Geochronology using radiogenic isotopes

Isotope geochemistry is a branch of the Earth Sciences that studies the isotopic signatures of rocks and minerals to study Earth processes. Two subclasses of isotope geochemistry are stable isotopes and radiogenic isotopes. The study of radiogenic isotopes examines how radioactive isotopes actively decay from a parent isotope into a daughter isotope. The rate of decay for the various radioactive isotopes can vary from fractions of a second to billions of years, depending on the particular isotope.

We can use the relative abundance of the parent and daughter isotopes in a given sample as a clock that records the amount of time since a rock formed. Likewise, you may have already heard about radiocarbon dating, which measures the relative abundance of carbon isotopes to determine the age once living organisms that died within the last few thousand years.

In radioactive decay, the time rate of decay of the parent isotope is proportional to its concentration. Thus, if P is a measure of the concentration of the parent isotope, we can state this mathematically as:

$$\frac{dP(t)}{dt} = -\lambda P(t), \tag{1}$$

λ is the decay constant. We can integrate this differential equation over time to obtain the radioactive decay equation:

$$P(t) = P_0 e^{-\lambda t}, \tag{2}$$

where P_0 is the initial concentration when $t = 0$. Now you can see that radioactive decay is an exponential process. The half-life $t_{1/2}$ is the time it takes for $P = P_0/2$, meaning when half of the original material has decayed. Inserting this into the decay equation, gives

$$\frac{P}{P_0} = \frac{1}{2} = e^{-\lambda t_{1/2}}. \tag{3}$$

From this you can see that if you measure the time $t_{1/2}$, you can solve for λ. Conversely, if you measure λ, you can solve for $t_{1/2}$.

Rearranging the decay equation (2) we can write an expression for P_0:

$$P_0 = P(t)e^{\lambda t} \tag{4}$$

The amount of daughter nuclides D^* produced by the radioactive decay is the difference between the initial concentration and the present day concentration of the parent isotope:

$$D^*(t) = P_0 - P(t) \tag{5}$$

Inserting P_0 from equation 4 gives:

$$D^*(t) = P(t)(e^{\lambda t} - 1) \tag{6}$$

The total number of daughter nuclides D is then the sum of those produced by decay of the parent isotope as well as any initial concentration in the sample D_0:

$$D(t) = D_0 + D^*(t) \tag{7}$$

The full expression is then

$$D(t) = D_0 + P(t)(e^{\lambda t} - 1) \tag{8}$$

While we can measure $D(t)$ and $P(t)$ in the lab, you can't extract the age of the rock t without also knowing the initial concentration of the daughter isotope D_0.

Consider the decay of uranium to lead. There are two radioactive uranium isotopes that decay:

$$^{238}\mathrm{Ur} \longrightarrow {}^{206}\mathrm{Pb} \tag{9}$$
$$^{235}\mathrm{Ur} \longrightarrow {}^{207}\mathrm{Pb} \tag{10}$$

^{238}Ur has a half-life of 4.47 billion years and ^{235}Ur has a half-life of 710 million years. The full expressions are then:

$$^{206}\mathrm{Pb} = {}^{206}\mathrm{Pb}_0 + {}^{238}\mathrm{Ur}(e^{\lambda_{238} t} - 1) \tag{11}$$
$$^{207}\mathrm{Pb} = {}^{207}\mathrm{Pb}_0 + {}^{235}\mathrm{Ur}(e^{\lambda_{235} t} - 1) \tag{12}$$

where λ_{238} is the decay constant for ^{238}Ur and λ_{235} is the decay constant for ^{235}Ur.

In certain minerals such as zircons that form from a magma, lead is initially excluded from the crystal structure and thus $^{206}\mathrm{Pb}_0$ can be neglected.

$$\left(\frac{^{206}\mathrm{Pb}}{^{238}\mathrm{Ur}}\right) = (e^{\lambda_{238} t} - 1) \tag{13}$$
$$\left(\frac{^{207}\mathrm{Pb}}{^{235}\mathrm{Ur}}\right) = (e^{\lambda_{235} t} - 1) \tag{14}$$

Taking the ratio of these equations gives:

$$\left(\frac{^{206}\mathrm{Pb}}{^{238}\mathrm{Ur}}\right) = \frac{(e^{\lambda_{238} t} - 1)}{(e^{\lambda_{235} t} - 1)} \left(\frac{^{207}\mathrm{Pb}}{^{235}\mathrm{Ur}}\right). \tag{15}$$

This equation is useful since it shows the linear relationship between the isotope ratios.

Since measuring the absolute abundances of isotopes is problematic with finite rock samples, often relative abundances, as commonly measured with mass spectrometers, are used. We did this in the above example where the initial concentration is zero. For other system where the initial concentration is non-zero, a non-radiogenic stable isotope is used for the relative comparison. For the Ur-Pb system, usually the stable ^{204}Pb isotope is used. The above equations are then rewritten as

$$\left(\frac{^{206}\text{Pb}}{^{204}\text{Pb}}\right) = \left(\frac{^{206}\text{Pb}}{^{204}\text{Pb}}\right)_0 + \left(\frac{^{238}\text{Ur}}{^{204}\text{Pb}}\right)(e^{\lambda_{238}t} - 1) \tag{16}$$

$$\left(\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right) = \left(\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right)_0 + \left(\frac{^{235}\text{Ur}}{^{204}\text{Pb}}\right)(e^{\lambda_{235}t} - 1) \tag{17}$$