Algoritmi di Parsing: Bottom-up Parsing

Slides based on material by Ras Bodik available at http://inst.eecs.berkeley.edu/~cs164/fa04

Welcome to the running example

· we'll build a parser for this grammar:

$$E \rightarrow E + T \mid E - T \mid T$$

 $T \rightarrow T^* \text{ int } \mid \text{ int}$

(non ambiguous grammar with no useless variables)

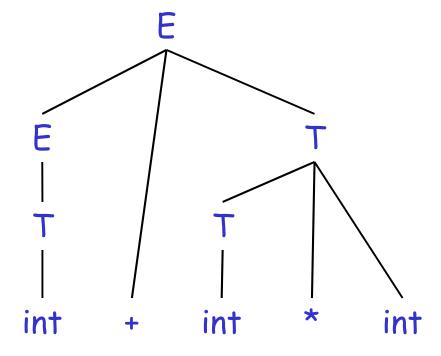
- · see, the grammar is
 - left-recursive
 - not left-factored
 - ... and our parser won't mind!

Gli algoritmi di parsing Bottom-Up (in particolare quelli LR) sono molto più potenti dei parser Top-Down (LL). I parser LR possono gestire grammatiche ricorsive a sinistra e molte grammatiche che non sono fattorizzate a sinistra senza necessità di riscriverle, rendendole ideali per la sintassi dei linguaggi di programmazione reali.

Example input, parse tree

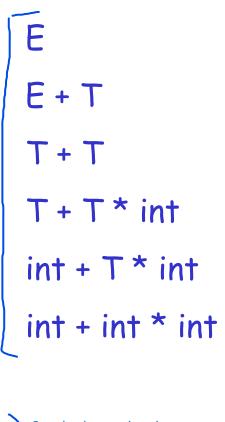
· input:

• its parse tree:

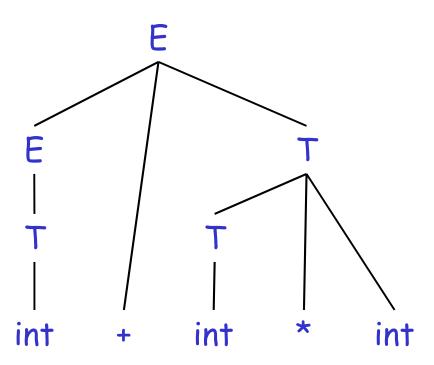


Key idea: build the derivation in reverse

(Riduzione da Corpo della Produzione)



Guardo al punto in cui sono arrivato ed applico una produzione al contrario (scelta a caso).



Algo: Guarda dove vuoi nella stringa (int + int * int). Trova il corpo di una produzione presente nella stringa. Riduci la stringa trovata alla testa. Continua fino a che ho il Parse Tree (cioè arrivo alla radice)

Chaotic bottom-up parsing

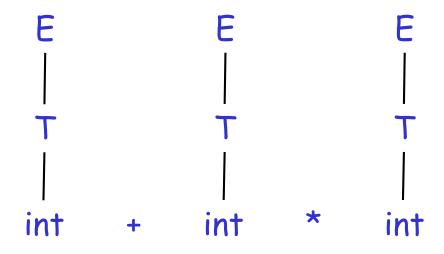
- The algorithm:
 - 1. stare at the input string s
 - · feel free to look anywhere in the string
 - 2. find in s a right-hand side r of a production $N \rightarrow r$ (trova una production $N \rightarrow r$) (tro
 - ex.: found int for a production $T\rightarrow$ int
 - 3. reduce the found string r into its non-terminal N (Riduzione)
 - ex.: replace int with T
 - 4. if all string reduced to start non-terminal
 - we're done, string is parsed, we got a parse tree
 - 5. otherwise continue in step 1

Don't celebrate yet! NON SEMPRE FUNZIONA L'ALGORITMO (funziona solo se si ha fortuna di scegliere la giusta produzione da ridurre)

- not guaranteed to parse a correct string

- is this surprising?
- example:

and we are stuck



Lesson from chaotic parser

· Lesson:

- if you're <u>lucky</u> in selecting the string to reduce next, then you will successfully parse the string

How to "beat the odds"?

- that is, how to find a lucky sequence of reductions that gives us a derivation of the input string?
- use non-determinism! (Prova tutte le possibili combinazioni di Riduzioni)



Non-deterministic chaotic parser Inefficiente

The algorithm:

- 1. find in input all strings that can be reduced
 - assume there are k of them (cioé su quella stringa attuale, posso effettuare k riduzioni diverse)
- 2. create k copies of the (partially reduced) input
 - it's like spawning k identical instances of the parser
- 3. in each instance, perform one of k reductions
 - and then go to step 1, advancing and further spawning all parser instances
- 4. stop when at least one parser instance reduced the string to start non-terminal (Termina quando una copia trova il Parse Tree)

Properties of the n.d. chaotic parser

Claim:

- the input will be parsed by (at least) one parser instance

But:

¹ exponential blowup: k*k*k*...*k parser copies

Also:

² Multiple (usually many) instances of the parser produce the correct parse tree (due to multiple corresponding derivations). This is wasteful.

Overview

- 1. Chaotic bottom-up parser
 - it will give us the parse tree, but only if it's lucky
- 2. Non-deterministic chaotic parser
 - creates many parser instances to make sure at least one builds the parse tree for the string
 - an instance either builds the parse tree or gets stuck
- 3. Non-deterministic LR parser (next)
 - restrict where a reduction can be made
 - as a result, fewer instances necessary

Non-deterministic LR parser

- → Vogliamo avere un unico successo e la possibilità di clonare i parser (mantenere il non-determinismo)
 - What we want:
 - create multiple parser instances
 - to find the lucky sequence of reductions
 - 1:2

but the parse tree is found by at most one instance

zero if the input has syntax error

Deve risolvere i due problemi dell'algoritmo precedente (Esplosione delle copie del parser e Multiple soluzioni)

Two simple rules to restrict # of instances

Shift (left to right) and Reduce (right to left)

1. split the input in two parts:

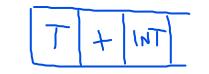
Divido l'input in due parti: il parser esamina solo la parte a sinistra della freccia e la riduzione ammessa é quella vicina al punto di divisione delle due parti

- right: unexamined by parser
- left: in the parser (we'll do the reductions here)

- Derivazione canonica a destra (quindi l'algoritmo trova un unica derivazione, se ha successo)

 reductions allowed only on part adjacent to split
- allowed: $T + \underline{int} \rightarrow * int$ after reduction: $T + \underline{T} \rightarrow * int$ not allowed: $\underline{int} + int \rightarrow * int$ after reduction: $\underline{T} + int \rightarrow * int$

hence, left part of string can be kept on the stack



Si può vedere come uno stack con cima a dx per capire quali riduzioni sono ammesse

Wait a minute!

Queste restrizioni non sono fatalmente severe?

Saren't these restrictions fatally severe?

nessuna istanza riesce

nessuna istanza riesce ad analizzare l'input the doubt: no instance succeeds to parse the input

No. Recall:

one parse tree corresponds to multiple derivations

- in n.d. chaotic parser, the instances that build the same parse tree each follow a different derivation

Le caratteristiche del parsing LR non impediscono di trovare il Parse Tree quando l'input è valido, perché:

- Se la stringa è valida, esiste un Parse Tree unico.
- Un Parse Tree unico implica una sola derivazione canonica a destra.
- L'algoritmo LR è progettato per trovare l'inverso di questa derivazione canonica a destra unica attraverso le sue operazioni di Shift e Reduce.

Wait a minute! (cont)

recall: two interesting derivations

- left-most derivation, right-most derivation

LR parser builds right-most derivation

- but does so in reverse: first step of derivation is the last reduction (the reduction to start nonterminal)
- example coming in two slides

hence the name:

- L: scan input left to right
- R: right-most derivation

so, if there is a parse tree, LR parser will build it!

- this is the key theorem

Se la stringa di input appartiene al linguaggio definito dalla grammatica (cioè, se esiste un albero di parsing per quella stringa). Allora, l'algoritmo LR è garantito per trovare la corretta sequenza di Shift/Reduce che corrisponde all'inverso della derivazione più a destra, e quindi accettare l'input.

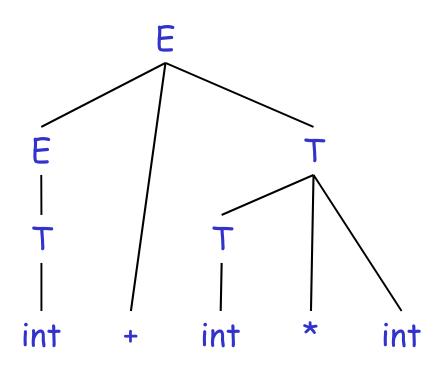
LR parser actions

- The left part of the string will be on the stack
 - the > symbol is the top of stack
- Two simple actions
 - reduce:
 - like in chaotic parser,
 - but must replace a string on top of stack
 - shift:
 - shifts > to the right,
 - which moves a new token from input onto stack, potentially enabling different reductions
- These actions will be chosen non-deterministically

Example of a correct LR parser sequence

A "lucky" sequence of shift/reduce actions (string parsed!):

```
E
E+T>
E + T * int >
E + T * ▶ int
E + T ▶ * int
E + int > * int
E + ▶ int * int
E + int * int
T → + int * int
int > + int * int
int + int * int
```



Example of an incorrect LR parser sequence

```
will be stuck after reduction to T + E!
why can't we reduce to E + T, instead?
T + T
T + T * int >
T + T * ▶ int
T + T > * int
T + int \rightarrow * int
T + \rightarrow int * int
T \rightarrow + int * int
int > + int * int
                                   int
                                                   int
                                                                   int
int + int * int
```

Where did the parser instance make the mistake?

C'é sempre una derivazione corretta (a meno che ci siano degli errori sintattici) ed é una derivazione canonica a destra (il non determinismo permette di avere al massimo un successo in una istanza del parser e le altre istanze diventano bloccanti/fallite)



Non-deterministic LR parser

The algorithm: (compare with chaotic n.d. parser)

Per gestire le scelte multiple, il parser si clona. Vengono create k nuove copie del parser, ognuna con lo stesso contenuto dello stack e lo stesso

puntatore all'input.

1. find all reductions allowed on top of stack

assume there are k of them

Il parser esamina la cima del suo stack. Se i simboli in cima allo stack corrispondono al corpo di k diverse regole di produzione della grammatica, quelle k riduzioni sono possibili in quel momento.

~2. create k new identical instances of the parser

in each instance, perform one of the k reductions; in original instance, do no reduction, shift instead

(In ogni istanza, esegue una delle k riduzioni; nell'istanza originale, non esegue riduzioni, esegue invece uno shift)

and go to step 1

4. stop when a parser instance reduced the string to start non-terminal

- Istanze 1 a k (Reduce): Ogni nuova istanza esegue una delle k possibili riduzioni.

- Istanza Originale (Shift): L'istanza da cui sono partite le clonazioni non esegue riduzioni. Tenta invece l'operazione di Shift (sposta il prossimo simbolo di input sullo stack). Questo passaggio copre tutti i possibili conflitti che possono sorgere nel parsing Bottom-Up:
- > Conflitto Shift/Reduce: Un simbolo permette sia una riduzione che uno shift. L'algoritmo crea due percorsi per esplorarle entrambe.
- > Conflitto Reduce/Reduce: Più riduzioni sono possibili contemporaneamente. L'algoritmo crea un percorso per ognuna.
- Tutte le istanze (le k nuove ridotte e l'originale shiftata) ripetono l'intero processo tornando al Passo 1.
- Il parsing ha successo quando una singola istanza è riuscita a eseguire tutte le riduzioni necessarie, fino a che l'intero input è rappresentato dal solo simbolo iniziale sullo stack e l'input è esaurito.

Overview

- 7
- Chaotic bottom-up parser
 - tries one derivation (in reverse)
- 2 · Non-deterministic chaotic parser
 - tries all ways to build the parse tree
- Non-deterministic LR parser
 - restricts where a reduction can be made
 - as a result,
 - only one instance succeeds (on an unambiguous grammar)
 - all others get stuck
- Generalized LR parser (next)
 - idea: kill off instances that are going to get stuck ASAP

Revisit the incorrect LR parser sequence

```
T+T>
T+T* int>
T+T* int
T+T* int
T+T > * int
T+ int * int
T+ int * int
T > + int * int
int > + int * int

int + int * int

int + int * int
```

Key question:

What was the earliest stack configuration where we could tell this instance was doomed to get stuck?

Doomed stack configurations

The parser made a mistake to shift to

rather than reducing to

The first configuration is doomed

- because the T will never appear on top of stack so that it can be reduced to E
- hence this instance of the parser can be killed (it will never produce a parse tree)

How to find doomed parser instances?

- Look at their stack!
- How to tell if a stack is doomed:
 - list all <u>legal</u> (non yet doomed) stack configurations
 - if a stack is not legal, kill the instance
- Listing legal stack configurations
 - list prefixes (stack content) of all right-most derivations
 - describe them as a DFA
 - if the stack configuration is not from the DFA, it's doomed

Our example grammar

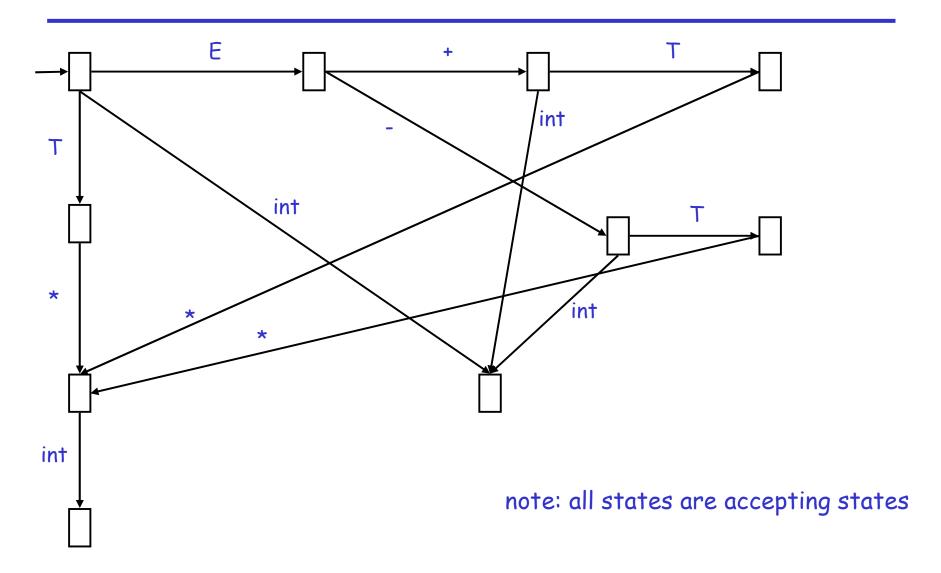
· we'll build a parser for this grammar:

$$E \rightarrow E + T \mid E - T \mid T$$

 $T \rightarrow T^* \text{ int } \mid \text{ int}$

- which are prefixes (stack content) of strings reached during right-most derivations?
 - E.g. those of length one.
 - · They are "E", "T" and "int"
 - Then think about those of length two, etc...

The stack-checking DFA



Simple LR parsing

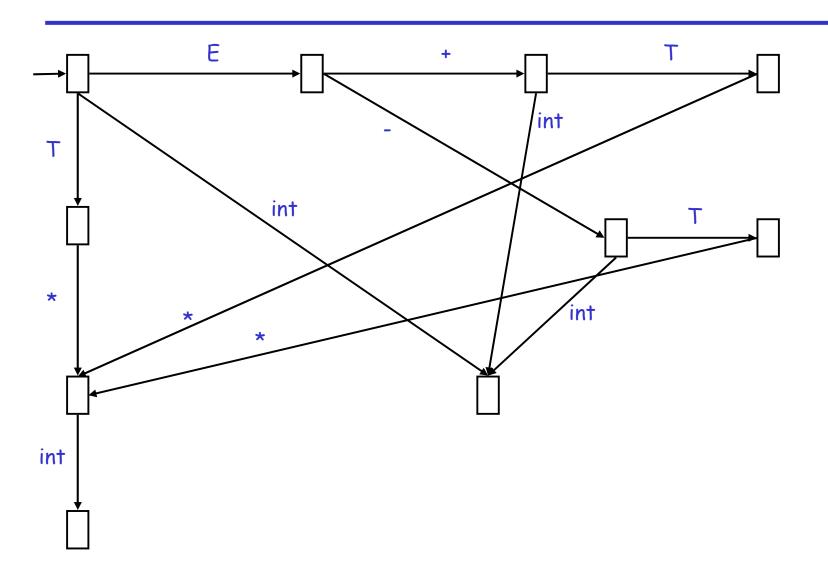
Introducing a new grammar initial variable

- In order to compute the stack checking DFA we preliminarily add to the grammar
 - a new initial variable E' and
 - a new production $E' \rightarrow E$ (with E being the old initial variable)
- Our example grammar thus becomes:

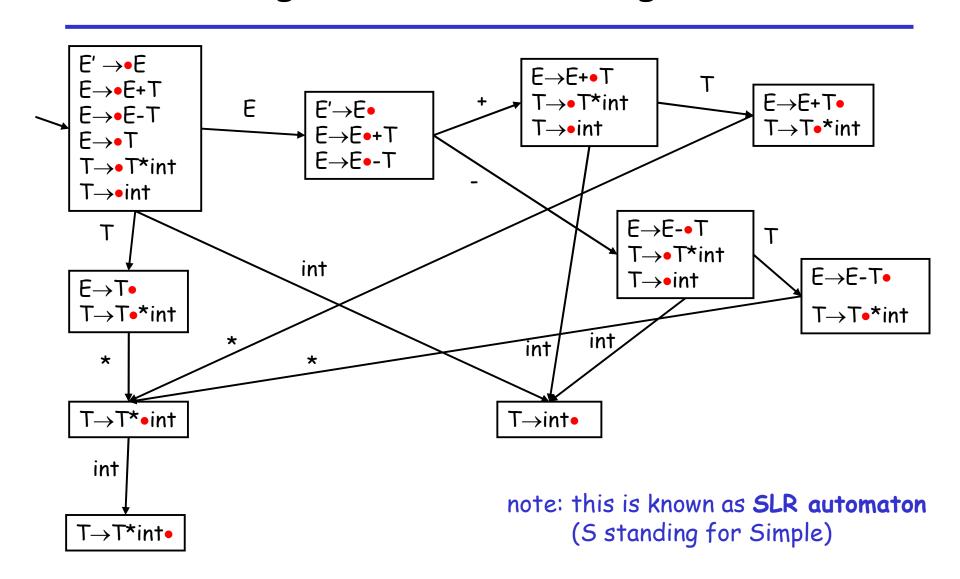
$$E' \rightarrow E$$

 $E \rightarrow E + T \mid E - T \mid T$
 $T \rightarrow T^* \text{ int } \mid \text{ int}$

The stack-checking DFA



Constructing the stack-checking DFA



SLR Items

· A SLR item has the form:

$$\begin{array}{c} \textbf{X} \to \alpha \bullet \beta \\ \text{with } \textbf{X} \to \alpha \beta \text{ being a production} \end{array}$$

- $X \rightarrow \alpha \bullet \beta$ describes context information
 - We are trying to find an X, and
 - We have α already on top of the stack
 - Thus we need to see next a string derived from β

DFA state: a set of SLR items

- A DFA state describes a parsing context:
 a set of SLR items representing possible
 productions to apply
 - We are trying to find $E \rightarrow E + \bullet T$
 - on the top of the stack we have E + already
 - Thus we are also trying to find $T \rightarrow \bullet$ T*int or $T \rightarrow \bullet$ int
 - · on the top of the stack we do not have anything yet

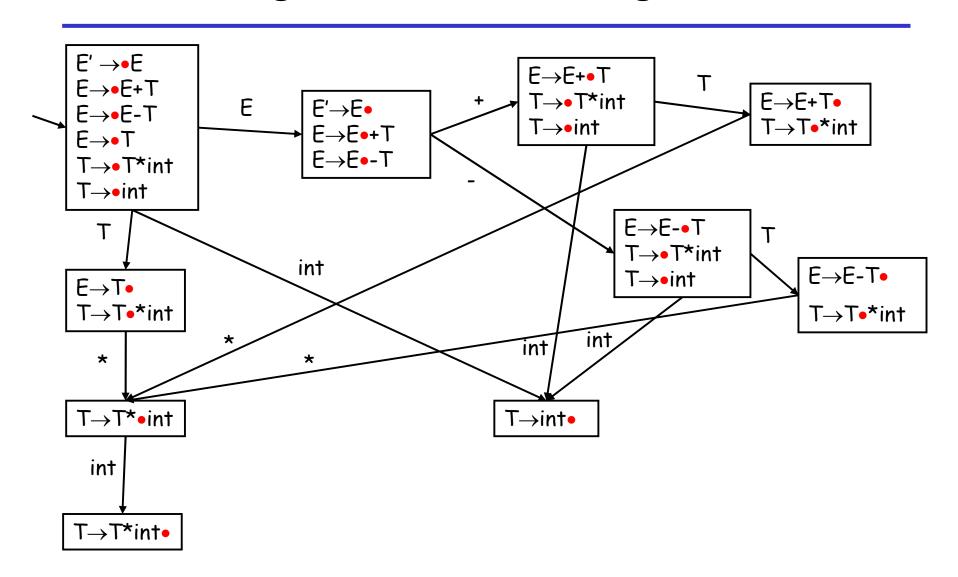
Note

- The symbol > was used before to separate the stack from the rest of input
 - α > γ , where α is the stack and γ is the remaining string of terminals
- In SLR items is used to mark a prefix of a production rhs:

$$X \to \alpha \bullet \beta$$

- Here β might contain non-terminals as well
- · In both cases the stack is on the left

Constructing the stack-checking DFA



Constructing the stack-checking DFA

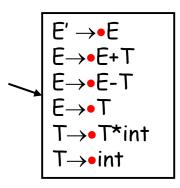
- A DFA state is a closed set of SLR(1) items
 - This means that we performed Closure
- The start state is Closure({E' → •E})

The Closure Operation

 The operation of extending the context with items is called the closure operation

```
Closure(Items) = repeat for each X \to \alpha \bullet Y\beta in Items for each production Y \to \gamma add Y \to \bullet \gamma to Items until Items is unchanged
```

Constructing the stack-checking DFA

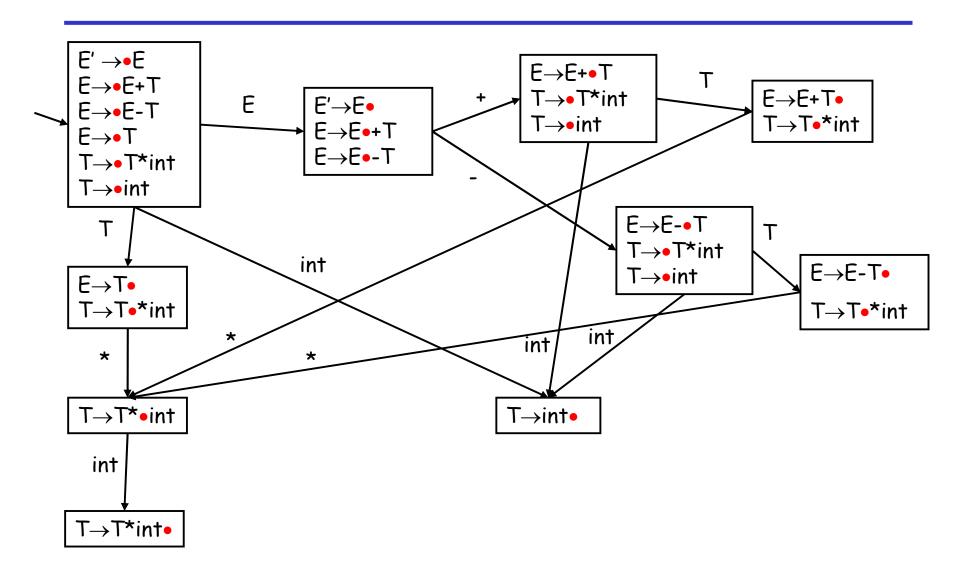


The DFA Transitions

- A state State that contains at least an item $X \to \alpha \bullet y \beta$ has a transition labeled y to a state that contains the items: Transition(State, y)
 - y can be a terminal or a non-terminal

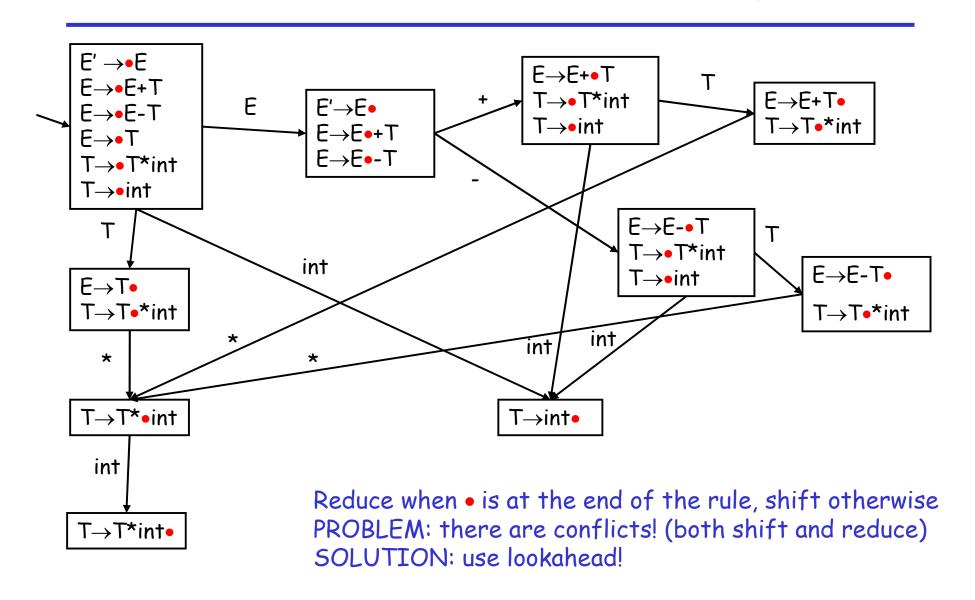
```
Transition(State, y) = Items \leftarrow \emptyset for each X \rightarrow \alpha \bullet y\beta \in State add X \rightarrow \alpha y \bullet \beta to Items return Closure(Items)
```

Constructing the stack-checking DFA



- The automaton indicates the correct stack configurations...
- ...but it also dictates when to do shift/reduce actions...

Shift/reduce based on stack-checking DFA

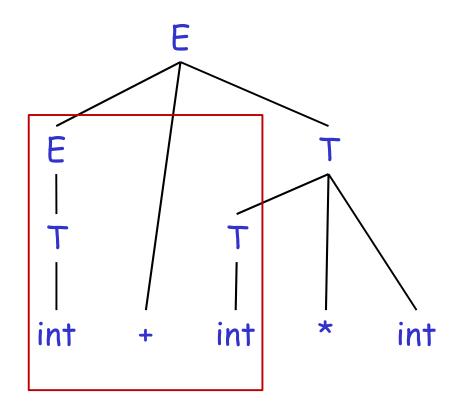


Example of a correct LR parser sequence

If we do not use lookahead ("*") we still have non-determinism! The lookahead, instead, tells that here we cannot reduce E+T to E

because it would cause E to be followed by "*" and we know * ∉ Follow(E)

```
E
E+T>
E + T * int >
E + T * ▶ int
E + T ▶ * int
E + int → * int
E + ▶ int * int
E ▶ + int * int
T + int * int
int + int * int
int + int * int
```

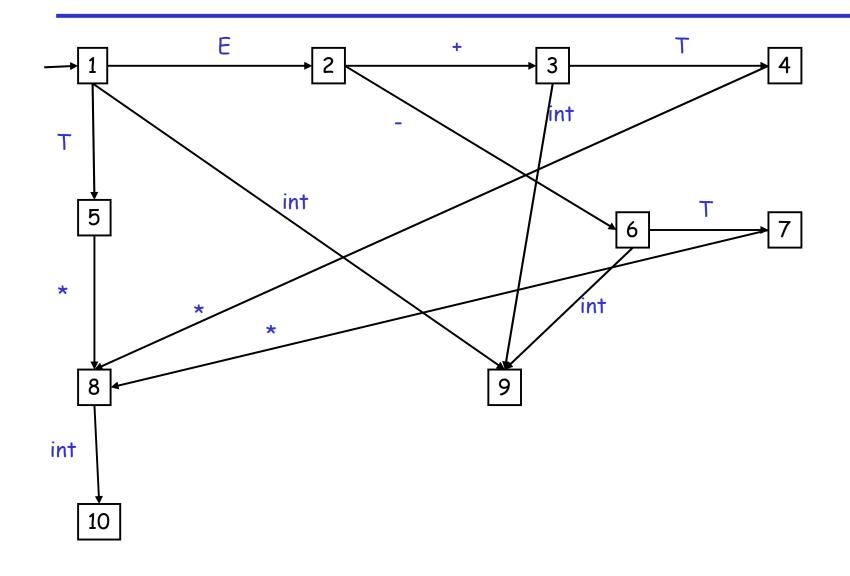


- · Lookahead for "shift":
 - "Shift" only on the expected terminals
- · Lookahead for "reduce":
 - "Reduce" only on terminals in the Follow set of the variable on the l.h.s. of the rule used to reduce

Computation of first and follow sets

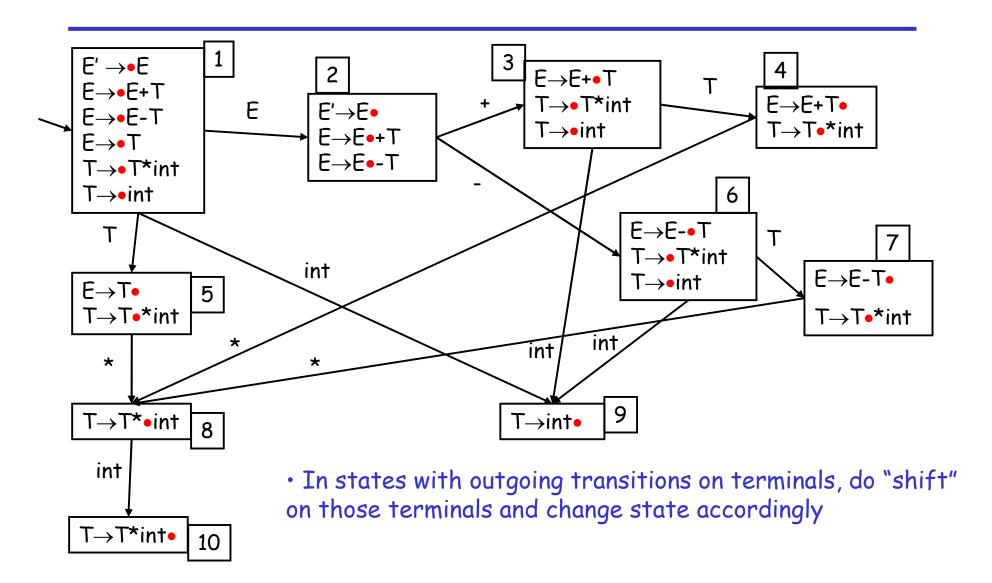
· Consider our example grammar

```
E' \rightarrow E
   E \rightarrow E + T \mid E - T \mid T
   T \rightarrow T^* \text{ int } | \text{ int }
First(E')={int} (no need to calculate it)
First(E)={int}
First(T)={int}
Follow(E')=\{\$\} (no need to calculate it, always \{\$\})
Follow(E)={$,+,-}
Follow(T)={$,+,-,*}
```



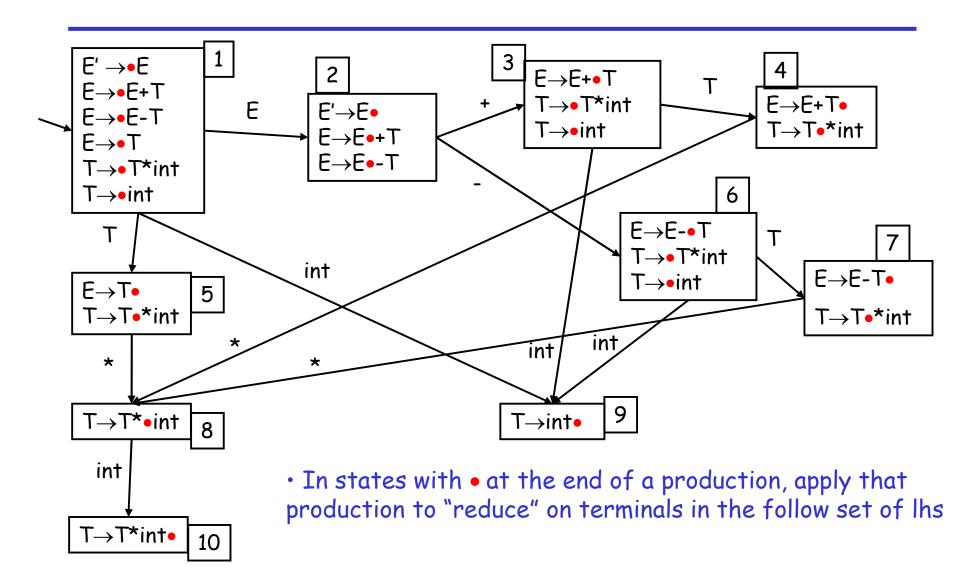
How to use the SLR automaton (tabular representation)

	int	+	-	*	E	T
1	9				2	5
2		3	6			
3	9					4
4				8		
5				8		
6	9					7
7				8		
8	10					
9						
10						



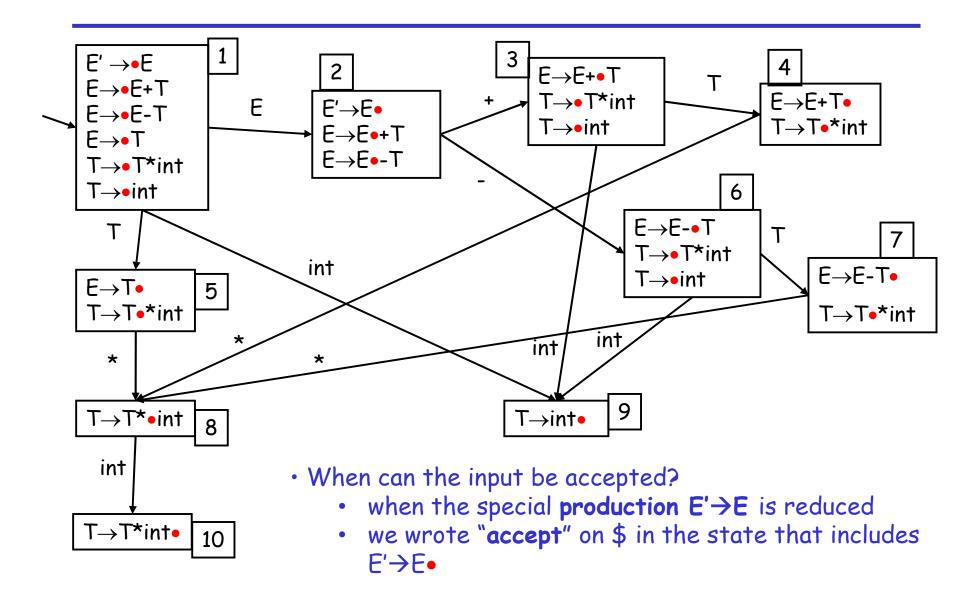
How to use the SLR automaton (tabular representation)

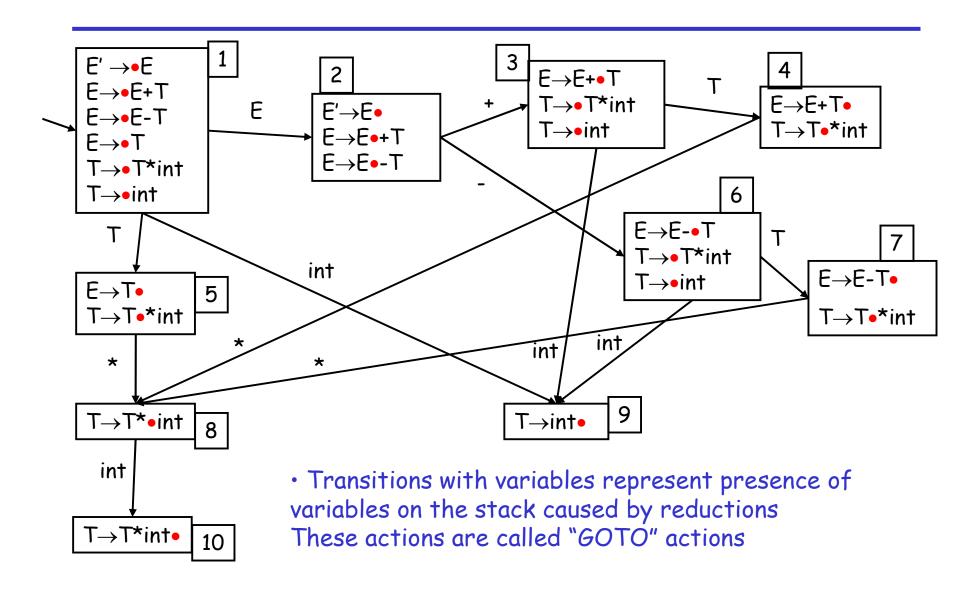
	int	+	-	*	Ε	T
1	shift,9				2	5
2		shift,3	shift,6			
3	shift,9					4
4				shift,8		
5				shift,8		
6	shift,9					7
7				shift,8		
8	shift,10					
9						
10						



How to use the SLR automaton (tabular representation, \$ column added)

	int	+	-	*	\$	E	Т
1	shift,9					2	5
2		shift,3	shift,6		accept		
3	shift,9						4
4		red,E→E+T	red,E→E+T	shift,8	red,E→E+T		
5		red, $E \rightarrow T$	red, $E \rightarrow T$	shift,8	red, $E \rightarrow T$		
6	shift,9						7
7		red,E→E-T	red,E→E-T	shift,8	red,E→E-T		
8	shift,10						
9		red,T→int	red,T→int	red,T→int	red,T→int		
10		red, T→T*int	red, T→T*int	red, $T \rightarrow T^*$ int	red, T→T*int		





How to use the SLR automaton (complete tabular representation)

	int	+	-	*	\$	E	Т
1	shift,9					goto,2	goto,5
2		shift,3	shift,6		accept		
3	shift,9						goto,4
4		red,E→E+T	red,E→E+T	shift,8	red,E→E+T		
5		red, $E \rightarrow T$	red, $E \rightarrow T$	shift,8	red, $E \rightarrow T$		
6	shift,9						goto,7
7		red,E→E-T	red,E→E-T	shift,8	red,E→E-T		
8	shift,10						
9		red,T→int	red,T→int	red,T→int	red,T→int		
10		red, $T \rightarrow T^*$ int	red, T→T*int	red, $T \rightarrow T^*$ int	red, T→T*int		

- The LR predictive algorithm governed by this table is called SLR(1) parsing algorithm
 - Note that it works only if each cell of the table contains at most one action! (i.e. no conflict)
 - In this case, the grammar is a SLR(1) grammar

Bottom-up parsing algorithm

Remember the idea of LR parsing

- LR parsing reduces a string to the start symbol by inverting productions:
- str such that str \rightarrow * input string of terminals while str \neq S:
 - Identify β in str such that $A \to \beta$ is a production and $S \to^* \alpha$ $A \gamma \to \alpha$ $\beta \gamma = str$
 - Replace β by A in str (so α A γ becomes new str)
- Stronger than top-down parsing!
 - make decisions after seeing all symbols β rather than before (LL(1) make decisions knowing at most one of those symbols, the lookahead)

Bottom-up parsing algorithm

- · Add \$ at the end of the input
- Execute the bottom-up parsing algorithm using an automaton (e.g. the SLR automaton) to decide whether to shift or reduce
 - Every time a decision should be taken, use the stack as input for the automaton
 - The automaton will communicate whether to shift or reduce (based on the lookahead)

A Running Example

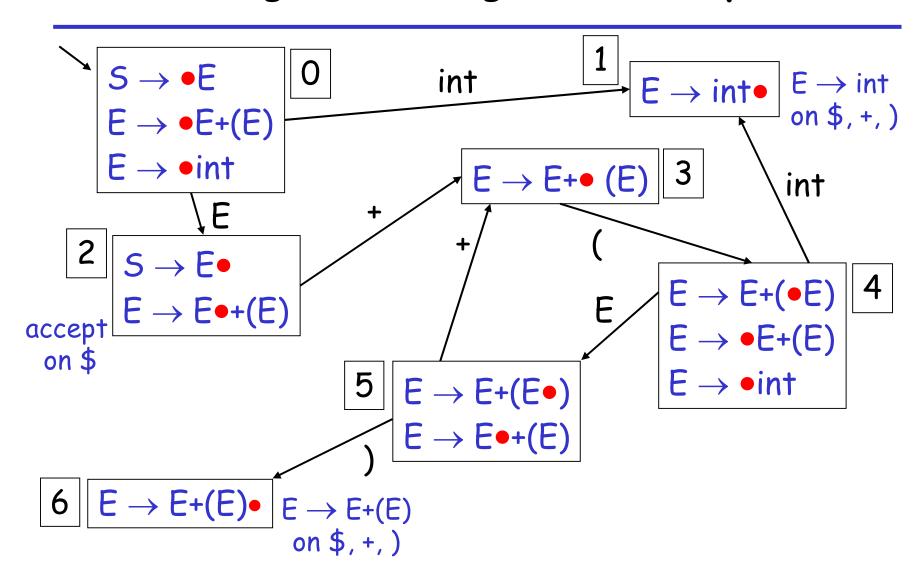
- We will study the bottom-up parsing algorithm considering
 - The grammar:

$$E \rightarrow E + (E) \mid int$$

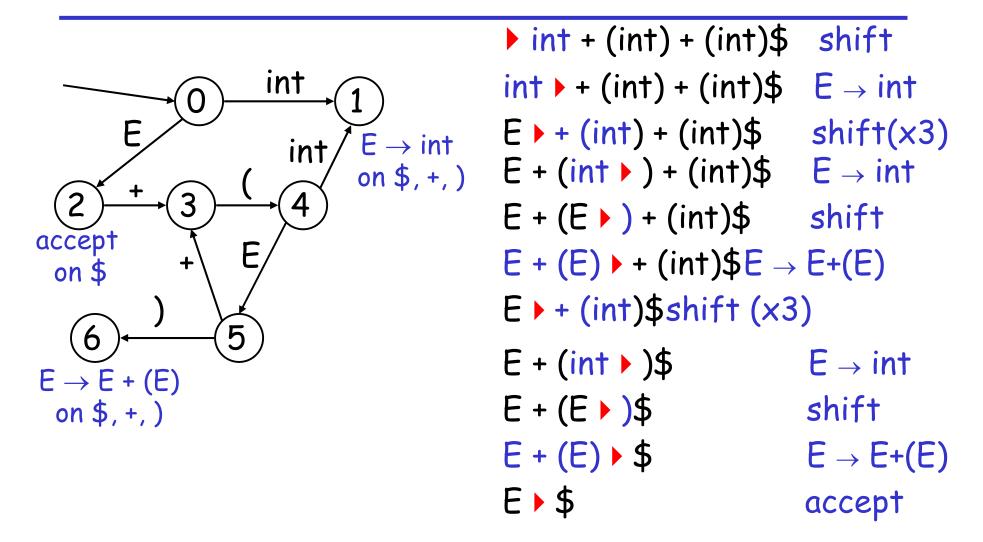
- An automaton/parsing table for such a grammar, i.e. its SLR(1) automaton

- First(E) = {int}
- Follow(E) = $\{\$,+,\}$

Constructing the Parsing DFA. Example.



Bottom-up parsing algorithm example

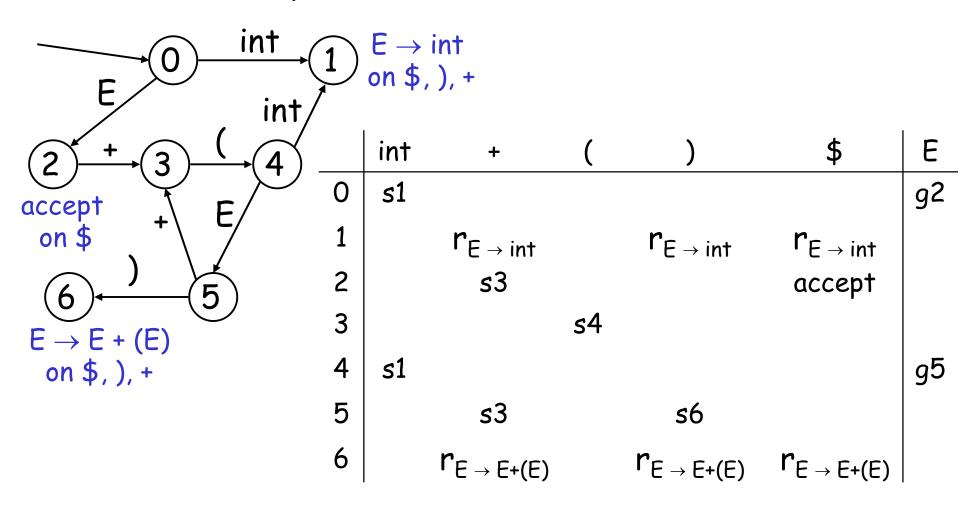


DFA table representation is actually used

- Parsers represent the DFA as a 2D table
 - As for table-driven lexical analysis
- Lines correspond to DFA states
- Columns correspond to terminals and nonterminals
- In classical treatments, columns are split into:
 - Those for terminals: action table
 - Those for non-terminals: goto table

Representing the DFA. Example

The table for our DFA:



The Bottom-up Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
 - This is wasteful, since most of the work is repeated
- So record, for each stack element, state of the DFA after that element
- Parser maintains a stack

```
\langle sym_1, state_1 \rangle \dots \langle sym_n, state_n \rangle state, is the final state of the DFA on sym_1 \dots sym_k
```

The Bottom-up Parsing Algorithm

```
Let I = w_1 w_2 ... w_n $ be initial input
Let j = 1
Let DFA state 0 be the start state
Let stack = \( \dummy, 0 \)
   repeat
         case action[ top_state(stack), I[j] ] of
                  shift k: push \langle I[j], k \rangle; j += 1
                  reduce X \rightarrow \alpha:
                      pop |\alpha| pairs,
                      push (X, goto[top_state(stack), X])
                  accept: halt normally
                  error: halt and report error
```

bison: a parser generator

- bison generates parsers building the LALR(1) automaton/table
 - slightly more complex variant of SLR(1) such that a conflict-free table is generated for more grammars
- bison input:
 - file with token description, priority and associativity rules, grammar rules, and some auxiliary C intructions
- Bison generates two files in output:
 - a C program containing the code for the parser
 - the LALR(1) table

bison: structure of bison input

```
%{
    C declarations
%}
    Bison declarations
%%
    Grammar rules
%%
    Programs
```

bison: example of input

```
%{
#include <stdio.h>
int yylex();
void yyerror(char *s){ printf("parser error") ; }
%}
%token ID WHILE BEGIN END DO IF THEN ELSE SEMI ASSIGN
%start prog
%%
          : stmlist;
prog
stmlist
           : stm
           | stmlist SEMI stm;
           : ID ASSIGN ID
stm
           | WHILE ID DO stm
           | BEGIN stmlist END
           | IF ID THEN stm
           | IF ID THEN stm ELSE stm;
```

bison: example of output

In the generated table we read:

```
state 18 contains 1 shift/reduce conflict.

state 18

stm -> IF ID THEN stm . (rule 7)

stm -> IF ID THEN stm . ELSE stm (rule 8)

ELSE shift, and go to state 19

ELSE [reduce using rule 7 (stm)]
```

- If there are no explicit precedence declarations, BISON follows the following:
 - In shift/reduce conflicts the shift is executed
 - In reduce/reduce conflicts the first rule is applied