Assignment 2: Linear Programming

Michael Toce

January 27, 2016

1 Part A: Minimize Calories

1.1 Food List: Original

Before I begin I would like to note that I removed beets, cabbage, and fish from the list because I planned on making the food list more personalized. I very rarely eat these foods so I figured it would be better to leave them off of the food list.

For the first problem, I minimized the Calories while keeping the recommended daily intake of 70g for fats, 310g for carbs, 50g for proteins, 1000mg for calcium, and 18mg for iron. In addition keeping in mind that the maximum amount of food that can be consumed in one day is 2kg

The Calories were minimized for someone with low amount of exercise per day or someone who is trying to eat less to lose weight.

First I made an array of the recommended daily values for each nutrient. After importing the groceries.txt file into my program, I made an array of all the different calorie amounts. Then I made a "food" array of all the other parameters which would serve as the x's in the zeta function and the constraint equations. I divided the calcium and iron values by 1000 and changed the daily intake from 1000mg to 1g of calcium and from 18mg to .018g of iron in order to account for changing the units. Then I divided all elements in the food array by 100 to make the elements dimensionless. I then used a built-in function in scipy to optimize the Calories with the the three arrays I created. The output is a list of the amount of each food in grams. It is shown in Figure 1.

The results from the minimization of Calories was somewhat surprising. It seemed very nonsensical that in order to minimize Calories you would have

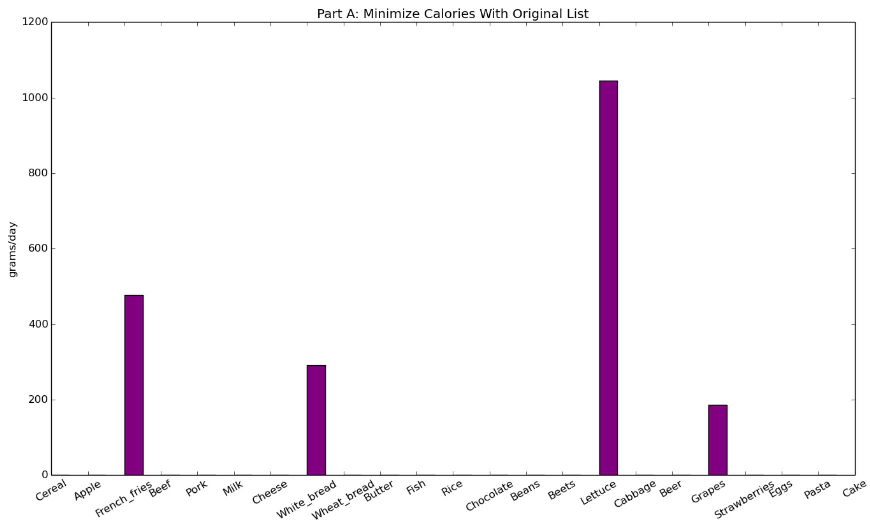

Figure 1: The grams/day for the original food list in order to minimize the amount of Calories

to eat almost 500g of french fries per day. On the other hand, eating over 1000g of lettuce, 300g of white bread, and 200g of grapes all make sense. Since we are minimizing Calories but calcium, iron, and carbs are important as well.

In order to find the amount of Calories, I took the amount of grams/day from the output of my minimization function and divided it by 100 so it was back to the amount of food/100g. Then I multiplied the amount of each food per 100g by its calorie counterpart and summed them all together.

$$Total_{Cal} = \sum_{i=0}^{n} (output[i]/100)(calorie[i])\textbf{n = length of food list} \quad (1)$$

$$\begin{aligned} Cal_{Original} = {} & (477.42100)(93) + (291.75/100)(266) \\ & + (1044.67/100)(17) + (186.16/100)(69) \\ & = 1526.10 Calories \end{aligned} \quad (2)$$

The total minimum Calories for the original food list is 1526 Calories. I thought this result was fairly low, especially considering a large portion of the food intake was french fries when fatty fried food is usually loaded in Calories. So, I looked further into this and replaced the french fries on the food list with McDonald's fries instead.

1.2 Food List: Change to McDonald's Fries

In order to fully understand the confusing result from the first part, I looked up the food stats for McDonald's french fries and used them instead of the ones currently on the list. The change for the french fries in the groceries.txt file was:

Calories: 93 kcal $\rightarrow$ 323 kcal
fats: 12g $\rightarrow$ 15.47g
carbs: 21.55g $\rightarrow$ 42.58g
proteins: 1.96g $\rightarrow$ 3.41g
calcium: 5mg $\rightarrow$ 19mg
iron: 0.35mg $\rightarrow$ 0.80mg

In order to fully understand the change, I simply switched the Calories of the original fries with McDonald's fries to see how the total Calories were impacted.

$$\begin{aligned} Cal_{FrySwap} = (477.42/100)(323) + (291.75/100)(266) \\ + (1044.67/100)(17) + (186.16/100)(69) \\ = 2624.17 Calories \end{aligned} \tag{3}$$

The change is exactly what is to be expected. Since the amount of Calories in 100g of fries increased almost threefold, the total Calories would be expected to jump to almost twice as much, given the amount of fries was 477g/day. This spike in Calories and carbs make french fries a questionable option for minimizing Calories. Thus the graph completely changes when running the optimization program on this new food list. As Figure 2 shows, the amount of lettuce goes down to make more room for other foods to make up the lost nutrients the french fries were providing. The cheese, butter and cake make up for the lost fat while the large increase in pasta makes up for the loss in carbs.

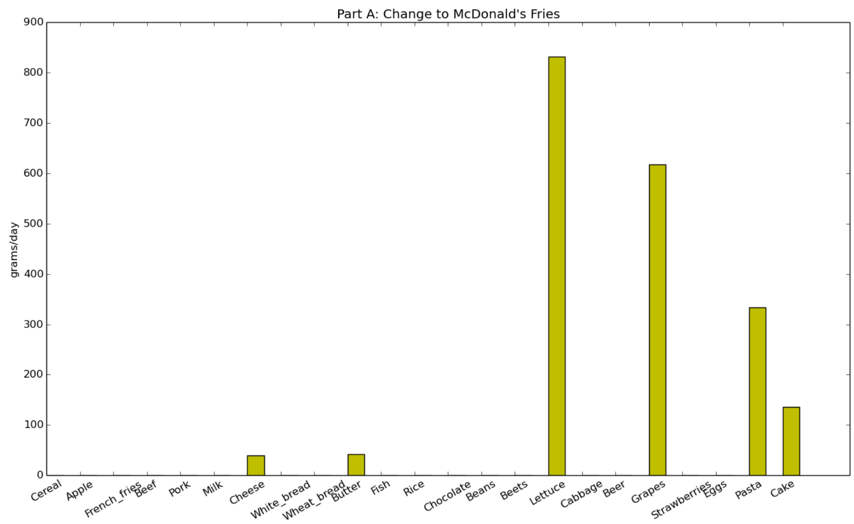

Figure 2: The grams/day for the change to McDonald's french fries in order to minimize Calories

$$\begin{aligned} Cal_{withMcD's} = (39.35/100)(357) + (41.64/100)(717) + (831.39/100)(17) \\ + (617.46/100)(69) + (333.76/100)(124) + (136.40/100)(399) \\ = 1964.52 Calories \quad (4) \end{aligned}$$

Clearly the 400 Calorie jump is expected since the Calories for the original fries were so unbelievably low. When they were swapped with McDonald's fries, other foods took the fries' place, such as cheese, butter and cake to make up for the lost fats. These foods are what caused the increase in Calories.

2 Part B: Minimize Fats

For the second problem, our goal is to minimize fats instead of Calories while having the lower limit of Calories be 2000. This was easily done by changing the minimization array to the fats column, and adding the Calories column to the food list while adding a parameter in the coefficients array that the Calories needed to be greater than 2000.

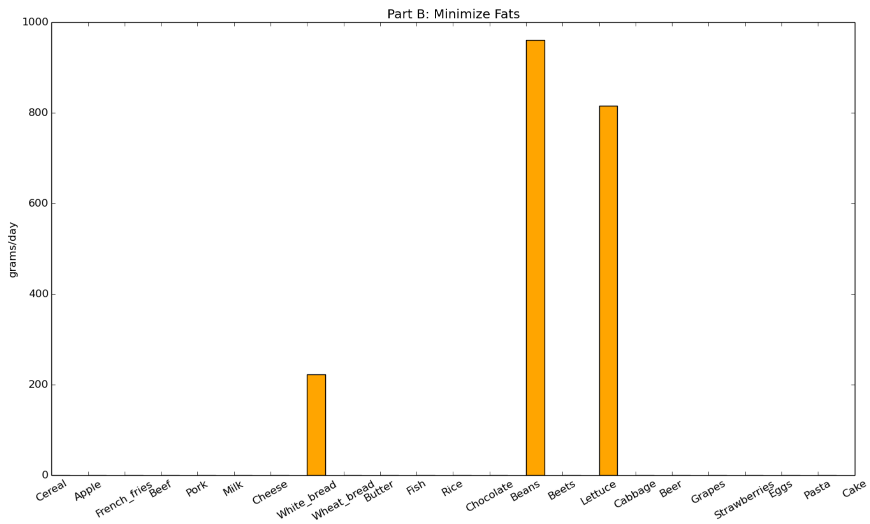

Figure 3: The grams/day of the food in order to minimize fats while keeping calories to at least 2000

For the rest of the problems the new McDonald's fries will be used in place of the old ones, since our results were much better and as expected given the correct amount of Calories for such a fatty food.

For Part B, the foods that were optimized were white bread, beans and lettuce as shown in Figure 3. Now this makes a lot of sense given how low in fat these foods are for the amount of Calories they give (excluding lettuce). The Total Fat was calculated the same way as the Total Calories from part A. Here are the calculations for the total fats and Calories from part B.

$$\begin{aligned} Fats_{partB} = (222.62/100)(3.64) + (961.45/100)(0.54) \\ + (815.93/100)(0.20) = 14.93grams \end{aligned} \tag{5}$$

$$\begin{aligned} Cal_{Rounded} = (222.62/100)(266) + (961.45/100)(132) \\ + (815.93/100)(17) = 1999.99Calories \end{aligned} \tag{6}$$

The total Calories in part B being less than 2000 is all in due to rounding error. When calculated without rounding, the result is as follows.

$$Cal_{noRound} = (222.62443439/100)(266) + (961.4479638/100)(132) + (815.92760181/100)(17) = 2000.0000000010998 Calories \quad (7)$$

3 Part C: Adding Constraints

3.1 Without Vitamins A + D

For this problem, I added constraints to the original 5 in order to hopefully create more variety in the number of foods being selected. The constraints I chose were sodium (Na), fiber, sugar, vitamin A, vitamin B6, vitamin B12, vitamin C, and vitamin D.

I ran into some trouble converting the international units the vitamin A and vitamin D are measured in into grams. Since the vitamins themselves are composed of even further substructures, things get very messy when converting and I could not get my units to work properly. Luckily, all of the other vitamins were measured in either mg or mcg.

I looked up the recommended daily dosage of the new constraints and got:

Na: 2400 mg
Fiber: 25 g
Sugars: 25g
Vitamin A: 2650 IU
Vitamin B6: 1.3 mg
Vitamin B12: 2.4 mcg
Vitamin C: 85 mg
Vitamin D: 60 IU

I converted these to grams and made them negative in my array of coefficients. Since I needed to make the quantities of food dimensionless before I input them into the function, I accounted for this by dividing all mg columns by 1000 and all mcg columns by 1E6. Then like before I simply divided everything by 100 to get rid of the 100g dependence so my result would be in grams/day.

The total calories for part C without vitamins A+D is shown here:

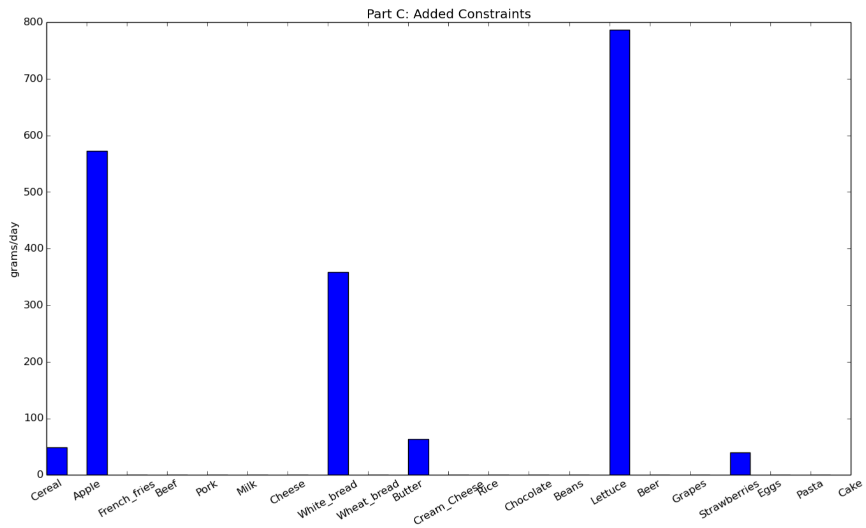

Figure 4: The grams/day while minimizing Calories with added constraints

As shown in Figure 4, the results were solid for what I was aiming for. There was more variety in the results, however lettuce still dominated like in almost every test.

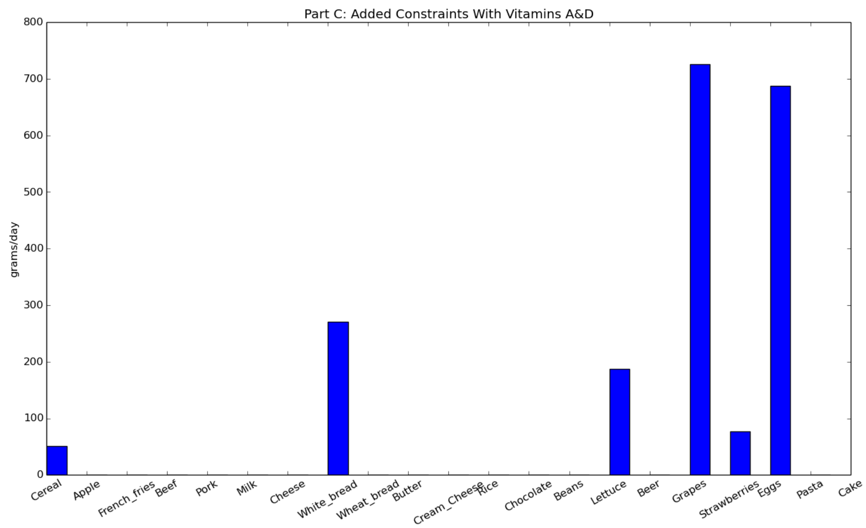

Figure 5: The grams/day with added constraints and Vitamins A and D before correction

$$Total_{Cal} = (48.78/100)(369) + (572.88/100)(52) + (358.35/100)(266) \\ +(63.27/100)(717)+(786.40/100)(17)+(39.54/100)(32) = 2031.09 Calories \tag{8}$$

3.2 With Vitamins A+D

Although the results were poor due to a conversion error, I still think that sharing the results in Figure 5 of my test with Vitamin A and Vitamin D are worth sharing. There even seemed to be more variety, however my minimum Calorie amount was way off of 2000 at 2812.71 Calories.

$$Cal_{before} = (50.54/100)(369) + (270.80/100)(266) \\ + (187.17/100)(17) + (726.28/100)(69) + (77.35/100)(32) \\ + (687.85/100)(196) = 2812.71 Calories \tag{9}$$

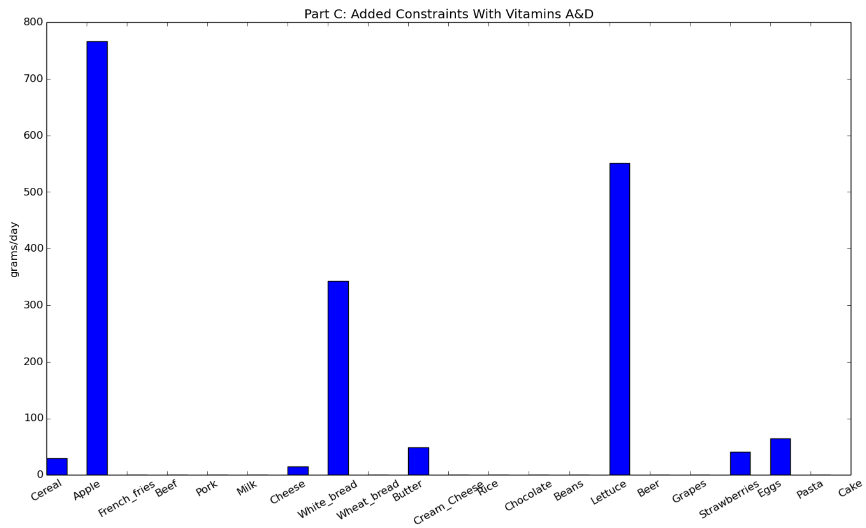

Figure 6: The grams/day with added constraints and Vitamins A+D after correction

The reason for this Calorie jump is due to the large number of eggs necessary to satisfy the constraint equations. I figured out that the reason was due to the constraint for vitamin D being too high and only eggs, cake and cheese have any decent amount of vitamin D in them.

So, there must have been an error in conversion OR more likely, we get vitamin D from the sun and this was included in the recommended daily dose. Therefore I cut the vitamin D by 10 percent to get a much better result worthy of an answer with some serious variety as shown in Figure 6.

$$\begin{multline} Cal_{after} = (29.46/100)(369) + (766.87/100)(52) + (14.86/100)(357) \\ + (342.74/100)(266) + (48.42/100)(717) + (551.71/100)(17) \\ + (40.62/100)(32) + (63.99/100)(196) = 2051.60 Calories \tag{10} \end{multline}$$

Clearly, the correction to the vitamin D constraint created much more variety in the foods. This is clearly the most optimal set of constraints that I tried to place on the food list.

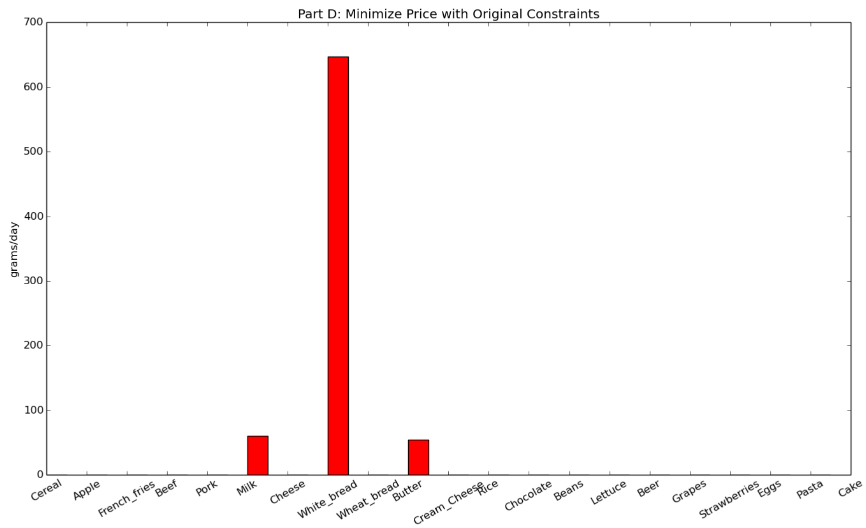

Figure 7: The grams/day with the original constraints when minimizing price

4 Part D: Minimize Cost

In order to minimize the cost, it makes much more sense to revert the constraints back to the simple original ones so the foods won't be bound by vitamins and fiber and sugar and should I dare say - less important things. The importance of fats proteins and carbs is extreme so i will focus on these original constraints along with the calcium and iron.

4.1 With Original Constraints

With the original constraints and minimizing price instead of calories, the variety and low caloric intake certainly was not there. As Figure 7 shows, only milk, white bread, and butter were of importance when it came to keeping the price low.

$$Cal_{partDorig} = (60.70/100)(60) + (646.65/100)(266) + (54.85/100)(717) = 2149.78 Calories \quad (11)$$

In addition to there being no variety, the number of calories was 150 above 2000, when we gave it the minimum of 2000 calories. When minimizing price, I would think the calories would stay around 2000 but they jumped up by 7.5 percent!

$$Price_{partDorig} = (60.70/100)(.0875) + (646.65/100)(0.315) + (54.85/100)(.727) = 2.49 dollars \quad (12)$$

This price means you can get by everyday eating like absolute shit for only 2.5 dollars per day! If that doesn't sound like a great deal I don't know what does...

Also of worthy note is the lack of lettuce being a factor on any scale whatsoever since I started minimizing the price. It must be too expensive to be deemed worthy by the optimization gods.

4.2 With Added Constraints

Now let's see how the minimization of price is affected when we constrain the food with all of those additional parameters. I wonder how high the price will get when all that vitamin, sugar, and fiber needs to be accounted for and with healthy food being so expensive.

As shown in Figure 8, the variety of foods is quite high again, like in part C. However, in order to account for the strict constraints for these vitamins and things that only healthy foods have, the price almost doubled!

$$Price_{partDnew} = (67.40/100)(.80) + (136.37/100)(.0875) + (520.04/100)(.315) + (39.25/100)(.727) + (12.17/100)(.157) + (132.28/100)(1.03) + (66.56/100)(.404) = 4.23 dollars \quad (13)$$

$$Cal_{partDnew} = (67.40/100)(369) + (136.37/100)(60) + (520.04/100)(266) + (39.25/100)(717) + (12.17/100)(112) + (132.28/100)(32) + (66.56/100)(196) = 2181.67 Calories \quad (14)$$

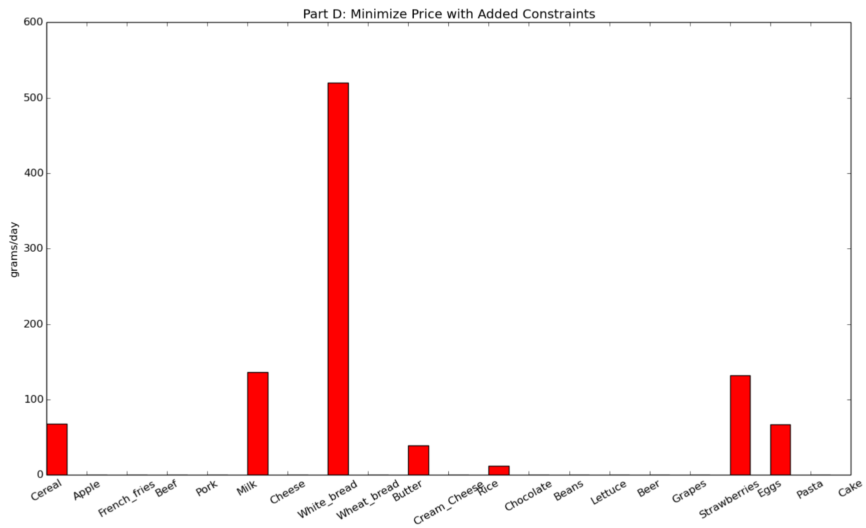

Figure 8: The grams/day of product with added constraints when minimizing price

The Calorie amount for someone minimizing their price with these strict constraints is going to end up spending a lot of money anyway. I wonder how this compares to eating completely healthy - by not overindulging those 130 calories. The price of the part C problem is:

$$Price_{partC} = (29.46/100)(.80) + (766.87/100)(.307) + (14.86/100)(.957) + (342.74/100)(.315) + (48.42/100)(.727) + (551.71/100)(.396) + (40.62/100)(1.03) + (63.99/100)(.404) = 7.03 dollars \quad (15)$$

Wow! This increase is huge! just to eat 130 less calories per day cost almost an extra 3 dollars! Also, the food is probably prioritized in a way that doesn't even take into account money at all in part C. Therefore, following the part D plan with added constraints would be the best option for anyone who wants to eat healthy but also can't spend too much money on food.

5 For the Triathlete's Among us

I wanted to include a section primarily for my own motivation with what to eat this semester when it comes to training for my half ironman race in June.

For this reason I have decided to add a few foods to the list that I either enjoy eating, or eat on a regular basis.

Hopefully this will guide me in my decisions over the semester in what foods will keep me training and injury free while maintaining a healthy body.

Since training itself loses calories simply from the exercise, I will set the calorie constraint at 2500 and minimize price, since as I saw from the results of parts C+D, it would be much better to save some money and still get all those good vitamins and nutrients. Here are the changed food stats for a triathlete in training:

fats: 70g $\rightarrow$ 100g

carbs: 310g $\rightarrow$ 500g

protein: 50g $\rightarrow$ 100g

However, I will keep the fats to 70g per day given I am trying to maintain a healthier body image and also lose weight in my training.

fats: 70g

Here are the changed foods:

-French Fries

-Chocolate

+Ramen Noodles

+Bananas

+Whey Protein Mix

+McChicken

+Mozz Sticks

+Mandarins

+Green Peppers

+Carrots

5.1 For the Triathlete: Minimize Price

The results for the minimization of price were quite interesting. As Figure 9 shows, bananas, white bread, milk, and eggs were the core of the diet. The calories were fairly decent at 2912.43 and at a daily price of 3.80 dollars, this diet is certainly not too bad.

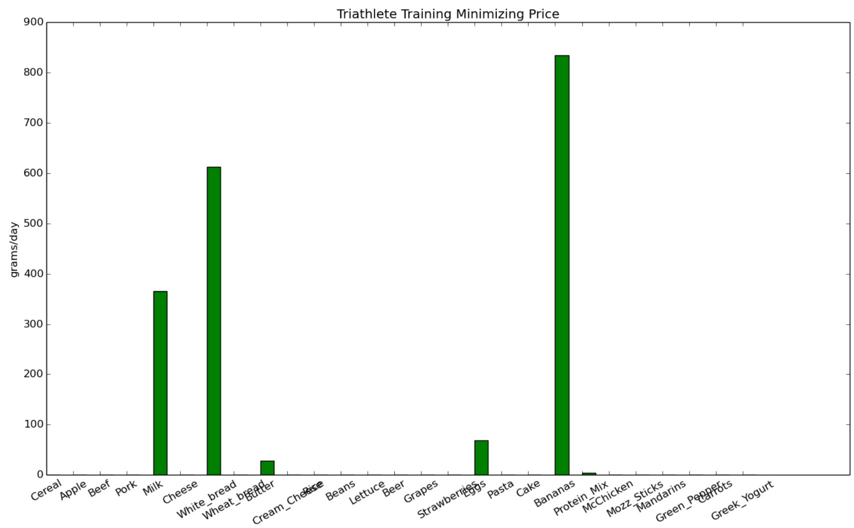

Figure 9: The grams/day for a triathlete in training who's on a budget

However, I am curious what the difference in diet would be if I minimized calories instead, and did not worry about the price.

5.2 For the Triathlete: Minimize Calories

As Figure 10 shows, there was a fairly large change in the foods that are accepted. This is most likely because pasta is just too expensive to make the cut in a price minimization problem. However, pasta isn't that expensive, and it would be worth the small increase in money from 3.80 dollars to 5.50 dollars for some real meals. The amount of calories with this diet is 2869.90 Calories. The larger amount of Calories for the triathlete's diet is due to the larger constraint on carbs and protein. After all, someone who is exercising quite a lot every day needs the energy to put the work in.

I am very pleased with these results because they basically tell me to eat like I have been eating since the start of the semester. Implementing the minimization on price caused a bit of an awkward situation because there weren't any foods that could be eaten as meals. However, by spending an extra 1.70 dollars/day, the pasta is well worth some real food.

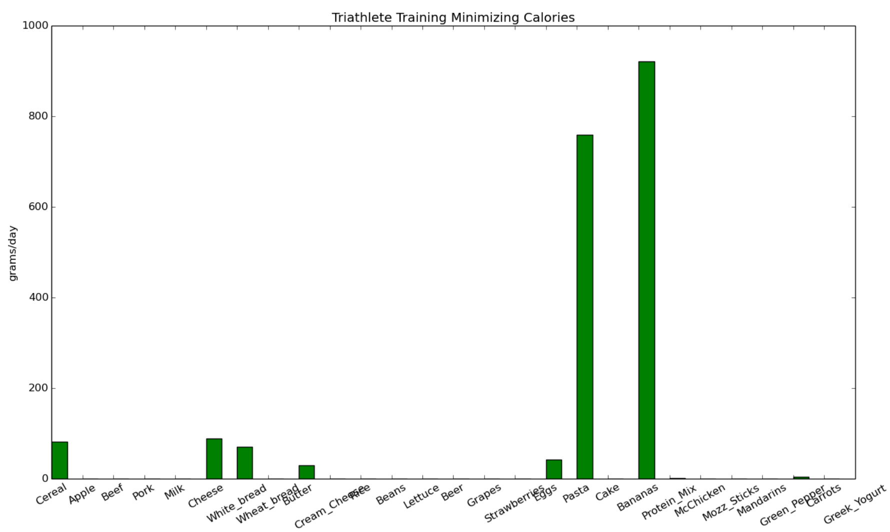

Figure 10: The grams/day for a triathlete in training who is minimizing calories