# Assignment 6: Dynamical Chaos

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## 1 Introduction

In non linear differential systems, arbitrarily close initial conditions result in an exponential divergence of trajectories. This means that as time goes on, initially close parameters of the system will diverge at an exponential rate. This exponential divergence is dependent on how robust the initial conditions are. Dynamical chaos refers to a system which is chaotic, but the disorder is also deterministic (you can figure out what the next step will be from the previous step). An ergodic system is one which is non-deterministic.

## 2 Part 1: Logistic map for Feigenbaum Series

The logistic map for the Feigenbaum series consists of many splits as $\mu$ approaches closer and closer to 4. The whole map is seen in Figure 1. As seen in Figure 2a, the first split comes at 3, the next at around 3.45 and the third at around 3.55, as shown in Figures 2b and 2c. The latter two figures depicting both of the forks from the first, to ensure that they both split at the same value.

As $\mu$ approaches closer and closer to 4, the number of splits in the series outnumber the amount of lines capable of coding it, so the plot looks like a bunch of dots.

In studying the stability of the attractors, as shown in Figure 3,I plotted the X-value of the system at each $\mu$ vs. the iterations to show how the chaos of the system unfolds over a number of iterations. You can clearly see the chaos grow as the width of the highest and lowest X-value increase as $\mu$ increases.

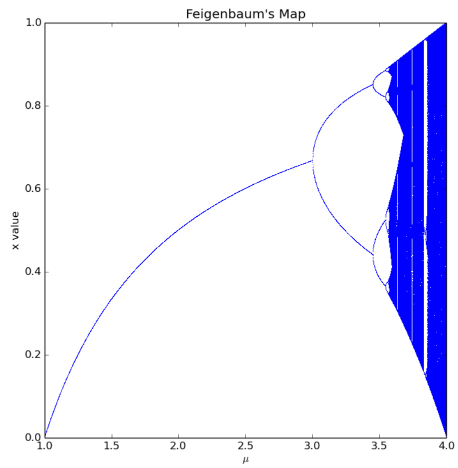
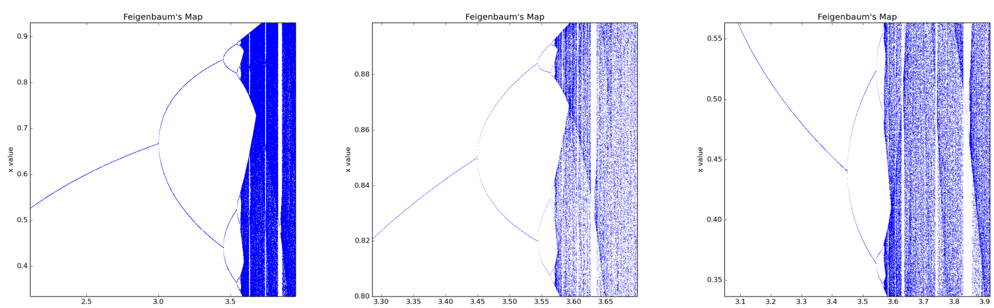


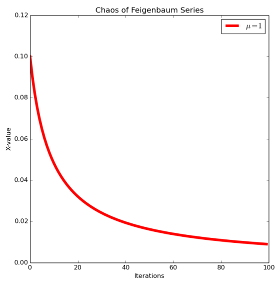

Figure 1: Feigenbaum's Map



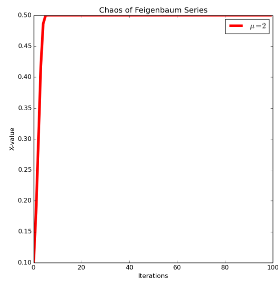

(a) 1st bifurcation (b) Top 2nd and 3rd (c) Bottom 2nd and 3rd

Figure 2: Feigenbaum's Map

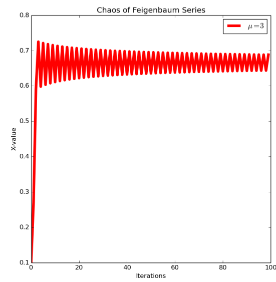
Also, as shown in Figure 4, the number of iterations after about 200 or so does not affect the chaos of the system. You cannot simply continue iterating and expect the chaos to go away. This is a lot like the Heisenberg uncertainty principle of quantum mechanics. You cannot know the exact location of a particle since you will disturb it's location by observing it too closely.




(a) $\mu = 1$

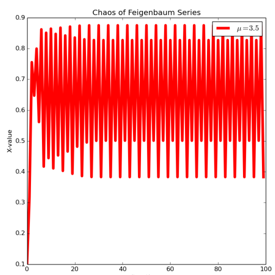



(b) $\mu = 2$

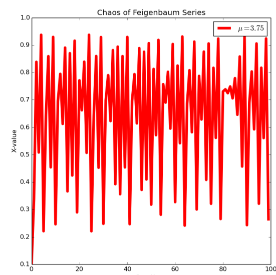



(c) $\mu = 3$

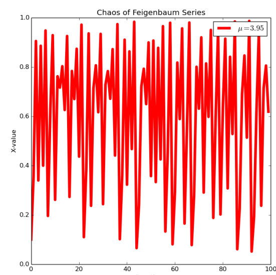
Figure 3: Chaos of Feigenbaum's Map for different values of $\mu$




(a) $\mu = 3.5$

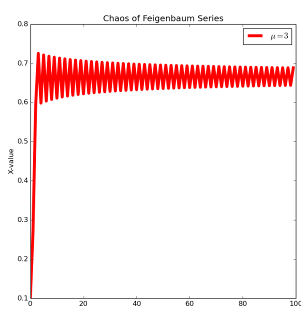



(b) $\mu = 3.75$

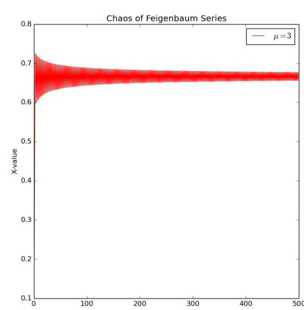



(c) $\mu = 3.95$

Figure 4: Chaos of Feigenbaum's Map for different values of $\mu$




(a) $\mu = 3$, $i = 100$




(b) $\mu = 3$, $i = 500$

Figure 5: Chaos depending on number of iterations

# 3 Part 2: Arnold's Cat Map

Arnold's cat map is based off of moving the pixels in such a way that will shuffle them in an oddly shaped torus. The shift in the image uses the gamma function, which is simply moving the $x$ by $2x + y$ and the $y$ by $x + y$. If this is done, the image will split into 4 sections and wrap themselves into 4 triangles along the square. However, as the pixels become more dissociated it is hard to believe that the image could possibly come back to normal. However, every image using Arnold's cat map method of using this torus will recover the base image after no more than 3N iterations (where N is the dimension of the image).

I applied Arnold's cat map on 4 different images which differ in size to see how the number of iterations changes, how the intermediate images vary, and if there is any correlation between the dimension of the image and the number of iterations it took to output the original image.

Firstly, as shown in Figure 8, the image is shown being torn apart and refigured again upside down, then after the same number of iterations ends up being right side up. All of my other images did similar things. The other midpoint y-axis inverted images are shown in Figures 9b,10b, and 11b.

Interestingly, all the maps seemed to invert along the x an y axis for the midpoint image except the $N = 480$ image (Figure 10). Also, in the $N = 500$ image (Figure 9), although it inverted along x and y, looked as if through cross-eyes. This was also true for the $N = 500$ image although it was not inverted at all for its midpoint image.

The stats for the dimension and number of iterations of each image is as follows:

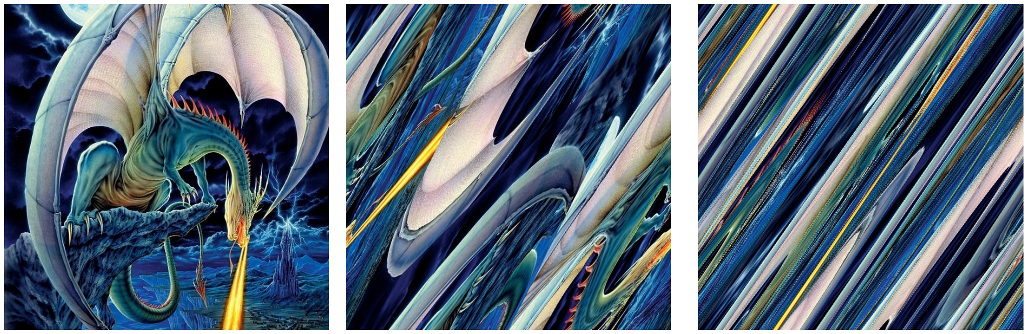
Dragon: N = 601, i = 120

Homunculus Seal: N = 500, i = 300

Geass Symbol: N = 480, i = 96

Simon TTGL: N = 250, i = 300

However much I tried, I simply could not correlate N with the number of iterations. I also could not correlate the strangeness of the midpoint images. It seems as though the system really does have a sort of deterministic chaos to it. We know we're going to get our image back, but we don't know how many iterations it will take unless we have already done an image of that size. It does intrigue me that for the $N = 601$ image and $N = 480$ image, $i \approx \frac{N}{5}$. It also intrigues me that for the $N = 500$ and $N = 250$ image, $i = 300$.



(a) Original Image (b) First Shift (c) Second Shift

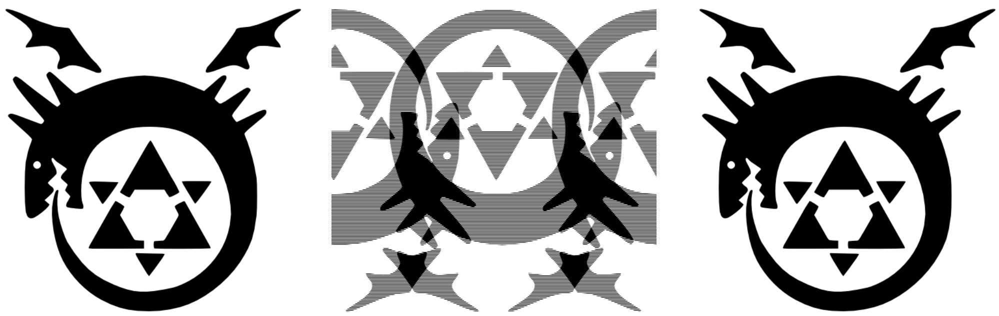
Figure 6

(a) 30th Shift (b) 60th Shift (c) 90th Shift

Figure 7

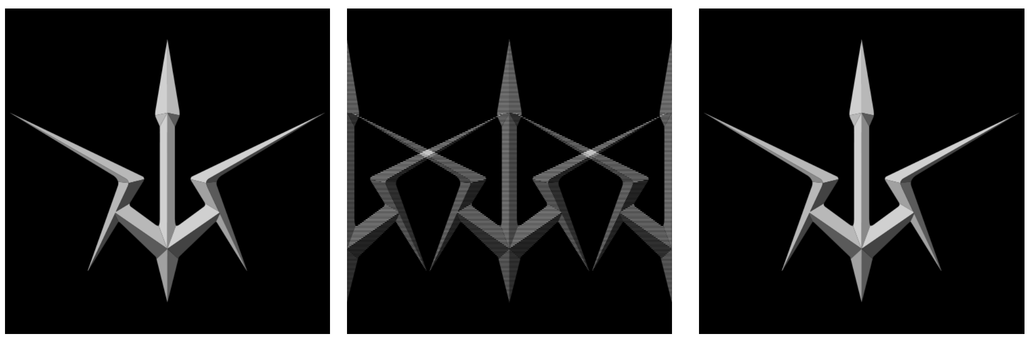
(a) 118th Shift (b) 119th Shift (c) 120th Shift

Figure 8: Arnold's Cat Map for N = 601 (prime number)



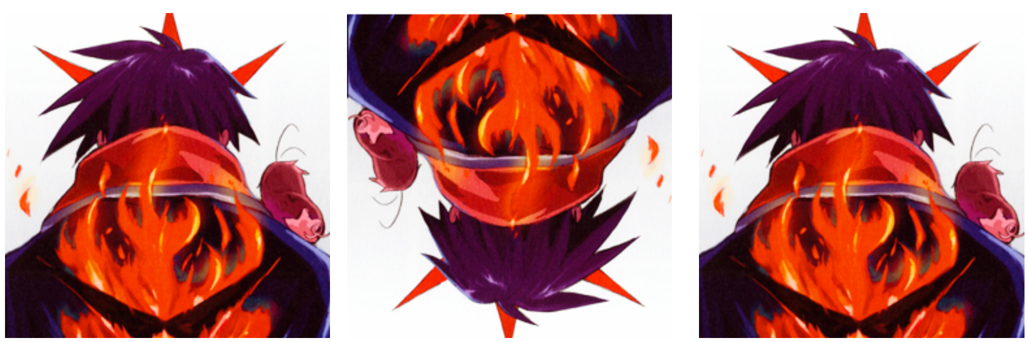
(a) Original Image (b) 150th Shift (c) 300th Shift

Figure 9: Arnold's Cat Map for N = 500



(a) Original Image (b) 48th Shift (c) 96th Shift

Figure 10: Arnold's Cat Map for N = 480



(a) Original Image (b) 150th Shift (c) 300th Shift

Figure 11: Arnold's Cat Map for N = 250

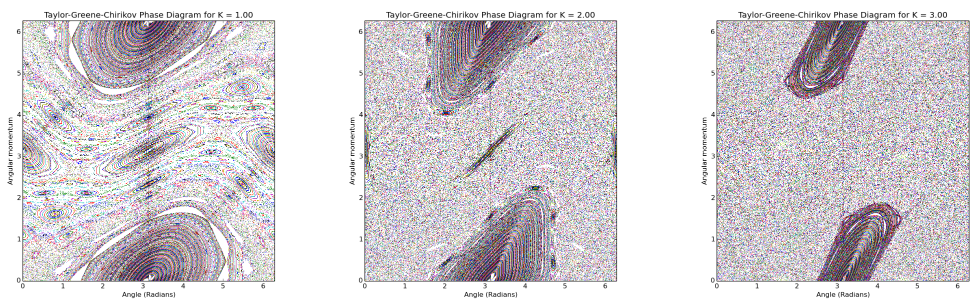
# 4 Part 3: The Standard Map

The equations used for the standard map are as follows:

$$\theta_{n+1} = \theta_n + p_{n+1} \tag{1}$$

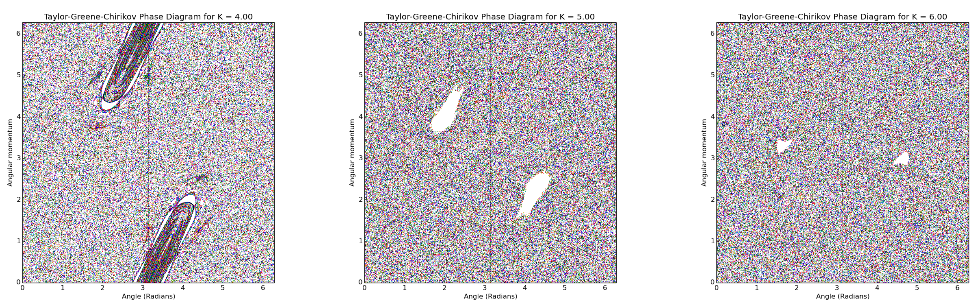
$$p_{n+1} = p_n + K sin(\theta_n) \tag{2}$$

As Figures 12 and 13 show, as the kick parameter K increases, the chaos of the system goes up. Clearly in Figure 12a the system has many fractal islands and seems to be much more stable than any other integer, therefore I will now search for an appropriate K between 0 and 1.




(a) $k = 1$ (b) $k = 2$ (c) $k = 3$

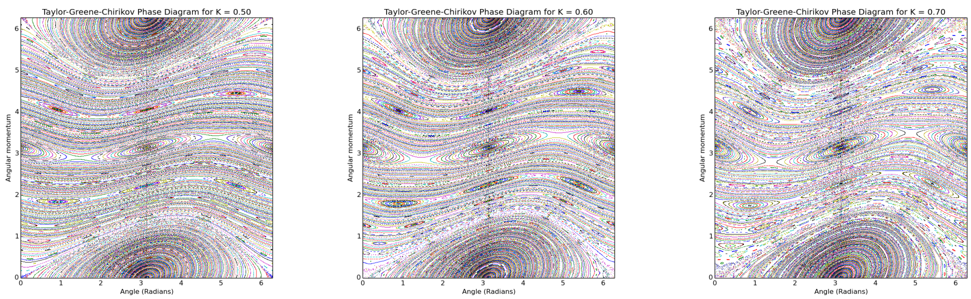
Figure 12




(a) $k = 4$ (b) $k = 5$ (c) $k = 6$

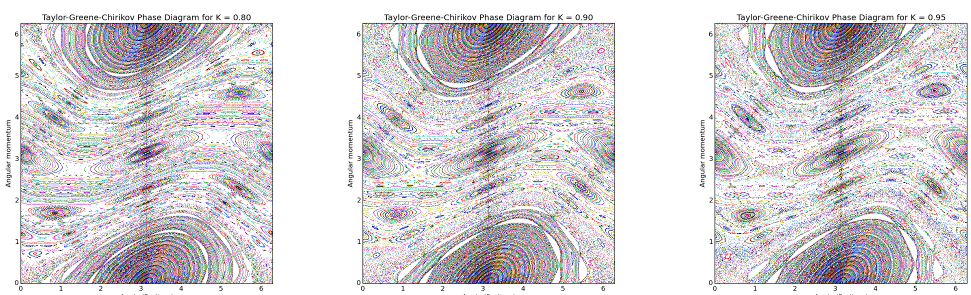
Figure 13: Standard Map for integral values of K

The number of fractal islands only goes up in these plots as K increases to 1. Therefore the critical kick parameter value is around $K = 0.95$ or $K = 1$. The maximum number of fractal islands in the $k = 0.95$ phase diagram is around 64. I would assume that the number of fractal islands is the number of attractors so the system has somewhere around 64 attractors. The value at which the fractal islands disappear is after 1.




(a) $k = 0.50$ (b) $k = 0.60$ (c) $k = 0.70$

Figure 14




(a) $k = 0.80$ (b) $k = 0.90$ (c) $k = 0.95$

Figure 15: Standard Map for $K < 1$