79 54 57 48 52 // 33 swaps with 48
15 33 48 54 57 79 52 // 48 swaps with 79
15 33 48 52 57 79 54 // 52 swaps with 54
15 33 48 52 54 79 57 // 54 swaps with 57
15 33 48 52 54 57 79 // 79 swaps with 57
```

After this is complete, the array is completely sorted. Note that for an array of size 6, we made 5 passes. Selection sort passes over the array $n-1$ time for an array of size n. This is because we need to pass over the full array one full time for each position except the last one, because by the time we reach it there is only one element remaining in the unsorted portion.

An implementation in C would look something like this:

```c
void selectionSort(int list[], in lo, int hi){
// sorts list[lo] to list[hi]
    for(int h = lo, h < hi; ++h){
        int s = getSmallest(list,h,hi);
        swap(list,h,s);
    }
}
```

Where `swap` and `getSmallest` are implemented as follows:

```
int getSmallest(int list[], int lo, int hi){
// returns index of smallest value
    int smallest = lo;
    for(int h = lo; h < hi; ++h)
        if(list[h] > list[smallest])
            smallest = h;
    return smallest;
}

void swap(int list[], int i, int j){
    int temp = list[i];
    list[i] = list[j];
    list[j] = hold;
}
```

Algorithm analysis: Selection Sort

The number of comparisons we make decreases by 1 each pass. In the first pass, we make n-1 comparisons, int he second we make n-2, etc. This series sums to (1/2)(n)(n-1), which is on the order of n^2. Thus, we say that selection sort is of the order O(n^2). Also, note that selection sort makes the same number of comparisons regardless of the data (so if it's input is already sorted, it makes no difference).

1.2 Sorting an Array: Insertion Sort

Consider the same array as above. We can also sort it by iterating through the array, placing a number in the correct location within our sorted array at each step. In practice, this means iterating through the array, moving each index into the spot above and stopping once we've found the correct spot:

```
void insertionSort(int list[], int n){
// Sorts first n items (list[0]..list[n-1])
    for(int i = 1; i < n; ++i){
        int key = list[h];
        int k = h-1;
        while(k >= 0 && key < list[k]){
            list[k+1] = list[k];
            --k;
        }
        list[k+1] = key;
    }
}
```

We exit the while loop when k and assign the key to its final location when it is no longer less than the element before.

Algorithm analysis: Insertion Sort

For each item j we consider, we make between 1 and $j-1$ comparisons. Therefore, on average we expect to make $(1/2)(j-1)$ comparisons per item on random data. This results in a runtime on the order of $O(n^2)$. Note that performance can vary depending on the nature of the data; sorting requires $n-1$ comparisons in the best case, $(1/4)n^2$ on average, and $(1/2)n^2$ in the worst case.

1.3 Inserting an Element in Place

The pattern of inserting an element into a sorted (sub)list as we do in insertion sort is fairly common. We can abstract this idea into a function `insertInPlace`. We assume that the array has enough place to hold the new element.

```
void insertInPlace(int newItem, int list[], int m, int n){
// list[m..n] is sorted
// after insertion, list[m..n+1] will be sorted
    int k = n;
    while(k >= m && newItem < list[k]){
        list[k+1] = list[k];
        --k;
    }
    list[k+1] = newItem;
}
```

1.6 Binary Search

Binary search is a fast searching method that relies upon the list being sorted. Throughout execution, we maintain a known subset where we know the element must exist, if it exists in the array. This contiguous space is called the *search space*, and we progress towards finding the element/termination by shrinking our search space in each iteration.

```
int binarySearch(int key, int list[], int lo, int hi){
    while(lo <= hi){
        int mid = (lo+hi)/2;
        if(key == list[mid]) return mid;
        if(key < list[mid]) hi = mid-1;
        else lo = mid+1;
    }
    return -1; // key wasn't found
}
```

1.9 Merging Ordered Lists

Sorted lists can be merged by repeatedly taking the smallest head of the arrays being merged. This merging process can also be seen in *merge sort*.

```
int merge(int A[], int m, int B[], int n, int C[]){
    int i = 0, j = 0, k = -1;
    while(i < m && j < n){
        if(A[i] < B[j]) C[++k] = B[i++];
        else C[++k] = B[j++];
    }
    if(i == m)
        for( ; j < n; j++) C[++k] = B[j];
    else
        for( ; i < m; i++) C[++k] = A[i++];
    return m + n;
}
```

As long as we have more elements to process in both arrays, we enter the loop. When this is no longer the case, we insert all the extra elements from the larger array directly.

5 Stacks and Queues

5.1 Abstract Data Types

We refer to implementations of data types whose operations can be performed by users without knowledge of the underlying implementation.

5.2 Stacks

A **stack** is a linear list where items are added and deleted from the same end. This means that stacks exhibit a "last in, first out" property. As an example, consider the following set of numbers:

```
36 15 52 23
```

If we want to print the following:

```
23 52 15 36
```

We would need to construct the following stack:

```
(top) 23 52 15 36 (bottom)
```

And we would remove numbers one at a time, printing them as we remove them. In psuedocode:

```
initialize empty stack S
read(num)
while a next number, num was read:
    push num onto S
    read(num)
while S is not empty:
    num = pop S
    print num
```

5.2.1 Implementing a Stack Using an Array

Stacks can be implemented with a struct containing an array and an int index of the top of the stack:

```
typedef struct {
    int top;
    int ST[MaxStack];
} StackType, *Stack;
```

Note that having StackType is useful for allocating a stack (general useful practice). When initialized, we'll set top to -1. Considering a stack variable, Stack S, we have access to top and the stack array, as we should based on our above definition of ADTs. Declaring a stack variable is therefore pretty simple:

```
Stack initStack(){
    Stack sp = malloc(sizeof(StackType));
    sp->top = -1;
    return sp;
}
```

And a user would simply initialize a new stack with Stack S = initStack().

For our stack, we'll require three basic functions isEmpty, pop, and push. isEmpty is simple:

```
bool isEmpty(StackS){
    return (S->top == -1);
}
```

When we push items to the stack (after checking that the stack isn't full), we increment top (this works nicely because we we initialize top as -1) and set ST[top] to point to the item we're adding:

```
void push(Stack S, int n){
    if(S->top == MaxSize-1){
        printf("Stack overflow");
        exit(1);
    }
    ++(Stack->top);
    S->ST[S->top] = n;
}
```

To pop off the top of the stack (after checking that the stack isn't empty), we decrement top and return the value we just popped off:

```
int pop(Stack S){
    if(isEmpty(S)) return RogueVal; // designated value to indicate empty
    int temp = S->ST[S->top];
    --(S->top);
    return temp;
}
```

Implementing a Stack Using a Linked List

The array implementation is simple and efficient, but doesn't allow for resizing. One implementation that does is a linked list. We add new items to the head of the list, and remove the head when popping. The stack is defined as follows:

```
typedef struct node {
    int num;
    struct node *next;
} Node, *NodePtr;

typedef struct {
    NodePtr top;
} StackType, *Stack;
```

To initialize an new empty stack, we allocate memory for a stack S and set S→top to null. Checking if a stack is empty simply requires checking S→top:

```
bool isEmpty(Stack S){
    return (S->top == NULL);
}
```

Top push an object onto the top of the stack, we allocate memory and set the appropriate values:

```
void push(Stack S, int n){
    NodePtr np = malloc(sizeof(Node));
    np->num=n;
    np->next=S->top;
    S->top=np;
}
```

To pop the top object off the stack (after checking that there is a value to pop), we set S→top to the second item on the stack and return the value we are popping off.

```
int pop(Stack S){
    if(isEmpty(S)) return RogueVal; // Same as above
    NodePtr tmp = S->top;
    int temp = S->top->num;
    S->top = S->top->next;
    free(tmp);
    return temp;
}
```

Note: 5.3-5.5 omitted from notes

5.6 Queues

A queue is a linear list where items are added to one end and removed from the other. The basic operations we want to perform on queues are:

- Adding an item to the queue (enqueue)

- Taking an item off the queue (dequeue)

- Checking if a queue is empty

- Examining the head of the queue

Implementing a Queue Using an Array

For implementations that use arrays, we need to know size at the time of allocation. In this discussion, MaxQ will be used to indicate this size. The data type is defined as follows:

```
typedef struct{
    int head,tail;
    int QA[MaxQ];
} QType, *Queue;
```

According to this definition, head and tail can vary between 0 and MaxQ-1 (note that here, we are not filling index 0). We define the queue initialization function as follows:

```
Queue initQueue(){
    Queue qp = malloc(sizeof(QType));
    qp->head = qp->tail = 0;
    return qp;
}
```

We add things to the queue at the tail, incrementing tail and placing the object at the tail index. When implementing queues as arrays, the head will point "just in front" of the actual head, while the tail points directly to the last element. We enqueue as follows:

```
void enqueue(Queue Q, int n){
    if(Q->tail == MaxQ-1) Q->tail = 0;
    else ++(Q->tail);
    if(Q->tail == Q->head){
        printf("\nQueue is full\n");
        exit(1);
    }
    Q->QA[A->tail] = n;
}
```

Tail is either incremented, or set to 0 (so we can recognize the queue as full). If the queue is not full, the value is set at the new tail. Now, we consider dequeue:

```
int dequeue(Queue Q){
    if(isEmpty(Q)){
        printf("Attempted to remove from an empty queue\n");
        exit(1);
    }
    if(Q->head == MaxQ - 1) Q->head = 0;
    else ++(Q->head);
    return Q->QA[Q->head];
}
```

We first check first check if the queue is empty. If it is not, we increment the head and return the value of the item pointed to (this works nicely because, as mentioned earlier, head points one behind the actual head).

Consider the example sequence that illustrates usage of an array-implemented queue:

1) We insert the values 5 8 1 9 (where 5 is inserted first). Head is 0, tail is 4 (incremented 1 for each insertion). 2) We call dequeue, which returns 5. Head is incremented to 1.

Implementing a Queue Using an Linked List

As usual, the advantage of implementing with a linked list is that we don't have to know the size of our structure beforehand. Two pointers point to the first and last nodes in the queue struct (each node contains queue data and a pointer to the next). The typedefs thus look as follows:

```
// Generic implementation of queue data, only holding an int here
typedef struct{
    int num;
} QueueData;

typdef struct node{
    QueueData data;
    struct node *next;
} Node, *NodePtr;

typedef struct{
    NodePtr head,tail;
} QueueType, *Queue;
```

Queue initialization follows from this structure:

```
Queue initQueue(){
    Queue qp = malloc(sizeof(QueueType));
    qp->head=NULL;
    qp->tail=NULL;
    return qp;
}
```

Head and tail are both null in the initially empty queue. To check if a queue is null, we just need to check its head.

```
int empty(Queue Q){
    return (Q->head == NULL);
}
```

To enqueue objects, we first allocate space for their node. If the queue is empty, we set the head and tail to the new pointer. Otherwise, we set next pointer in the current tail to the new pointer and then set the new tail:

```
void enqueue(Queue Q, QueueData d){
    NodePtr np = malloc(sizeof(Node));
    np->data = d;
    np->next = NULL;
    if(isEmpty(Q)){
        Q->head = np;
        Q->tail = np;
    }
    else{
        Q->tail->next = np;
        Q->tail = np;
    }
}
```

When we dequeue, we first check that the queue is not empty. If it isn't, we store the head in a temporary variable (for later freeing) and increment head to head→next. If head→next is empty, we know the queue is now empty and set Q→tail to NULL. The data within the previous head node, which should be stored as soon as we know the queue isn't empty, is returned.

```
QueueData dequeue(Queue Q){
    if(isEmpty(Q)){
        printf("Tried to dequeue from an empty queue");
        exit(1);
    }
    QueueData hold = Q->head->data;
    NodePtr temp = Q->head;
    Q->head = Q->head->next;
    if(Q->head == NULL) Q->tail = NULL;
    free(temp);
    return hold;
}
```

11 Hashing

11.1 Hashing Fundamentals

Hashing is a fast method for searching for an item in a table where each item's key determines its placement. Keys are converted to numbers (if non-numeric) and then **hashed** (mapped) to a table location. If multiple keys hash to the same location, we have a **collision**, which we must resolve.

11.1.1 The Search and Insert Problem

Problem statement: Given a (possibly empty) list of items, search for a given item. If not found, insert it.

There are many ways we can implement the list:

1. As an array, where integers are placed in the next available position and searching is sequential. Fast insertion by slow searching.

2. As an array, where each item is inserted such that the array remains sorted. Searching is faster than (1), but insertion is slower.

3. As an unsorted linked-list, which must be searched sequentially.

4. As a sorted linked list

Alternatively, we can use **hashing**, which allows for fast search *and* easy insertion.

11.2 Solving the Search and Insert Problem by Hashing

Suppose we have 12 spots (indexed 1-12) to place numbers and we want to insert 52. A simple hash function might be `x % 12 + 1`, where 52 would be placed in slot 5. If we try to add 16 next, however, we have a collision. One option (*linear probing*) is to just place it (16) in the next open spot. Our searching is thus slightly more complicated:

- Apply our hash function and check the slot

- If the slot is occupied by a different number, try the next location. Keep going until it's found or an empty slot is found.

Roughly as code:

```
loc = hash(key)
while(num[loc] && num[loc] != key)
    loc = loc % n + 1 //n == number of slots
if(!num[loc]){
    num[loc] = key;
}
```

Note that the above while loop never terminates if that table is full (which we never allow happen in practice). Also, rather than `!num[loc]`, we'll assign a default 'empty' value (ex. -1 or 0).

11.2.1 The Hash Function

Looking beyond the previous example, sometimes we encounter keys that are non-numeric. When handling these, we first need to convert them to some numeric value. Considering strings, we can add up numeric char values:

```
int h = 0, wordNum = 0;
while(word[h] != '\0') wordNum += word[h++];
loc = wordNum % n + 1; //loc between 1 and n
```

The problem here is that words with the same letters hash to the same location. To counter this, we can apply a weight to each letter based on position in the word.

```
int h = 0, wordNum = 0;
while(word[h] != '\0'){
    wordNum += w * word[h++];
    w = w + 2;
}
loc = wordNum % n + 1;
```

Generally, we want to spread our data out in the table but avoid having a costly function.

Deleting an Item from a Hash Table

We can't simply delete values, or our resolved collisions might erroneously not be found. Instead, we can *mark* values as deleted (ex. -1).

11.3 Resolving Collisions

The above collision-resolution process is an example of **linear probing**. Other options include **quadratic probing**, **chaining**, and **double hashing**.

11.3.1 Linear Probing

Essence: `loc += 1`. Downsides: Clustering (long chains tend to get longer, and connect to other chains). *Primary clustering* is when keys hash to different locations but trace the same path looking for an empty location (ex. keys that hashes to 5 and 6). *Secondary clustering* is when keys that hash to the same location trace the same path.

Trying to solve this problem by substituting `k` for `1` can be *worse* if not all locations end up being checked. However, s o long as table size `m` and `k` are coprime, we know all locations will be generated. Note: not that important in practice since we don't want full tables anyways.

When evaluating the speed that results from collision-resolution methods, we consider **search length**, which is a function of **load factor** f where:

Using this f, we have:

as the average number of steps for successful searches and:

as the average number for unsuccessful searches. Trying some figures, we can easily see that filling the table above 75% capacity is not optimal (0.75: 8.5/unsuccessful vs. 0.90: 50.5/unsuccessful).

11.3.2 Quadratic Probing

Essence: `loc += ai + bi^2`

Rough implementation:

```
loc  = hash(key);
int i = 0;
while(num[loc] && num[loc] != key){
    ++i;
    loc = loc + a*i + b*i*i;
    while(loc > n) loc = loc - n; // n is table length
}
if(!num[loc]) num[loc] = key;
```

Note that powers of 2 for *n* are quite bad, only a small fraction will be tried. For prime *n*, half can be reached (usually sufficient).

11.3.3 Chaining

Items that hash to the same index are held within a linked list. Thinking about implementation, we start with a basic linked list implementation:

```
typedef struct node{
    int num;
    struct node *next;
} Node, *NodePtr
```

and a function to create new nodes:

```
Node newNode(int n){
    Node temp;
    temp.num = n;
    temp.next = NULL;
    return temp;
}
```

and hash would be an array of these nodes as follows:

```
NodePtr hash[MaxItems]; // 0-index
```

which we initialize with NULLs:

```
for(int i = 0; i < MaxItems; ++h) hash[h] = NULL;
```

Putting this together, we might have an implementation that like this:

TODO: Add a full implementation with creation, searching, and search + insert.