Landscape Evolution Modeling

Landscape evolution modeling studies how the shape, or geomorphology, of Earth's surface changes over time due to tectonic and erosive forcings, which act to increase or decrease elevation locally.

First list consider some tectonic and other forcings that act to increase or decrease Earth's surface elevation. Where two continents collide, Earth's surface can be forced up, such as is presently occurring in the Himalaya Mountains, where the India and Asian continents are colliding; the present rate of uplift is 1 cm/year in the location that is rising the fastest. Conversely, where a tectonic plate is being stretched apart and thinned, Earth's elevation will decrease; an example of this is the East African Rift valley. Large scale changes in elevation can also occur from the relative motion along fault zones, which can result in some sections of crust being uplifted and others dropped down, either through abrust changes during a large earthquake, or due to slow but continuous creep along the fault. Another cause of uplift is post-glacial rebound, where the Earth's crust was formerly pushed down into the mantle due to the weight of over a kilometer of ice during the last ice age, and removal of the ice mass at the end of the ice age happened faster than the crust and mantle can rebound (i.e spring back up like silly putty), resulting in a slow uplift of the surface over time, such as is happening in the northern parts of North America. While these forces are generally very slow, with elevation change rates on the order of mm or cm per year, over tens or hundreds of thousands of years they can results in large elevation changes.

Erosion occurs due to the weathering of rock into regolith, which is unconsolidated rocky material. Over long time scales weathering occurs due to the chemical reactions of groundwater and rain water with the minerals composing a rock, but over shorter time scales is accelerated by freezing and thawing cycles that use the expansion of freezing water as a slow but persistent lever that forces open cracks in otherwise solid rocks. Wind, water and glacier ice then act under the force of gravity to transport sediment generally in a downhill direction.

In the sections below we will take an introductory look at how to use landscape evolution modeling to study how uplift and erosion acts to shape a landscape over time.

Conservation of Mass

A fundamental component landscape evolution modeling is the conservation of mass, which is expressed by the equation:

$$\frac{\partial z}{\partial t} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}\right) \tag{1}$$

where z is the local elevation, t is time and q is the volumetric flux of sediments (with units m^2 per unit time). This equation states that the rate at which the elevation increases or decreases is equal to the negative of the divergence of the volumetric flux of that quantity.

Uplift

Perhaps the simplest model of landscape evolution over time concerns the regional or local uplift due to tectonic forces. This can be expressed mathematically as:

$$\frac{\partial z}{\partial t} = u \tag{2}$$

where u is the rate of uplift in m per unit time. Note that in general $u = u(x, y)$ and so $z = z(x, y)$. For simple problems u may be constant over time, whereas more specific cases my consider $u = u(x, y, t)$, meaning that the uplift rate may vary in both space and time.

For the simple case where u is constant in time, z after some time Δt is simply:

$$z = z_0 + u\Delta t \tag{3}$$

where z_0 is the original elevation.

Colluvial Erosion and Deposition

Over long time scales, the flux of material that erodes due to colluvial processes such as creep, which do not involve mass transport by water, has been observed empirically to be described by a linear dependence on the slope:

$$q_x = -\kappa\frac{\partial z}{\partial x} \tag{4}$$

$$q_y = -\kappa\frac{\partial z}{\partial y} \tag{5}$$

Conventiently, this results in a diffusion equation that can be solved to model colluvial processes:

$$\frac{\partial z}{\partial t} = \kappa \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) \tag{6}$$

The figure below shows a 1D example of this type of erosion and deposition computed using the FTCS finite difference method with $\kappa = 1000$ m^2/year.

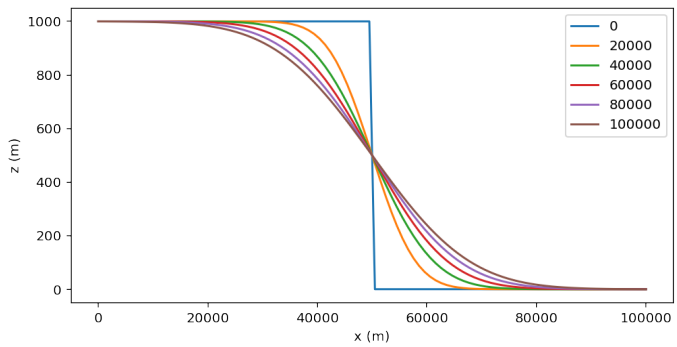

Figure 1: Example of diffusion modeling of colluvial erosion and deposition of a 1000 m cliff over a time span of $100,000$ years. The top side of the cliff is slowly eroded over time, while material is deposited at the base of the cliff.

Erosion and Deposition by Channel Flow

Channel flow refers to how precipitation on Earth's surface flows in the downhill direction and generally results in water streams merging together into increasingly larger channels with distance downhill. For example, think of a network of small streams at high elevations in a watershed, which gradually merge together to form a larger stream, which eventually could merge with other streams and form a river. The ability of a stream or river to transport sediment generally depends on the speed of the river, with fast moving rivers being able to erode more quickly than others. Similarly, a wide and deep river with a large flux of water can transport more sediment than narrow and shallow rivers. While a fast moving river has energy to carry suspended sediments and even large rocks and boulders for very fast currents in steep rapids or during a flood, slow moving rivers will generally result in the deposition of sediments.

Here we will use a simple model of the effects of channel flow on surface elevation. We start with the concept of sediment flux, denoted as Q, which is the total volume flux of

sediment being carried by the water (with units m^2 per unit time). Along a flow path (ranging from puddle to stream to river...), the change in surface elevation from the water flow is described as

$$\frac{\partial z}{\partial t} = -\frac{\partial Q}{\partial l} \tag{7}$$

where ∂l denotes an infinitesimal increment of distance along the flow path. Thus as Q increases, meaning the river picks up more sediment due to erosion, the surface elevation will decrease due to the loss of material. Conversely, when Q decreases, material is being deposited and the elevation increases.

Next we introduce the concept of carrying capacity Q_e, which is the maximum flux that can be supported by a given discharge of water. We can then restate the formula above as

$$\frac{\partial z}{\partial t} = -\frac{Q_e - Q}{L_e} \tag{8}$$

where L_e is some characteristic scale length over which the deposition or erosion occurs. There are two scenarios. When $Q > Q_e$ deposition occurs and the elevation increase according to:

$$\frac{\partial z}{\partial t} = \frac{Q - Q_e}{L} \tag{9}$$

Conversely, when $Q < Q_e$, meaning the flowing water can still hold more sediment, then erosion occurs. Again according to the equation

$$\frac{\partial z}{\partial t} = \frac{Q - Q_e}{L}. \tag{10}$$

In class we will implement the channel flow equation in 1D using a finite time step. To do this, we divide up the lateral dimension x into a series of n cells at locations x_i with spacing Δx. In each cell, we assume a precipitation rate v in units of m/year. Thus the height of precipitation in a given unit time is $v\Delta t$ and over the length of a cell, the total cross sectional area of precipitation in a time step is then:

$$q_{i,new} = v_i \Delta t \Delta x \tag{11}$$

The precipitation rate v can be a constant in simple models while more complex models might have v vary over time and space, for example precipitation could be made larger in mountains to account for orographic uplift. In our 1D model, we will assume the topography slopes down from left to right without any local depressions. Thus the water will flow from left to right. The total amount of water flowing across a cell is then the sum of the locallly precipitated water $q_{i,new}$, plus the water that flows in from the left.

$$q_i = q_{i,new} + q_{i,left} \tag{12}$$

where $q_{i,left}$ is the water that flows in from the left. Thus when modeling this, we will start on the left and gradually build up the volume of water in q; this is analogous to how rivers generally increase in water volume as you head downstream.

Now we need to consider the sediment carrying capacity of the water stream Q_e. We will assume the sediment load generated locally in the cell is

$$Q_e = \kappa_r s_i q_i \tag{13}$$

where κ_r is a scaling constant and s_i is the local slope in the downward direction:

$$s_i = \left| \frac{z_{i+1} - z_i}{x_{i+1} - x_i} \right| \tag{14}$$

Thus steeper slows can carry a larger sediment load, and the larger the water flux q_i, the larger the sediment carrying capacity.

Now you are probably asking, what about the sediment load Q_i? We start out by initializing the sediment load to be zero everywhere, ie. $Q_i = 0$ for all $i = 0$ to $n - 1$.

In each time step, we update q_i starting on the left most cell. We then update z_i and Q_i as follows:

If $Q_i > Q_{e,i}$ the water is carrying more sediment than it can hold and so deposition occurs according to:

$$z_i = z_i + \frac{Q_i - Q_{i,e}}{\Delta x} \tag{15}$$

We then set

$$Q_i = Q_{i,e} \tag{16}$$

Conversely, when $Q_i < Q_{e,i}$ the water can hold more sediment and so we allow it to start eroding the that cell with the updated elevation:

$$z_i = z_i + \frac{Q_i - Q_{i,e}}{\Delta x} \frac{\Delta x}{L_e} \tag{17}$$

where $Q_i - Q_{i,e}$ is negative and thus z will locally decrease. The scaling factor on the right is used to slowly increment the erosion, rather than having it occur all at once in a single cell. The sediment load is then increased according to:

$$Q_i = Q_i + (Q_i - Q_{i,e}) \frac{\Delta x}{L_e} \tag{18}$$

For both the deposition or erosive cases, we then pass q_i and Q_i to the next cell to the right (i.e. the water flows one step over).

Once we have swept from left to right across all cells, the time step is completed and we start a new time step, step with q and Q initialize to zero.

2D Channel Flow

For more realistic 2D scenarios, the channel flow equations become a bit more complex to implement. First, the water is generally assumed to flow from one cell to another down the direction of steepest slope. Thus you need to compute the slope everywhere in the model and know which direction is the steepest. Also, the flow is generally driven from the highest elevation to the lowest, so that the water discharge from precipitation and the load of sediments is integrated from cell to cell as the water flows downhill. Thus, while you already know how to model diffusion in 2D so that colluvial erosions should be easy to model in Python, channel flow modeling will require quite a bit more complexity in your code. Don't worry, I'll supply the codes that compute the difficult parts.

More on Thursday!