Assignment 7: Real Pendulum

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1 Introduction

The real pendulum is a pendulum system which includes gravity, air damping, and a spring force exerted by the string.

1.1 Choosing a Coordinate System

If a Cartesian coordinate system is chosen, then the motion of the bob wouldn't be an issue since the origin of the coordinate system could move with the bob. However, the forces then would be far too complicated to work out numerically, since they would be changing directions in the x-y coordinate system over time, especially the force of tension. Also, the sum of the forces would be the sum of the real and the sum of the fictitious forces:

$$\sum \vec{F} = \sum \vec{F}_{real} + \sum \vec{F}_{fictitious} \tag{1}$$

If a cylindrical coordinate system is chosen, then the motion of the bob would need to be calculated over time via differential equations. However, the forces are much easier to work out numerically, such as the tension force which now is in the radial direction.

2 Part 1: Ideal Mathematical Pendulum

2.1 Period of the Simple Pendulum

The differential equation for the ideal mathematical pendulum is:

$$\ddot{\theta} + \frac{g}{l} sin(\theta) = 0 \tag{2}$$

In previous years, we would use the $sin(\theta) = \theta$ approximation for small angles of θ. However, for a large θ, this approximation is inaccurate and we must use the full differential equation shown above.

For the small angle approximation, the period of the pendulum is:

$$P_{small\theta} = 2\pi\sqrt{\frac{l}{g}} \tag{3}$$

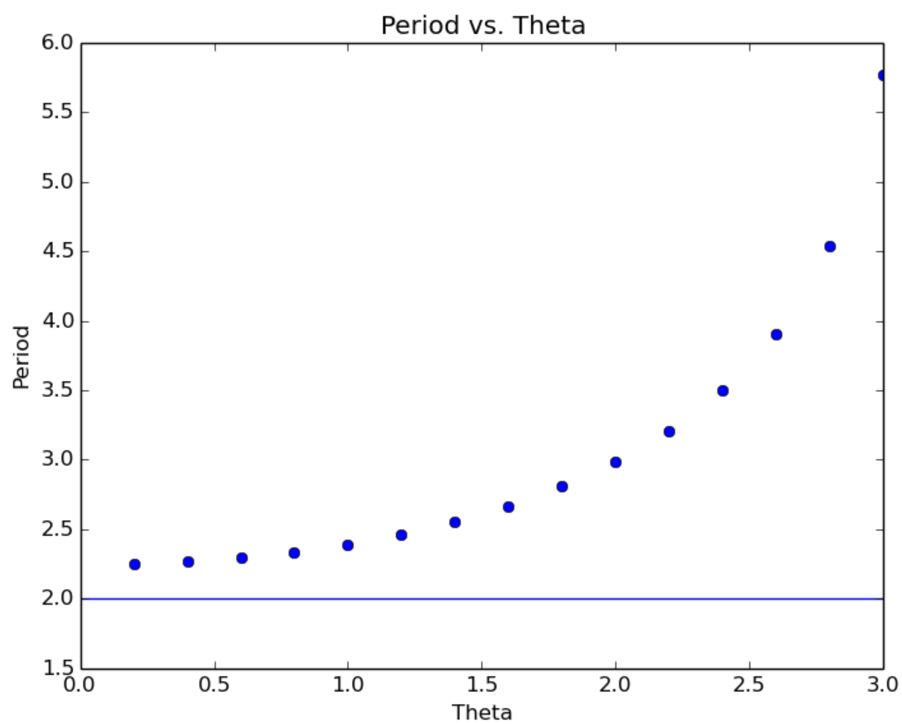


Figure 1: Period vs. Angle for a string of length 1. The horizontal line is the small angle period approx.

As Figure 1 shows, the period follows an exponential growth as theta increases.

2.2 Chaos of Simple Pendulum

The chaos of the simple pendulum should be non-existent. I looked at the chaos of the simple pendulum to ensure this was the case for future additions to the code. For the dynamical chaos, I followed two initial conditions, one was $L_0 = 1.0$, the other was $L_0 = 1.01$. I then plotted the difference of the length, which I calculated using the pythagorean theorem. As Figure 2 shows, the difference of the two lengths remains basically constant up to the thousandths place throughout 10000 iterations.

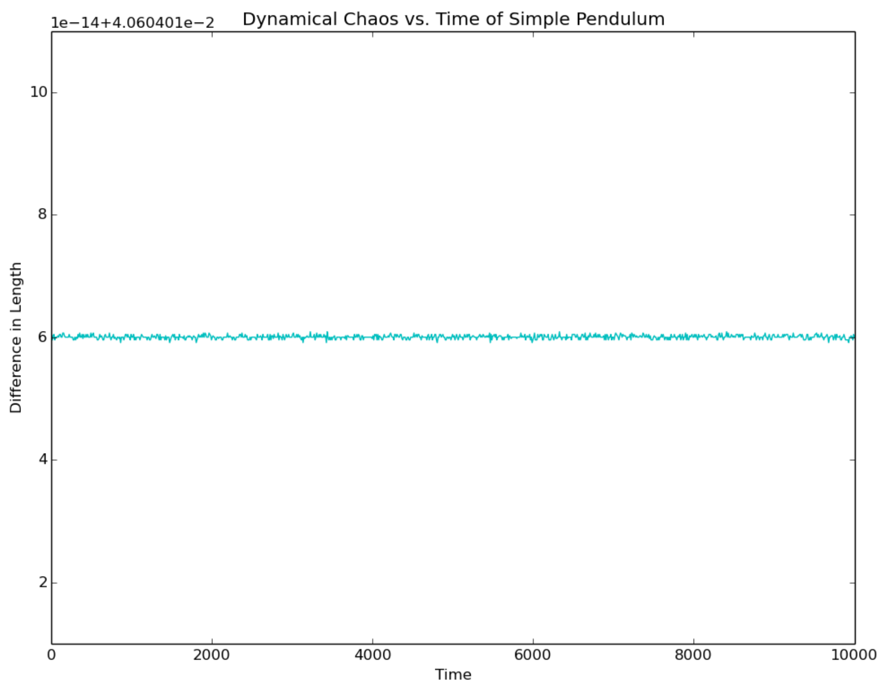


Figure 2: Dynamical Chaos of the Simple Pendulum based on initial condition L_0 and measuring the length

2.3 Phase Diagram of Simple Pendulum

For the phase diagram of the simple pendulum, I simply plotted ω vs. θ, as shown in Figure 3. The diagram shows that there is an equilibrium point at the center of the diagram, which is at (0,0).

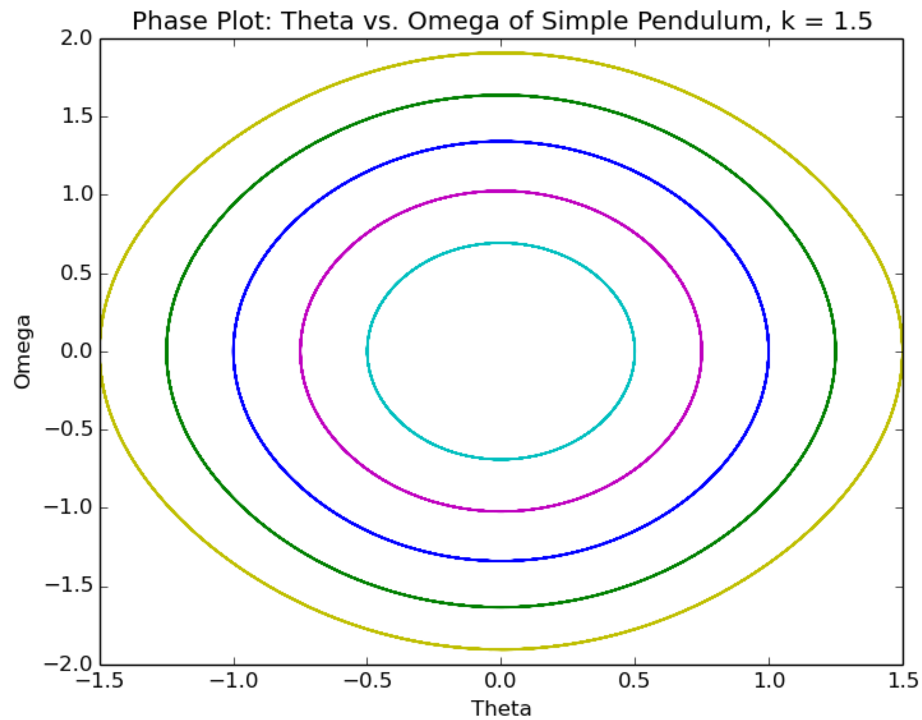


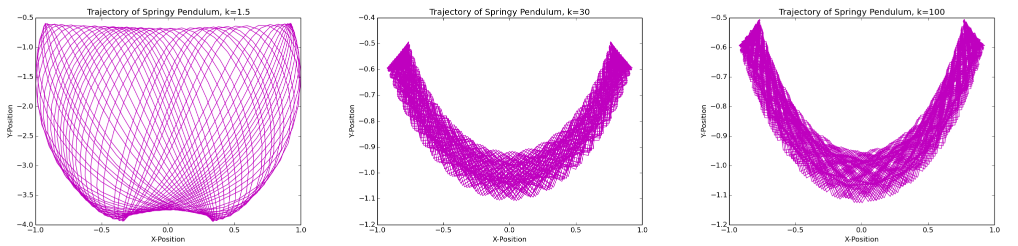
Figure 3: Phase Diagram of ω vs. θ for the Simple Pendulum

3 Part 2: Springy Pendulum

The pendulum with a springy string is an interesting system since the bob will now act in a slightly more random way. For the spring constant of the string, I thought a value of $k = 1.5$ would be fairly realistic, however I tested different spring constants to show different trajectories and how the spring constant itself affects the randomness of the system (how chaotic it is).

3.1 Trajectories of Springy Pendulum

As Figure 4 shows, the more realistic values for k are ones which are closer to 0, since a string is not that springy. For higher values of k, the trajec-



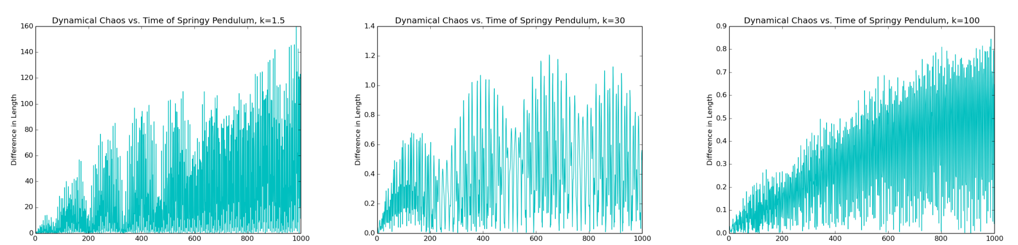
(a) Trajectory for $k = 1.5$ (b) Trajectory for $k = 30$ (c) Trajectory for $k = 100$

Figure 4: Trajectories of Springy Pendulum

tory is more controlled by the spring force, and there are more bounces and randomness than lower values of k. Also, it's interesting to note how the trajectory will never stray from a certain area of the coordinate plane.

3.2 Dynamical Chaos of Springy Pendulum

I wanted to plot the dynamical chaos for the different spring constants and see how the spring constant affected the chaos of the system. Unfortunately, I was only able to iterate up to 1000 since going higher caused the functions to actually malfunction.



(a) Chaos for $k = 1.5$ (b) Chaos for $k = 30$ (c) Chaos for $k = 100$

Figure 5: Dynamical Chaos for different values of k.

My hypothesis about the large spring constants causing more chaos in the system could not be more wrong. At least Figure 5 shows, that the lower the spring constant, the greater chaos exists in the system. I would have liked to integrate this over a larger timestep, but 1000 will have to do. The difference in length is over 100 times larger when $k = 1.5$, which is very interesting and the opposite result I thought I would get.

3.3 Phase Plots of Springy Pendulum

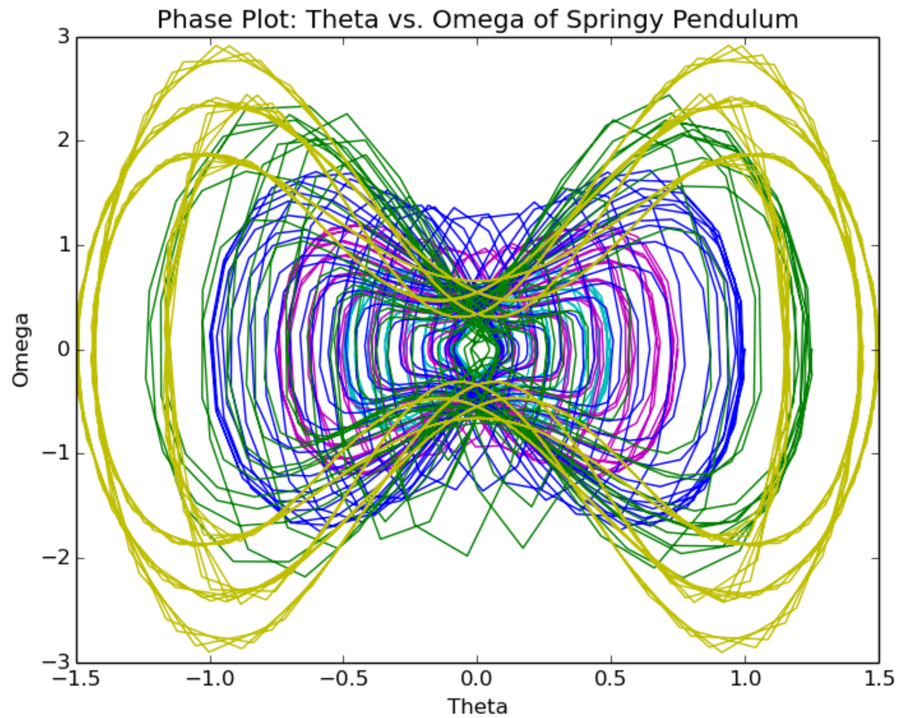


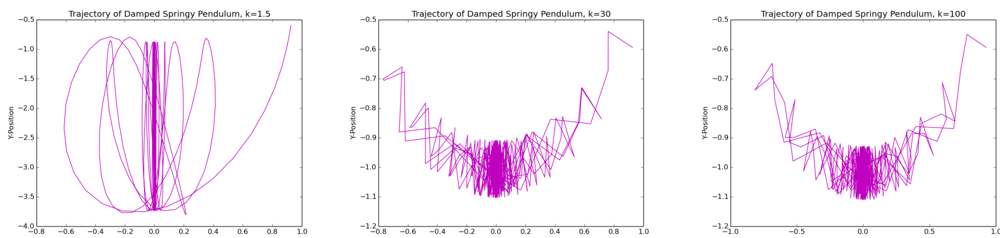
Figure 6: Phase Plot: ω vs. θ of Springy Pendulum

As Figure 6 shows, the phase diagram is much more crazy and the lines for one specific theta and omega don't overlap all too well. However, the equilibrium point maintains intact and at the center (0,0) so the phase plot is doing its job.

4 Part 3: Damped Springy Pendulum

Finally, I will dampen the system. Unfortunately, I could not figure out free fall in cylindrical coordinates with my code. Also, the dampening of the system was done on omega instead of the cartesian components of the velocity, since I was using cylindrical coordinates. This caused a slight problem where the system would be damped until theta stayed constant, but nothing would dampen the spring force or the y-component further, so the bob would simply continue bobbing up and down forever. I could not find a way to fix this so I figured I would dampen it the best I could by dampening omega.

4.1 Trajectories of Damped System



(a) Trajectory for $k = 1.5$ (b) Trajectory for $k = 30$ (c) Trajectory for $k = 100$

Figure 7: Trajectories of Damped Springy Pendulum

As Figure 7 shows, once the system was dampened enough for theta to remain constant, there was nothing left to dampen the system the rest of the way.

This can be seen by looking at Figure 8, the "chaos of the system". Which in this case shows more so how the system stopped being damped, and the y-coordinate of the length stays around while the x-coordinate goes to 0.

5 Energy of the Systems

Even though I plotted the total energy as the hamiltonian (the same way we did it in class), I could not get it to remain constant. Here are my attempted total energy plots for each section.

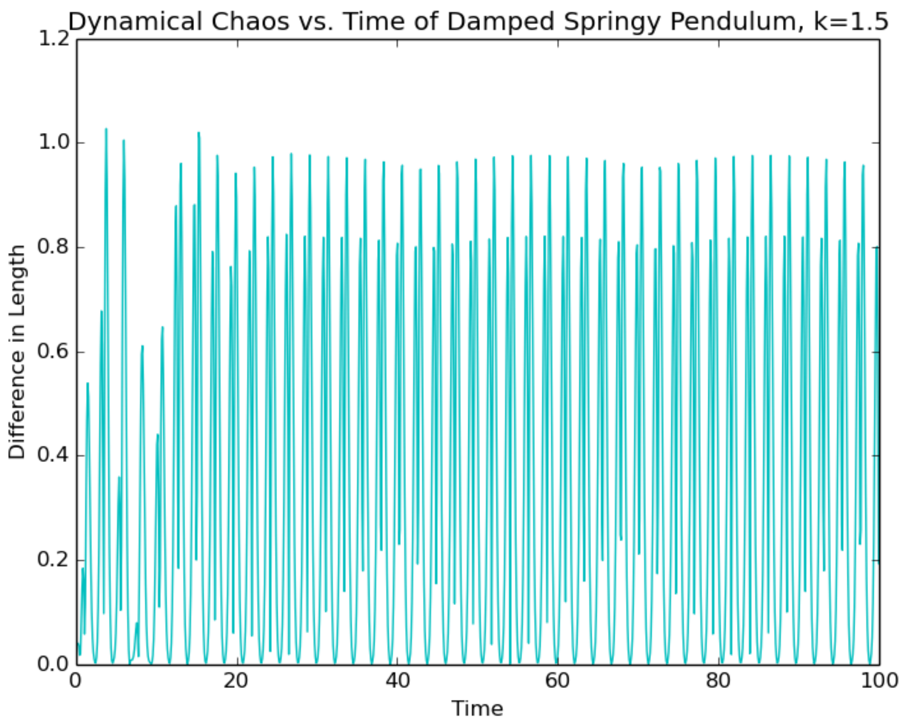
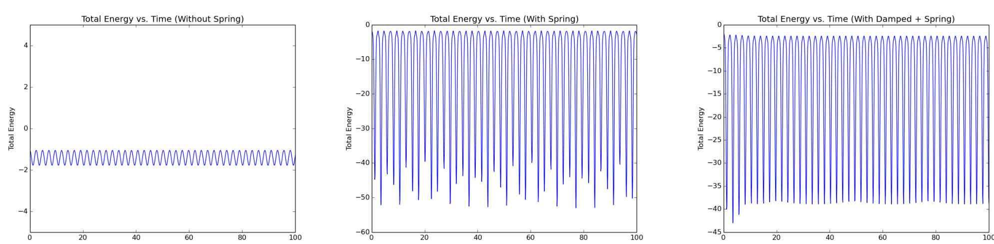


Figure 8: Y-term dominates due to lack of dampening



(a) Simple Pendulum (b) Springy Pendulum (c) Damped Pendulum

Figure 9: Total Energies of the different systems