Assignment 4: Population Models

February 9, 2016

1 Part 1: Logistic Equation

1.1 Exponential Growth Population Model

The exponential growth population model was thought to be the most accurate model for population growth. However, it did not take into account resource limitations. A population cannot grow continuously exponentially as there will always be an upper bound, whether it be predators, space, food and habitat limits, etc.

The basic exponential function was used to model population growth and is given here:

$$P = P_0 e^{rt} \tag{1}$$

where P is the final population, P_0 is the starting population, r is the increase rate, and t is the time.

The exponential model is shown in Figure 1. Clearly, the model is below the actual population. This is because the starting and ending points were used in the calculation. Since the population starts to linearize after 1950, an exponential growth function is not the correct method for a population model.

In Figure 2, the percent error of the exponential growth function as compared to the actual population was plotted. The largest percent error is more or less a bell curve, since the fit includes all points equally, including the data after 1950.

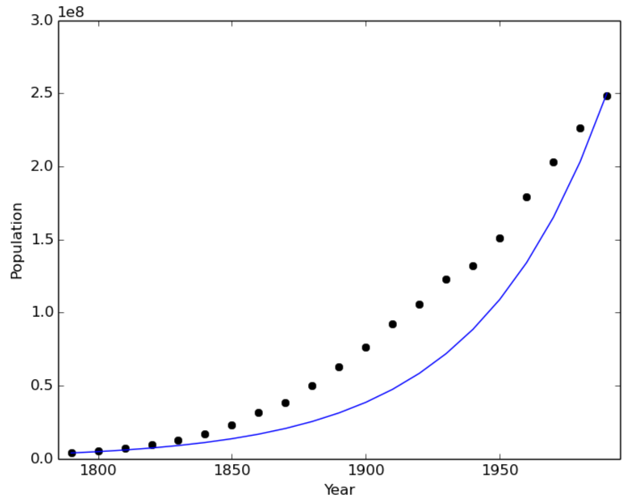

Figure 1: The Exponential Growth Model

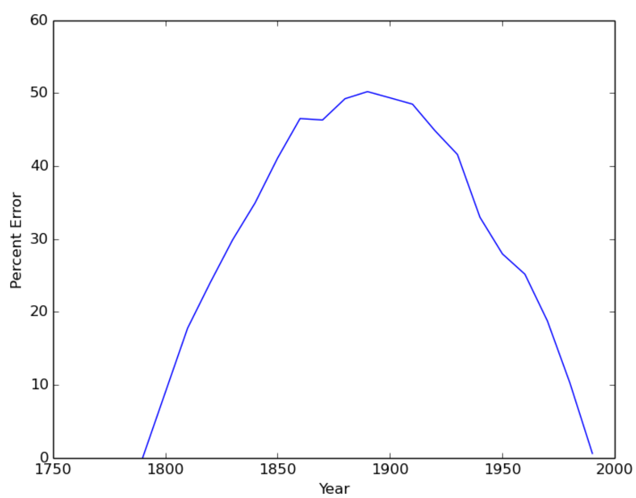

Figure 2: Percent Error of the Exponential Growth Model

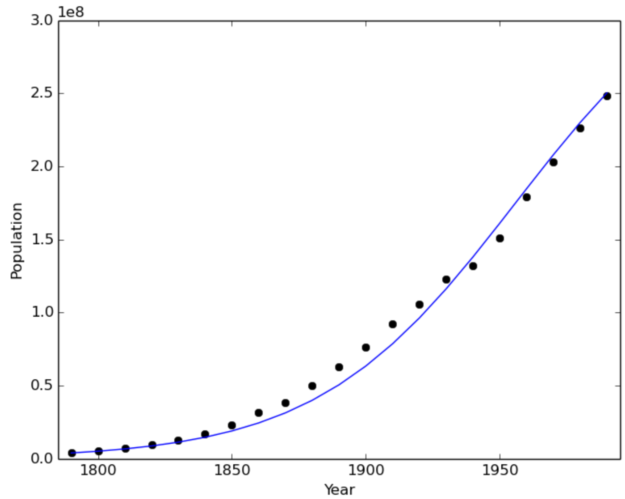

Figure 3: The Logistic Equation Method

1.2 The Logistic Equation Growth Model

Other than the exponential growth model, the logistic equation accounts for the resource limitations that hinder population growth. This equation is known as the Verhulst model.

$$\frac{dP}{dt} = kP(1 - \frac{P}{N}) \tag{2}$$

In order to work my code properly, I used the solution to the differential equation given to us.

$$P = \frac{k}{1 + Ae^{-rt}} \tag{3}$$

As Figure 3 shows, the logistic equation is much more suitable for a population growth model. It clearly takes into account the change in slope due to resource limitations.

The values for k and r, instead of k and N, since I used the solution equation of the differential equation, were $k = 3.5E8$ and $r = 2.7E-2$. This yielded a percent error according to Figure 4.

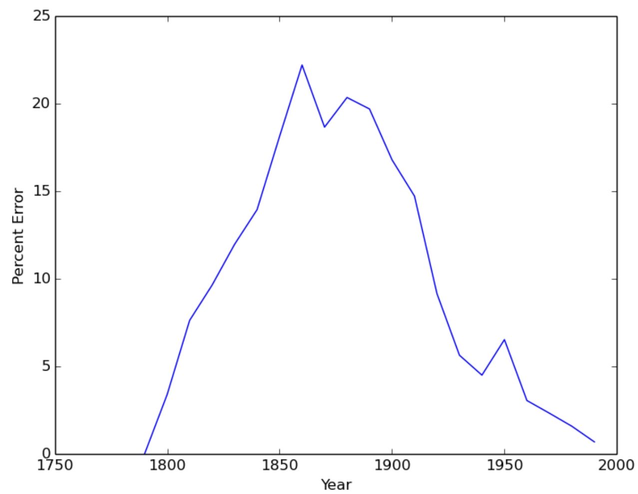

Figure 4: Percent Error of the Logistic Equation Method

The sharp peaks in the percent error are due to the sudden change in the rate of growth of the population. The data for years 1860 and 1940-1950 make quick jumps which yield in a small peak in percent error at these values. However, overall the error is much lower than the exponential growth population model.

1.3 Comparison Graphs

In Figure 5 and 6 the comparison between the two models are shown, simply overlayed to portray the difference in effectiveness and percent error.

2 Part 3: The Epidemic Equation

2.1 Original Parameters

First, I simply used the three groups given (healthy, sick, and immune), to model the epidemic's affect on the population.

I realized that the equations we had worked on in class were wrong since the sum of the groups over time was not always equal to 100 (my maximum).

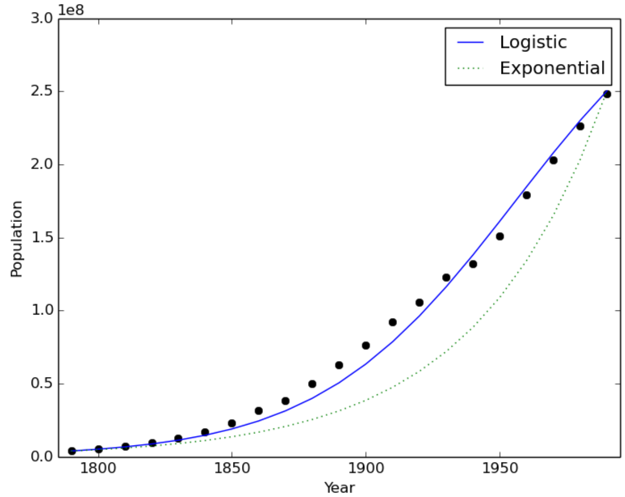

Figure 5: Comparison of the Exponential and Logistic Models

Figure 6: Comparison of the Percent Errors

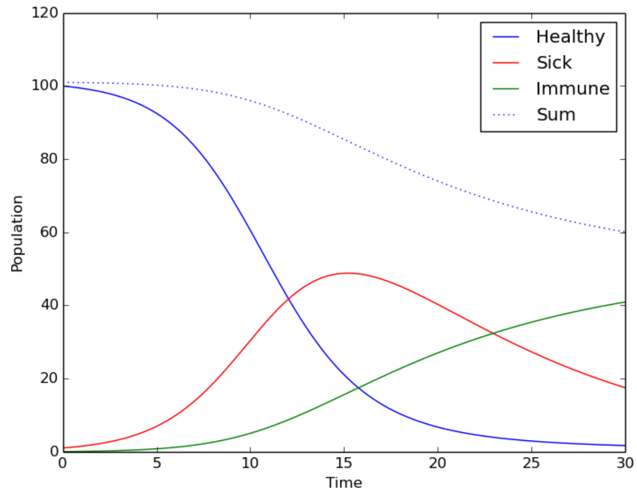

Figure 7: Test of Parameters from Class

This can be seen in Figure 7. This is likely due to the equations not being balanced. Clearly, the change in something needs to yield the exact same increase in one of the other parameters. So, these were the equations I used. (H = healthy, S = sick, I = immune)

$$dH = -aHS - cH \tag{4}$$

$$dS = aHS - bS \tag{5}$$

$$dI = cH + bS \tag{6}$$

In Figure 8, the sum of all the groups of people adds up to the maximum number of 100 now. Therefore the dats is much more accurate, as I am counting all living and dead as people.

2.2 Adding Vaccinations

So, now the immune category will be split up into two types: those who are dead and those who are vaccinated. The equations for these new parameters are as follows: (D = dead, v = vaccinated)

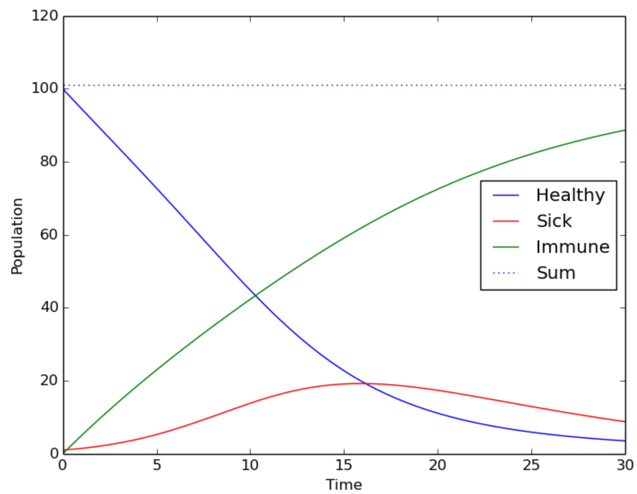

Figure 8: Epidemic's Affect on Population Over Time

$$dH = -aHS - cH \tag{7}$$

$$dS = aHS - bS \tag{8}$$

$$dD = bS \tag{9}$$

$$dV = cH \tag{10}$$

As Figure 9 shows, the sum of all four types of people remains 100, which means the equations hold up. Interesting to note is that splitting the immune categories up does not affect the number of healthy or sick. I'm not sure whether this is an issue with my equations or whether this physically makes sense, but it is of note.

As you can see from Figures 8 and 9, the number of sick people tends to peak at just after half of the time span that the healthy die, are vaccinated, or die. Also, the peak percent of the population that is ever sick is around 20 percent.

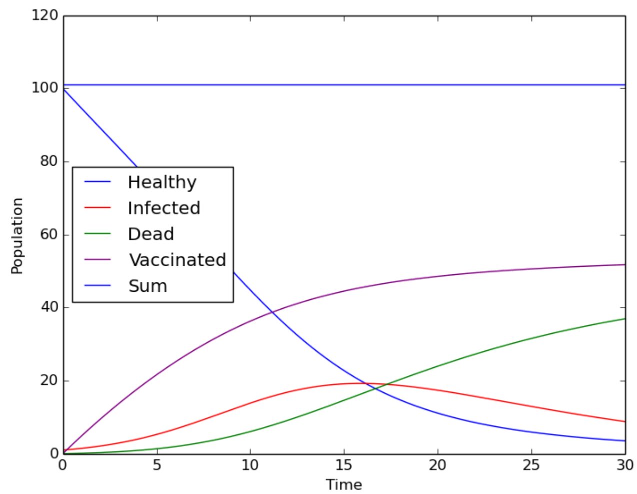

Figure 9: Epidemic's Affect with Vaccinations