Gravity and Seismology

Earth's Gravity

Precise measurements of variations in Earth's gravity field are used in the geosciences to map geology and geologic processes on a range of spatial scales. Gravity measurements are sensitive to lateral variations in rock density and so gravity surveys can be used to study the distribution of mass within Earth. One of the earliest uses of gravity measurements was for prospecting for ore bodies, where metal oxide and sulphide minerals that are much denser than their host rocks create excess gravity that can be measured with highly sensitive gravimeter instruments. Gravity can be measured on the ground with small sensors, but also can be made with airborne and satellite systems, allowing for the study of mass anomalies arising from features as small as hidden tunnels and ore bodies to large scale features such as regional and continental scale tectonic structures.

The force of gravity that is exerted from one object onto another is described quantitively by Newton's law of universal gravitation. For two point masses m_1 and m_2, the force is

$$\overrightarrow{F} = -G\frac{m_1 m_2}{r^2}\hat{r} \tag{1}$$

where

- $\overrightarrow{F}$ is the force applied on mass m_2 by mass m_1. Note that here the force is a vector, so it has both a magnitude and a direction. It's units are [N (Newtons) = kg m s^{-2}]
- G is the gravitational constant ($G = 6.674 \times 10^{-11}$) [m^3 kg^{-1}s^{-2}]
- r is the distance between the centers of the two point masses [m].
- $\hat{r}$ is a unit vector pointing from m_1 to m_2.

Since $\hat{r}$ is the direction from m_1 to m_2, the negative sign means that the force $\overrightarrow{F}$ is in the opposite direction.

We also know from Newton's second law of motion is that the force is equal to the mass times its acceleration, or $F = ma$. Equating the vector form of Newton's second law of

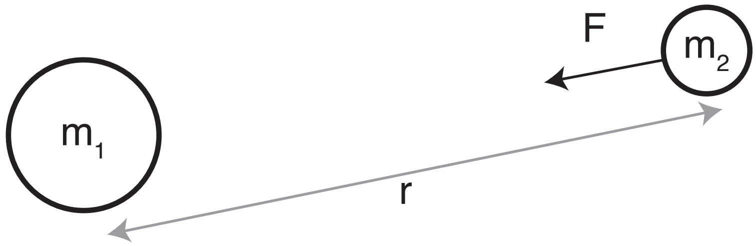

Figure 1: Force of gravity exerted on mass m_2 due to mass m_1.

motion with the formula above, and assuming the mass m_1 is the Earth's mass m_e, we can state

$$\overrightarrow{F} = -G\frac{m_e m_2}{r^2}\hat{r} = m_2\overrightarrow{a} = m_2\overrightarrow{g} \tag{2}$$

where we define the acceleration due to gravity as

$$\overrightarrow{g} = -G\frac{m_e}{r^2}\hat{r} \tag{3}$$

Given the Earth's mass $m_e = 5.976 \times 10^{24}$ kg and the equatorial radius $r = 6.37816 \times 10^6$ m, the acceleration from gravity is $g = 9.8$ m s^{-2}.

Given equation 2, we can see that Earth's gravity will generate a force that is proportional to the mass m_2 being acted upon. Since the mass of a body is simply the product of its density ρ and volume V, we can say

$$\overrightarrow{F} = m\overrightarrow{g} = \rho V\overrightarrow{g} \tag{4}$$

So for a fixed volume of material in Earth, the density is what controls the gravitational field produced by that volume.

Some densities of common materials in geology are shown in Table 1. Density of porous rocks will also depend on the degree of saturation, which is the amount of the pore space occupied by a fluid such as water or oil.

A formula for the gravitational acceleration of a sphere can be found through the application of Gauss' law for the flux through a surface enclosing the sphere. The derivation is a nice exercise in mathematical physics, but we don't have space for it here, so I will simply state the formula as:

$$\overrightarrow{g} = -G\frac{m}{r^2}\hat{r} = -G\frac{\rho V}{r^2}\hat{r} \tag{5}$$

Table 1: Densities of some rocks and minerals

Material	Density ρ, kg m^{-3}
glacier ice	917
water	1000
silt	1300-1800
sandstone	2000-2600
granite	2600-2900
basalt	2800
gabbro	2800-3100
Hematite, Fe_2O_3	5200
Galena, PbS	7500

where m is the sphere's mass, ρ is its density and V is the volume. This is precisely the same formula introduced earlier for a point mass. Thus we can see that the gravity is *non-unique* with respect to the density and volume of the mass; so long as the product ρV is the same, the gravity anomaly will be the same.

While the above formula is for the full gravity vector, it is most common to measure only the vertical component of gravity g_z, where $g_z = |\overrightarrow{g}|\sin\theta$, as shown in Figure 3. Further, since gravity is produced by all the rocks in the ground, it is only the excess mass (or mass deficit) from the anomalous body that produces the change in gravity. Thus we can replace ρ above with the density contrast $\Delta\rho = \rho_{body} - \rho_{background}$. Since $\sin\theta = z/r$ and $r = \sqrt{x^2+z^2}$, we have

$$g_z = G\frac{\Delta\rho V z}{r^3} \tag{6}$$

Since the volume of the anomalous regions is $V = \frac{4}{3}\pi R^3$, the gravity *anomaly* above the spherical body is then

$$g_z = G\Delta\rho\frac{4}{3}\pi R^3\frac{z}{(x^2+z^2)^{3/2}} \tag{7}$$

The anomaly for a cylinder (of infinity extent in the y direction) has a similar, but slightly different formula:

$$g_z = G\Delta\rho 2\pi R^2\frac{z}{x^2+z^2} \tag{8}$$

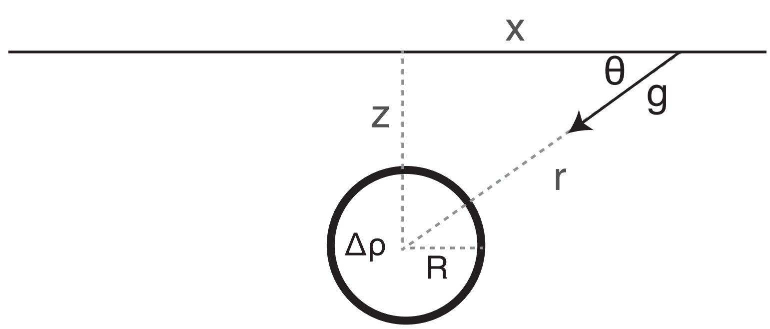

Figure 2: Geometry for a sphere or cylinder of radius R and density anomaly $\Delta\rho$ with respect to the background geology.

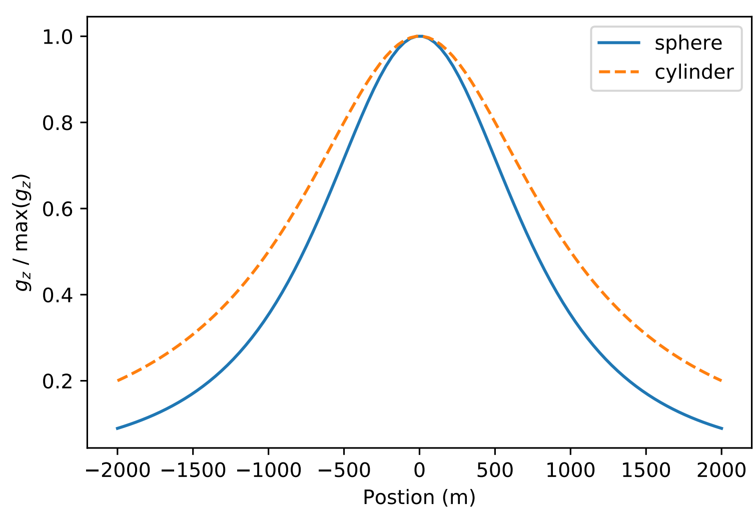

Figure 3: Gravity anomalies for a sphere and cylinder of radius R and anomalous density $\Delta\rho$.

Seismology

coming soon...