Description of Pairs Trading Strategy

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1. Methodology

The methodology of pairs trading strategy is mainly related with two points: 1) How to trigger a long/short position based on pairs trading strategy in each stock and 2) How to evaluate the performance of the trading signals. Such topics are covered next.

1.1 Pairs Selection

In the pairs formation phase, the idea is to bring all prices from the different assets to a particular unit and, after that, search two stocks that move together. Quantitatively speaking, this can be done in different ways. The approach used on this paper is the minimum absolute distance rule, meaning that, for each stock, will be searched a corresponding pair that offers the minimum absolute distance between the normalized price series.

The reason for the unit transformation is simple, each stock has its own unit. The use of original prices (without normalization) would be a problem for the case of the distance rule since two stocks can move together but still have a high absolute distance between them. After the normalization, all stocks are brought to the same standard unit and this permits a quantitatively fair formation of pairs.

The transformation employed is the normalization of the price series based on its mean and standard deviation, Equation [1].

$$P_{it}^{*} = \frac{P_{it} - E(P_{it})}{\sigma_i} \qquad [1]$$

The value of P_{it}^{*} is the normalized price of asset *i* at time *t*, $E(P_{it})$ is just the expectation of P_{it}, in this case the average, and σ_i is the standard deviation of the respective stock price, where both indexes are calculated within a particular moving window of the time series. With the use of Equation [1] all prices are going to be transformed to the same normalized unit, which will permit the use of the minimum abslute distance rule.

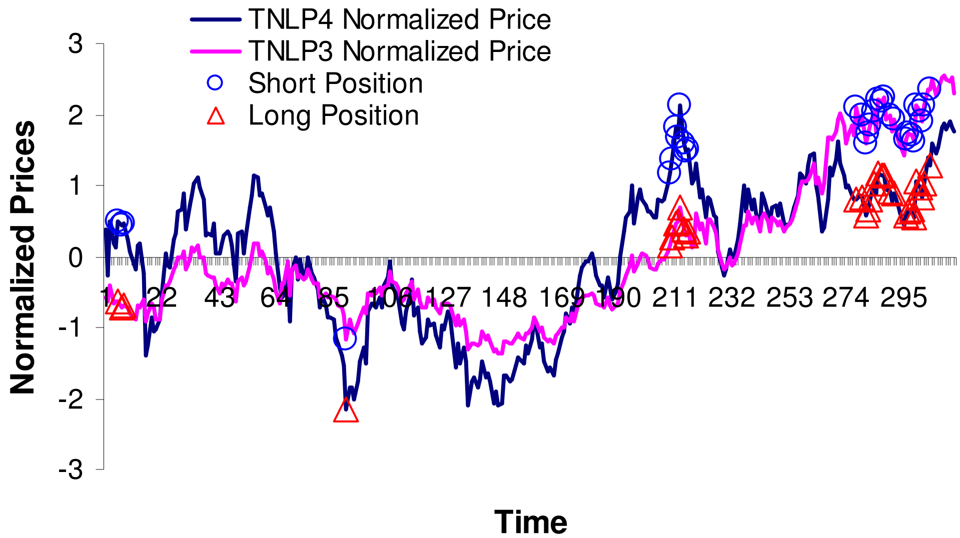
The next step is to choose, for each stock, a pair that has minimum absolute distance between the normalized prices. The normalized price for the pair of asset *i* is now addressed as p_{it}^{*}. After the pair of each stock is identified, the trading rule is going to create a trading signal every time that the distance between P_{it}^{*} and p_{it}^{*} is higher than *d*. The value of *d* is arbitrary, and it represents the filter for the creation of a trading signal. It can't be too high, otherwise only a few trading signal are going to be created and it can't be to low or the rule is going to be too flexible and it will result in too many trades and, consequently, high value of transaction costs.

After a trading sign is created, the next step is to define the positions taken on the stocks. According to the pairs trading strategy, if the value of P_{it}^{*} is higher (lower) than p_{it}^{*} then a short (long) position is kept for asset *i* and a long (short) position is made for the pair of asset *i*. Such positions are kept until the absolute difference between the normalized prices is lower than *d*.

The main logic behind the expected profits of pairs trading strategy is: if the movements between the pairs are going to continue in the future then, when the distance between an asset and its pair is higher than a particular threshold value (*d*), there is a good possibility that such prices are going to converge in the future, and this can be explored for profit purposes. If the distance is positive, then the value of P_{it}^{*}, according to the logic expressed earlier, probably will reduce in the future (short position for asset *i*) and the value of p_{it}^{*} is probably going to increase (long position for the pair of *i*). The same logic is true for the cases where the distance is negative. The situations where pairs trading fails to achieve profit are: a increase in the distance between P_{it}^{*} and p_{it}^{*}, where the market goes the opposite way of the expectation and also a decrease (increase) on the price of the long (short) position.

As an example, Figure 1 shows the pairs trading strategy for weekly prices of asset TNLP4 and its pair, TNLP3.

Figure 1 – Example of Pairs Trading with TNLP4 and TNLP3 with *d=1*



In Figure 1, TNLP3 is the pair of TNLP4 based on the minimum squared distance criteria. It's possible to see that both normalized prices have a similar behavior. On the points that have a blue circle or red triangle the absolute difference in the normalized prices have crossed the value of *d*, meaning that a trade has taken place. The blue circles (red triangles) are the short (long) positions created. This happens every time the absolute distance is higher than 1 and the value of the analyzed asset is higher (lower) than it's pair. Every time the absolute difference uncrosses the value of *d*, the positions are closed. If the assets, after the opening of a position, move back to the historical relationship, then the one with the higher price should have a decrease in the prices and the one with the lower price should have an increase. Since a short position was made for the first asset and a long position for the second, then, if both prices behave historically, a profit will arise from this trading case, and that's the whole idea behind pairs trading, making profits out of market corrections towards the average behavior.

1.2 Assessing Performance of the Strategy

One of the concerns of this study is to evaluate the performance of pairs trading strategy against a naïve approach. For that purpose, two methods are employed here. The first is the computation of the excessive return of the strategy over a properly weighted portfolio and the second is the use of bootstrap methods for evaluating the performance of the trading rule against the use of random long and short signals for each stock.

1.2.1 Calculation of Strategy's Returns

The calculation of the strategy total return is going to be executed according to the next formula, Equation [2].

$$R_E = \sum_{i=1}^{n}\left[\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}\right] + \left(\sum_{i=1}^{n}\left[\sum_{t=1}^{T} Tc_{it}\right]\right)\left[\ln\left(\frac{1-C}{1+C}\right)\right] \qquad [2]$$

Where:

R_{it} Real return of asset *i* on time *t*, calculated by $\ln\left(\frac{P_{it}}{P_{it-1}}\right)$;

$I_{it}^{L\&S}$ Dummy variable that takes value 1 if a long position is created for asset *i* at time *t*, value -1 if a short position is created and 0 otherwise. When a long position is made, this variable is going to be addressed as I_{it}^{L} and as I_{it}^{S} for short positions.

Tc_{it} Dummy variable that takes value 1 if a transaction is made for asset *i* on time *t* and zero otherwise.[1]

C Transaction cost per operation (in percentage);

T Number of observations on the whole trading period.

For Equation [2], the basic idea is to calculate the returns from the strategy accounting for transaction costs. The first part of [1], $\sum_{i=1}^{n}\left[\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}\right]$, calculates the total raw return of the strategy. Every time a long and short position is created for asset *i*, the raw return of the strategy on time *t*, for asset *i*, is R_{it}. Since *t* goes from 1 to T, is necessary to sum such returns, which creates $\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}$ (total return for asset *i*). Because of the fact that there are

[1] It's important to distinguish the values of $I_{it}^{L\&S}$ (long and short positions) from Tc_{it} (transaction dummy). The values of Tc_{it} are derived from the vector $I_{it}^{L\&S}$, but they are not equal. For example, suppose a long position is created for asset *i* on time t-1 and also on time *t,* only. The vector of I_{it}^{L} is going to have values of 1 to time *t-1* and *t*, but the vector of Tc_{it} has only value 1 for time *t-1*, since for *t*, the asset was already in the portfolio, so there is no need to buy it again. The same is true for short positions.

more than just one asset, is necessary to sum all total returns from the stocks, which gives the final result for the first part of [2], $\sum_{i=1}^{n}\left[\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}\right]$.

The second part of Equation [2] has the objective of accounting for transaction costs. For example, suppose that the trading cost of buy and selling one stock is *C*, which is expressed as a percentage of the transaction price. If such stock is purchased at price P_B and sold at price P_S, then the real buy and sell prices, including transaction costs, are $P_B(1+C)$ and $P_S(1-C)$. Taking the logarithm return of the operation results on the formula $R = \ln\left(\frac{P_S(1-C)}{P_B(1+C)}\right)$. Using logarithm properties, the previous equation becomes $R = \ln\left(\frac{P_S}{P_B}\right) + \ln\left(\frac{1-C}{1+C}\right)$. It's possible to see from this result that the return for this operation has two separate components, the logarithm return from difference between selling and buying price and also the term $\ln\left(\frac{1-C}{1+C}\right)$, which accounts for the transaction cost on the whole operation. This exemplified result basically states that the transaction cost for one operation of buying and selling one stock is $\ln\left(\frac{1-C}{1+C}\right)$.

Returning to the analysis of the second part of Equation [2], since $\ln\left(\frac{1-C}{1+C}\right)$ is the transaction cost of one operation, logically the term $\left(\sum_{i=1}^{n}\left[\sum_{t=1}^{T} Tc_{it}\right]\right)$ is just the number of operations made by the trading strategy. It's important to notes that, since $\frac{1-C}{1+C}$ is always less than one because C is always positive and higher than zero, then the value of $\ln\left(\frac{1-C}{1+C}\right)$ is always negative, meaning that the transaction costs are going to be subtracted from the strategy' returns, which is a intuitive result.

1.2.2 Evaluation of Strategy' Returns

In order to evaluate the performance of the strategy, is necessary to compare it to a naïve approach at investing. If the strategy performs significantly better than an out-of-skill investor, then such trading rule has value. This is the main idea that will conduct both methods used in this research to evaluate the performance of pairs trading strategy for Brazilian financial market. The approaches used here are computation of excessive return over a naïve buy&hold rule and the more sophisticated bootstrap method of random trading signals.

1.2.2.1 Computation of Excessive Return over a Benchmark Portfolio

The calculation of excessive return is the simplest approach to evaluate a trading strategy. The idea is quite simple: verify how much of returns does the strategy tested exceeds a naïve rule. In this case, the naïve rule is the buy&hold of a properly weighted portfolio for comparison with the long positions and a "sell&unhold" for the short positions.

The return of the naïve approach is based on the following formula, Equation [3].

$$R_{NE} = \sum_{i=1}^{n}\left[P_i^L \sum_{t=1}^{T} R_{it}\right] + \sum_{i=1}^{n}\left[P_i^S \sum_{t=1}^{T} R_{it}\right] + 2n\ln\left(\frac{1-C}{1+C}\right) \qquad [3]$$

For Equation [3], the value of P_i^L and P_i^S is just the proportion of days, related to the whole trading period, that the strategy created long and short positions for asset *i*. Formally, $P_i^L = \frac{\sum_{t=1}^{T} I_{it}^L}{T}$ and $P_i^S = \frac{\sum_{t=1}^{T} I_{it}^S}{T}$. Notes that, in the calculation of P_t^S, the sum of the short positions is always negative or equal to zero, since I_{it}^S takes values -1 and 0, only.

Since pairs trading strategy uses two different types of position in the stock market, long for the hope of a price increase and short for the hope of a price decrease, it's necessary to construct a naïve portfolios that also takes use of such positions. This is the function of the terms $\sum_{i=1}^{n}\left[P_i^L \sum_{t=1}^{T} R_{it}\right]$ and $\sum_{i=1}^{n}\left[P_i^S \sum_{t=1}^{T} R_{it}\right]$, where the first simulates a buy&hold (long positions) of a properly weighted portfolio and the second simulates a "sell&unhold" (short position) scheme for another properly weighted portfolio. The weights in both terms are derived from the number of long and short positions taken on each asset, as was showed before. The higher the number of long and short signals a strategy makes for asset *i*, higher the weight that such stock will have on the simulated portfolio. It's clear to see from Equation [3] that, if $P_i^S = P_i^L$, which is a perfectly hedged position for asset *i* in the benchmark portfolio, the terms $\sum_{i=1}^{n}\left[P_i^L \sum_{t=1}^{T} R_{it}\right]$ and $\sum_{i=1}^{n}\left[P_i^S \sum_{t=1}^{T} R_{it}\right]$ nulls each other and the accumulated return for this respective asset in the benchmark portfolio is zero.

As can be seen from Equation [3], one of the premises of the research is that the transaction cost per operation is the same for long and short positions. The last term of [3] is the transaction costs for opening positions (making the portfolio) and trade them at the end of the period. In this case, the number of trades required to form and close the two portfolios is $2n$, where n is the number of researched assets.

The excessive return for the strategy is given by the difference between [2] and [3], which forms the final formula for computing excessive return, Equation [4].

$$R_E^* = \sum_{i=1}^{n}\left[\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}\right] - \sum_{i=1}^{n}\left[P_i^L \sum_{t=1}^{T} R_{it}\right] - \sum_{i=1}^{n}\left[P_i^S \sum_{t=1}^{T} R_{it}\right] + \left(\sum_{i=1}^{n}\left[\sum_{t=1}^{T} Tc_{it}\right] - 2n\right)\left[\ln\left(\frac{1-C}{1+C}\right)\right] \quad [4]$$

Analyzing Equation [4], the maximization of R_E^*, which is the objective of any trading strategy, is given by the maximization of $\sum_{i=1}^{n}\left[\sum_{t=1}^{T} R_{it} I_{it}^{L\&S}\right]$, minimization of $\sum_{i=1}^{n}\left[P_i^L \sum_{t=1}^{T} R_{it}\right]$ and $\sum_{i=1}^{n}\left[P_i^S \sum_{t=1}^{T} R_{it}\right]$ and also minimization of $\left(\sum_{i=1}^{n}\left[\sum_{t=1}^{T} Tc_{it}\right] - 2n\right)$, since $\left[\ln\left(\frac{1-C}{1+C}\right)\right]$ is a constant. The conclusion about this analysis is intuitive because the strategy is only going to be successful if it efficiently creates long and short positions on the stocks, keeping the transaction costs and the benchmark returns at low values. Short story, make more with money less trades.